Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore-641 043
Bachelor's Degree Examination - November 2017

V Semester

Class : III UG — **Time :3 Hours**
Major : Mathematics — **Max. Marks: 100**

15BMAC14 Real Analysis-I
Part-A — **10 x 1=10**
Choose the correct answer

1. A set S is said to be countably infinite if it is equinumerous with the set of all ________
   a. real numbers b. complex numbers c. positive integers d. irrational numbers.

2. The complement of A relative to B is defined as the set
   a. $\{ x : x \in B \text{ but } x \notin A \}$ b. $\{ x : x \in B \text{ but } x \notin A \}$
   c. $\{ x : x \in B \text{ but } x \notin A \}$ d. $\{ x : x \in B \text{ but } x \notin A \}$

3. The distance between x and y where $x, y \in R^n$ is given by
   a. $\|x+y\|$ b. $\|x-y\|$ c. $\|x.y\|$ d. $\|x-y\|^2$

4. A set S is closed if and only if ________
   a. $S = \text{int}\, S$ b. $S = S'$ c. $S = \phi$ d. $S = \overline{S}$

5. The collection of all intervals of the form $1/n < x < 2/n$, $n = 2,3,4,....$ is an open covering of the interval
   a. $1 < x < 2$ b. $1 \le x \le 2$ c. $0 < x < 1$ d. $0 \le x \le 1$

6. A collection F of sets is said to be a covering of a given set S if
   a. $S \subseteq \bigcup_{A \in F} A$ b. $S \supseteq \bigcup_{A \in F} A$ c. $F \subseteq \bigcup_{A \in S} A$ d. $F \supseteq \bigcup_{A \in S} A$

7. In Euclidean space R^1, the ball $B(0;1)$ is the interval
   a. (-1, 0) b. (0, 1) c. (-1, 1) d. [0, 1]

8. If every open covering of a set S in R^n has a finite sub cover, then S is
   a. complete b. closed c. compact d. bounded

9. The convergence and divergence of a sequence depends on
   a. the underlying space only b. the metric only
   c. both underlying space and on the metric d. none of these

10. If a sequence $\{x_n\}$ in a metric space converges, then $\lim_{n \to \infty} x_n$ is _____
    a. 0 b. n c. unique d. not unique.

Part B **5 X 6=30**

Answer the following

Answer should not exceed 400 words or two pages

11.a. Define the following sets and give examples:

(i) finite set (ii) infinite set (iii) uncountable set.

(or)

11.b. Define a countable set. Prove that the set Q of all rational umbers is countable.

12.a. Prove that the union of any collection of open sets is an open set.

(or)

12.b. If x is an accumulation point of S, then prove that every n-ball B(x) contains infinitely many points of S.

13.a. Let G = $\{A_1, A_2, ...\}$ denote the countable collection of all n-balls having rational radii and centers at points with rational coordinates. Assume x ϵ R^n and let S be an open set in R^n which contains x. Then prove that $x \in A_k \subseteq S$ for some A_k in G.

(or)

13.b. Define a covering. Give an example for (i) a countable covering and (ii) a covering which is not countable.

14.a. Let M be a nonempty set. For x, y ϵ M, let d(x, y) = 0 if x = y and d(x, y) = 1 if x ≠ y. Then prove that (M, d) is a metric space.

(or)

b. Define the following: (i) Boundary of a set and (ii) Euclidean metric in R^n.

15.a. In a metric space (S, d), assume $x_n \to p$ and let $T = \{x_1, x_2, ...\}$ be the range of $\{x_n\}$. Then prove that (i) T is bounded and (ii) p is an adherent point of T.

(or)

b. In a metric space (S, d), prove that a sequence $\{x_n\}$ converges to p if and only if every subsequence $\{x_{k(n)}\}$ converges to p

Part C **5 x 12=60**

Answer the following

Answer should not exceed 800 words or four pages

16.a. Prove that the set of all real numbers is uncountable.

(or)

b. (i)If $F = \{A_1, A_2, ...\}$ is a countable collection of disjoint sets where each A_n is countable, then prove that the union $\bigcup_{k=1}^{\infty} A_k$ is also countable.

17.a. Define component interval. Prove that every point of a nonempty open set S belongs to one and only one component interval of S.

(or)

b. State and prove the representation theorem for open sets on the real line.

18.a. If a bounded set S in R^n contains infinitely many points, prove that there exists atleast one point in R^n which is an accumulation point of S.

(or)

b. State and prove the Cantor intersection theorem.

19.a. State and prove the Heine-Borel theorem.

(or)

b. Let S be a subset of R^n. Then prove that the following statements are equivalent:

(i) S is compact

(ii) S is closed and bounded

(iii) Every infinite subset of S has an accumulation point in S.

20.a. Prove that every Cauchy sequence is convergent in Euclidean space R^k.

(or)

b. Let p be an accumulation point of A and let b ϵ T. Then prove that $\lim_{x \to p} f(x) = b$ if and only if $\lim_{n \to \infty} f(x_n) = b$ for every Cauchy sequence of points in A – {p} which converges to p.

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