Review of Literature

The idea of fuzzy set theory, proposed by L.A. Zadeh (1965) plays a significant role as it handles uncertain or vague information in decision making, characterized by a membership function which assigns a membership value ranging between zero and one. The development of the fuzzy logic as the straight forward fuzzification, the generalization, the standardization, the axiomatization and the L-fuzzification. The definition of fuzzy set can be altered by the replacement of I with a complete lattice L (1967). Zadeh developed the fuzzy related concepts: fuzzy algorithm (1968), fuzzy languages (1969), fuzzy semantics (1971a), fuzzy control systems (1972), fuzzy probabilities (1984), fuzzy logic (1996) and fuzzy information (1997). However, in some actual environment, the fuzzy set theory has some limitations when decision maker deals with some uncertain information or vagueness.

Intuitionistic fuzzy set (IFS), proposed by Atanassov (1986) is characterized by a membership and non-membership function satisfies the condition that the sum of membership and non-membership is less than or equal to one. Hence it describes more precisely than fuzzy set. He extended other results of intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, elements of intuitionistic fuzzy logic (1999). Mendel et al. (2002) discussed a new representation for type-2 fuzzy set. Torra (2010) introduced Hesistant fuzzy sets and some basic operations and also proved that the envelope of the hesistant fuzzy sets is an intuitionistic fuzzy set.

Yager (2013) proposed a brand-new extension of fuzzy set called Pythagorean fuzzy set (PFS), which has been successfully applied in many fields for decision making procedures. PFS is characterized by a membership and non-membership function satisfies the condition that the square sum of membership and non-membership is less than or equal to one. It is noted that not all Pythagorean fuzzy set are intuitionistic fuzzy set but an intuitionistic fuzzy set must be a Pythagorean fuzzy set. Thus, various extensions have been introduced over the past years and widely applied in some branches of artificial intelligence.

Spherical fuzzy set is a generalization of picture fuzzy set and Pythagorean fuzzy set. There is a need of spherical fuzzy set to tackle an interesting scenario emerge when picture fuzzy sets and Pythagorean fuzzy sets both failed to handle. We can study the neutral degree in spherical fuzzy set where as in Pythagorean fuzzy sets and picture fuzzy sets it doesn't. In spherical fuzzy set, membership degrees are gratifying the condition $0 \le P^2(x) + I^2(x) + N^2(x) \le 1.$ (2019).

The idea of neutrosophic set is introduced by Smarandache (1999) which is a generalization of the fuzzy set, intuitionistic fuzzy set. The neutrosophic sets are characterized by a truth membership function (T), an indeterminate membership function (I) and a false membership function (F) independently. The new concepts of neutrosophic perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras and Applications is introduced by Smarandache (2017). The neutrosophic set and its extensions play a vital role to deal with incomplete and inconsistent information that exist in real world. Smarandache introduced neutrosophic quadruple sets and neutrosophic quadruple numbers. Furthermore special operations for set valued neutrosophic quadruple numbers and other triplet structures also defined and applied in the medical field (2019).

The concept of cubic set is characterized by fuzzy set and interval valued fuzzy set, which is an important tool to deal with uncertainty and vagueness. The hybrid platform of cubic set contains more information than a fuzzy set. Neutrosophic set combined with cubic sets gave the new concept of neutrosophic cubic set introduced by Jun et.al (2012).

The bipolar fuzzy set theory combines both polarity and fuzziness into a unified model and provides a theoretical basis for bipolar clustering, decision analysis and coordination. Zhang (1994) initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy set (BFSs) is described by a pair of numbers $\{\mu^+, \mu^-\}$ as the truth value where μ^+ is the degree to which we believe in a given statement and μ^- is the degree to which we believe in its negation. Bipolar fuzzy graph is an extension of fuzzy graph whose membership degree range is [-1, 1]. In an bipolar fuzzy graph, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0, 1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1, 0) of an element indicates that the element somewhat satisfies the implicit counter-property. For example, if sweetness of the food has been given as positive membership values then bitterness food is for negative membership values. Other tastes like salty, sour, pungent etc are irrelevant to the corresponding property. So these foods are taken as zero membership values.

A graph is a convenient way of representing the data in which the objects are vertices and their relations are edges. Many real life problems can be solved by using graphs. Based on Zadeh's fuzzy relation (1971b), Kaufmann (1973) introduced the idea of fuzzy graphs. Rosenfeld (1975) studied the structure of fuzzy graphs by obtaining various fuzzy analogs such as cycles, paths and connectedness. Al-Hawary (2011) considered certain concepts of fuzzy graphs. He also characterized several classes of fuzzy graphs and showed that parallel connection and series connection of balanced fuzzy graphs need not be balanced and they are balanced in case the original graphs are induced by cycles (2017). Further he defined some basic definitions related to picture fuzzy graphs and explored several properties of the above graphs (2018). Parvathi and Karunambigai (2006) extended the concept of intuitionistic fuzzy graph from the fuzzy graphs and also discussed some properties of intuitionistic fuzzy graphs. Naz et al (2018) introduced the idea of Pythagorean fuzzy graphs, an extension of intuitionistic fuzzy graphs.

Akram et al (2012) discussed the notion of strong intuitionistic fuzzy graphs and investigated some of their properties and introduced the concept of intuitionistic fuzzy line graphs. Akram (2013) investigated the concepts of neighbourly irregular bipolar fuzzy graphs, neighbourly totally irregular bipolar fuzzy graphs, highly irregular bipolar fuzzy graphs and highly totally irregular bipolar fuzzy graphs and the notion of bipolar fuzzy digraphs also introduced. Akram et al (2018c) proposed planar graphs under Pythagorean fuzzy environment. Akram et al (2020a) examined decision making methods based on spherical fuzzy graphs. Akram et al (2020b) extended the graph-theoretic concepts under a spherical fuzzy environment and discussed some operations, properties of irregular and edge-irregular spherical fuzzy graph with examples. The new idea of bipolar neutrosophic cubic graphs and bipolar spherical neutrosophic cubic graphs were studied in (2020) and some of their properties were discussed and also minimum spanning tree algorithm with numerical examples was presented.

In the book of neutrosophic graphs Kandasamy et.al (2015) introduced the concept of neutrosophic graphs. The different aspect of neutrosophic graphs is discussed by Akram et al. (2018a). Recently, neutrosophic methods introduced by Ye (2004) is used to find minimum spanning tree of a graph, where nodes are represented by single valued neutrosophic set and distance between two nodes represents the dissimilarity between the corresponding samples has been derived. A double-valued neutrosophic minimum spanning tree clustering algorithm is used to cluster double-valued neutrosophic information is introduced by Kandasamy (2016).

Prim and Kruskal algorithm are the two most common algorithms for finding the minimum spanning tree in classical graph theory (1956). Ye (2014) presented a method to obtain minimum spanning tree of a graph where nodes (samples) are represented in the form of single valued neutrosophic set and distance between two nodes which represents the dissimilarity between the corresponding samples has been derived. A new theory is introduced called single valued neutrosophic graph theory (SVNGT) and their extensions finds its applications in diverse fields (2016g).

Kandasamy (2016) introduced a Double-Valued Neutrosophic Minimum Spanning Tree (DVN-MST) clustering algorithm to cluster the data represented by double-valued neutrosophic information. Mandal and Basu (2016) proposed a solution approach of the optimum spanning tree problems considering the incompleteness, indeterminacy and inconsistency of the information. Mullai (2017) developed the minimum spanning tree problem on a graph in which an bipolar neutrosophic number is associated to each edge as its edge length, and illustrated it by a numerical example. Further Smarandache et.al (2018) introduced the new idea of neutrosophic cubic graphs and their fundamental operations such as cartesian product, union and join of neutrosophic cubic graphs, composition, degree and order of neutrosophic cubic graphs and some results. Recently, the new concept of bipolar neutrosophic cubic graphs and single-valued bipolar neutrosophic cubic graphs is introduced and discussed some of their algebraic properties and present minimum spanning tree algorithm with numerical example (2020).

Graph coloring is assigning the colors to the edges or vertices or both edges and vertices of the graph based on some conditions. A vertex coloring of a graph G is assigning colors to each vertex such that no two adjacent vertices share the same color. It is applicable in many real life problems such as computer networks, bioinformatics, telecommunications, scheduling etc. Sometime in real-world one has to deal with impreciseness and uncertainty relation between points, in that case fuzzy technique arises. Munoz et al (2005) proposed the chromatic number of fuzzy graph.

Later Eslahchi et al (2006) extended fuzzy graph coloring. Based on the f-core of G, Zhang et al (2008) proposed some sufficient conditions for G of class-1 for f-colorings. Hakimi et al (1999) suggested that the coloring of multigraphs G often leads to improved upper bounds for the chromatic index. Verma et al (2018) suggested some basic operations of Pythagorean fuzzy graphs and also proved complementary with strong Pythagorean fuzzy graphs. Prasanna et al (2017) described a vertex, edge and total coloring for strong and complete intuitionistic fuzzy graphs with suitable examples and also obtained the chromatic number as a crisp number.

Rohini et al (2019) introduced the bounds of single-valued neutrosophic vertex coloring of single-valued neutrosophic graph, complement of single-valued neutrosophic graph and discussed some operations on single-valued neutrosophic graph. Yamuna et al (2020) suggested the Pythagorean graph and proposed the bounds for that coloring number. Also presented and explored the new idea of color excellence by taking the Pythagorean fuzzy color partition into consideration.

FUZZY SETS

[Zadeh, 1965]

In this article, the author has introduced a new type of sets namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement, etc., with respect to the fuzzy sets.

FUZZY GRAPHS IN FUZZY SETS AND THEIR APPLICATIONS

[Rosenfeld, 1975]

In this article, the author reviewed some basic properties of fuzzy relations and generalized to the case where the underlying set is a fuzzy set. Fuzzy analogues of several basic graph-theoretic concepts (e.g., bridges and trees) are defined, and some of their properties are established.

INTUITIONISTIC FUZZY SETS

[Atanassov, 1986]

In this article, the author has provided the notion of intuitionistic fuzzy sets. This is considered to be the generalization on fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

MORE ON INTUITIONISTIC FUZZY SETS

[Atanassov, 1989]

In this article, the author has analyzed some new results on intuitionistic fuzzy sets. Two new operators on intuitionistic fuzzy sets are defined and their basic properties are studied.

INTUITIONISTIC FUZZY GRAPHS

[Parvathi and Karunambigai, 2006]

In this research article, a new definition for intuitionistic fuzzy graph is defined. Some properties of intuitionistic fuzzy graphs are considered and the authors introduced the notions of various concepts.

PYTHAGOREAN MEMBERSHIP GRADES IN MULTICRITERIA DECISION MAKING

[Yager, 2014]

In this research article, the author introduced a class of nonstandard Pythagorean fuzzy subsets whose membership grades are pairs, (a,b) satisfying the requirement $a^2 + b^2 \leq 1$. A variety of aggregation operations for these Pythagorean subsets are also obtained.

AN INTRODUCTION TO BIPOLAR SINGLE VALUED NEUTROSOPHIC GRAPH THEORY

[Said Broumi, Florentin Smarandache, Mohamed Talea and Assia Bakali, 2016g]

The concept of bipolar single valued neutrosophic graphs as the generalization of bipolar fuzzy graphs, N-graphs, intuitionistic fuzzy graph, single valued neutrosophic graphs and bipolar intuitionistic fuzzy graphs are studied in this article.

SINGLE VALUED NEUTROSOPHIC GRAPHS

[Said Broumi, Mohamed Talea, Assia Bakali and Florentin Smarandache, 2016a]

In this paper, the authors introduced certain types of single valued neutrosophic graphs (SVNG) and also investigated some of their properties with proofs and examples.

SHORTEST PATH PROBLEM UNDER BIPOLAR NEUTROSOPHIC SETTING

[Said Broumi, Assia Bakali, Mohamed Talea, Florentin Smarandache and Ali, 2016n]

In this paper, the authors developed an algorithm to find the shortest path on a network in which the weights of the edges are represented by bipolar neutrosophic numbers

CHROMATIC NUMBER OF BIPOLAR FUZZY GRAPHS

[Tahmasbpour and Borzooei, 2016]

In this paper, two different approaches to chromatic number of a bipolar fuzzy graph are introduced. The first approach is based on the alpha cuts of a bipolar fuzzy graph and the second approach is based on the definition of Eslahchi and Onagh for chromatic number of a fuzzy graph. Finally, the authors characterized the bipolar fuzzy vertex chromatic number and the edge chromatic number of a complete bipolar fuzzy graph.

STRONG INTUITIONISTIC FUZZY GRAPH COLORING

[Prasanna, Rifayathali and Ismail Mohideen, 2017]

In this paper, the authors introduced the concept of coloring the strong intuitionistic fuzzy graph and complete intuitionistic fuzzy graph.

ON BIPOLAR SINGLE VALUED NEUTROSOPHIC GRAPHS

[Said Broumi, Mohamed Talea, Assia Bakali and Florentin Smarandache, 2016k]

In this article, the authors combined the concept of bipolar neutrosophic set and graph theory and also introduced the notions of bipolar single valued neutrosophic graphs, strong bipolar single valued neutrosophic graphs, complete bipolar single valued neutrosophic graphs, regular bipolar single valued neutrosophic graphs and investigated some of their properties.

BIPOLAR NEUTROSOPHIC GRAPH STRUCTURES

[Muhammad Akram and Muzzamal Sitara, 2017]

In this research study, the authors introduced the concept of bipolar single-valued neutrosophic graph structures and also discussed certain notions of bipolar single-valued neutrosophic graph structures with examples. Some methods of construction of bipolar single-valued neutrosophic graph structures are defined and also some of their properties are presented.

ON INTUITIONISTIC FUZZY GRAPHS AND SOME OPERATIONS ON PICTURE FUZZY GRAPHS

[Al-Hawary, Mahmood, Jan, Ullah and Hussain, 2018]

In this study, the authors described some basic definitions related to picture fuzzy graphs and explored several properties of these fuzzy graphs. The operations union, join, Cartesian product and composition of picture fuzzy graphs and their properties are also studied.

PYTHAGOREAN FUZZY GRAPH WITH VERTEX COLORING

[Yamuna, Arun Prakash and Indra Kumar, 2020]

In this research article, the authors are enforced the definition of Pythagorean fuzzy graph vertex coloring implemented with illustrative examples and also introduced a new concept called color excellence Pythagorean fuzzy graph.

SPHERICAL FUZZY GRAPHS WITH APPLICATION TO DECISION MAKING

[Muhammad Akram, Danish Saleem and Talal Al-Hawary, 2020]

In this research study, the authors discussed two operations on spherical fuzzy graphs (SFGs), namely, symmetric difference and rejection; and also developed some results regarding their degrees and total degrees. Certain concepts of irregular SFGs with several important properties are described and also presented an application of SFGs in decision making.

DECISION MAKING METHOD BASED ON SPHERICAL FUZZY GRAPHS

[Muhammad Akram, 2020]

Certain concepts of spherical fuzzy graphs and various methods of their construction are presented. The author computed degree and total degree of spherical fuzzy graphs and some of their important properties are studied in this paper.

Notations

FS	-	Fuzzy set
FR	-	Fuzzy relation
FG	-	Fuzzy graph
IFS	-	Intuitionistic fuzzy set
IFR	-	Intuitionistic fuzzy relation
IFG	-	Intuitionistic fuzzy graph
PFS	-	Pythagorean fuzzy set
PFR	-	Pythagorean fuzzy relation
PFG	-	Pythagorean fuzzy graph
NS	-	Neutrosophic set
SVNS	-	Single valued neutrosophic set
BNS	-	Bipolar neutrosophic set
BFG	-	Bipolar fuzzy graph
NCS	-	Neutrosophic cubic set
NCG	-	Neutrosophic cubic graph
SFS	-	Spherical fuzzy set
SFG	-	Spherical fuzzy graph
MST	-	Minimum spanning tree
BSFS	-	Bipolar spherical set
BSFG	-	Bipolar spherical fuzzy graph
BSFMST	-	Bipolar spherical minimum spanning tree

BSFNCS	-	Bipolar spherical neutrosophic cubic set
BSFNCG	-	Bipolar spherical neutrosophic cubic graph
SNG	-	Spherical neutrosophic graph
CSNG	-	Complete spherical neutrosophic graph
SSNG	-	Strong spherical neutrosophic graph
BSNG	-	Bipolar spherical neutrosophic graph
BCSNG	-	Bipolar Complete spherical neutrosophic graph
BSSNG	-	Bipolar Strong spherical neutrosophic graph
O(G)	-	Order of graph G
deg(A)	-	Degree of a vertex A
tdeg(A)	-	Total degree of a vertex A
deg_S(A)	-	Number of edges incident on vertex that are strongly adjacent to A
BSFNCMST	-	Bipolar spherical neutrosophic cubic minimum spanning tree