REVIEW OF LITERATURE

Over the decades, classical set theory plays a vital role in modelling real physical situation but still its has proven to be inadequate in modelling imprecise and ambiguous informations. This has led to the development of numerous generalized frameworks, including fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, and their extensions. Also decision-making under uncertainty is an important research area in fields of engineering, computer science, management, economics, etc. In order to trace the evolution of these frameworks and understand the contributions made by various researchers, a review of the existing literature hs been done and some significant works are presented below.

The notion of fuzzy sets, introduced by Zadeh L. A. (1965) [96], represents the pioneering approach for handling uncertainty through membership grades. Goguen J. (1967) [27] contributed by generalizing fuzzy sets to L-fuzzy sets, providing a categorical framework with lattices as the range of membership functions. Zadeh L. A. (1975) [97] introduced linguistic variables to model approximate reasoning, laying the groundwork for fuzzy control systems and approximate decision-making.

Later on, Atanassov K. T. (1986) [8] introduced intuitionistic fuzzy sets (IFSs), which include both membership and non-membership degrees to better capture uncertainty. Afterwards, Atanassov K. T. (1991) [9] proposed temporal intuitionistic fuzzy sets to model time-varying uncertainty, an important feature for dynamic systems. Type-2 fuzzy logic systems, developed by Karnik N. N et al. (1999) [37], and their interval type-2 extension (Liang Q & Mendel J.M, 2000) [51], allow the modelling of uncertainty in the membership function itself. Yılmaz S. and Çuvalcıoğlu G. (2014) [95] introduced two different types of operators using inf or sup for temporal intuitionistic fuzzy sets, extending their universes and properties.

Smarandache F. (1998) [74], (1999) [75], (2005) [76] proposed neutrosophy as a generalization of fuzzy and intuitionistic fuzzy theories by introducing the concept of indeterminacy alongside degrees of membership and non-membership. This led to the definition of neutrosophic sets capable of representing incomplete, inconsistent, and indeterminate information. Wang H. et al. (2010) [89] addressed the need for a more precise technical specification of neutrosophic sets by introducing single-valued neutrosophic sets (SVNSs). Kim J et al. (2018) [42] defined set-theoretic operators for SVNSs and established various properties related to their operations and relations. Salama A. A. et al. (2014) [68] introduced three types of neutrosophic crisp sets, defined their operations and properties, and developed the concept of neutrosophic crisp relations. Salama A.A and Smarandache F. (2015) [69] further expanded this work by presenting a comprehensive theory of neutrosophic crisp sets. Hur. K et al. (2016) [29] established a category-theoretic framework for neutrosophic sets, formalizing their structure and relationships through morphisms. Subsequently, Hur. K et al. (2017) [30] extended this approach to neutrosophic crisp sets, providing a categorical foundation for modeling crisp yet indeterminate information. These works strengthen the theoretical basis for neutrosophic systems and their applications in uncertainty modeling.

Jun Y. B. et al. (2012) [32] introduced cubic sets, which integrate fuzzy and interval-valued fuzzy information. Subsequently, Chinnadurai V. et al. (2016) [21] provided additional characterizations of cubic sets, such as P-union, P-intersection, R-union, and R-intersection operations. Kang J. G. and Kim C. S. (2016) [33] further explored mappings of cubic sets. Smarandache F. et al. (2017) [80] extended cubic concept to neutrosophic set and define neutrosophic cubic sets. Building on this, Ali M. et al. (2016) [2] proposed neutrosophic cubic sets, and introduced internal and external neutrosophic cubic sets, established key properties, developed a neutrosophic cubic decision-making method, and applied it to pattern recognition. Recently, Broumi S. et al. (2022) [15], developed Fermatean neutrosophic graphs and interval-valued Fermatean neutrosophic graphs, enabling the modelling of uncertain relationships.

Pythagorean fuzzy sets, proposed by Yager R. R. (2013) [92], are characterized by the condition that the squared sum of membership and non-membership degrees does not exceed one. Neutrosophic Pythagorean sets with dependent components and improved correlation coefficients (Radha R. et al., 2021) [61] is presented. Akram M. et al. (2021) [1] introduced complex Pythagorean Dombi fuzzy operators to enhance decision-making processes. Spherical fuzzy sets, introduced by Kutlu Gundogdu F. & Kahraman C. (2019) [48], extend Pythagorean fuzzy sets to allow a spherical constraint on membership, non-membership, and hesitation. Mahmood T. et al. (2019) [53] applied spherical fuzzy sets to medical diagnosis. Ullah K. et al. (2018) [87] introduced T-spherical fuzzy sets, with associated similarity measures for pattern recognition. Dhanalakshmi V. (2023) [24] advanced risk analysis using trapezoidal spherical fuzzy numbers. Saeid Jafarzadeh Ghoushchi et al. (2023) [26] used spherical fuzzy SWARA-CODAS for evaluating clean energy barriers.

Later on, Fermatean fuzzy sets, introduced by Senapati T. and Yager R. R. (2020) [71], further generalize Pythagorean fuzzy sets by relaxing the squared sum constraint to a sum of cubes of the membership and non-membership degrees. Li Y. et al. (2025) [50] developed distance measures for Fermatean fuzzy sets, with applications in pattern classification and decision-making. Xu C. & Shen J. (2021) [91] focused on similarity measures for Fermatean fuzzy sets in MCDM and pattern recognition. Mengyao Zhan & Mu Zhang (2024) [98] proposed a Fermatean fuzzy TOPSIS method based on prospect theory, highlighting its application in credit assessment. Antony Crispin Sweety C. & Jansi R. (2021) [6] introduced Fermatean neutrosophic sets, further incorporating indeterminacy. Broumi S. et al. (2022) [15] applied Fermatean neutrosophic sets to graphs. Some recent research on Fermatean neutrosophic topological spaces, spherical Fermatean neutrosophic sets, is presented in [10], [64].

Topology plays a crucial role in formalizing continuity, convergence, and compactness in fuzzy contexts. Chang C. L. (1968) [18] proposed one of the earliest definitions of fuzzy topological spaces who defined fuzzy open sets as fuzzy subsets satisfying axioms similar to those of classical open sets. Lowen R. (1976) [52] refined this approach to better preserve classical topological properties such as compactness. Sostak A. (1985) [82], (1996) [83] generalized fuzzy topology further, introducing categorical approaches and graded openness. Chattopadhyay K. C. et al. (1992) [19] discussed gradation of openness, while Hohle U. and Sostak A. (1995) [28] developed a comprehensive theory of fuzzy topological spaces that unifies various definitions. Coker D. and Dimirci M. (1996) [22], (1997) [23] and Kubiak T. and Sostak A. (1997) [44], (2009) [45] also advanced intuitionistic fuzzy topology, including (L, M)-fuzzy topologies. Karatas S. and Kuru. C (2016) [36] expanded this by formalizing neutrosophic topology with operations and properties like closure, interior, and boundary. Tharani A. P. and Sujitha H.(2022) [84, 85] studied generalized closed sets in bi-topological and tri-topological spaces, contributing to the development of new structures in topology. Recently, research on neutrosophic topological spaces (2024) [70] and new types of neutrosophic topologies (2023) [81] were developed.

Recent developments include temporal intuitionistic fuzzy topologies in both Chang's (Kutlu F., 2019) [46] and Sostak's senses (Kutlu F., Bilgin T., 2015) [47], capturing dynamic changes in topological structures. Topological structures for neutrosophic sets extend the fuzzy topological concepts of Chang, Lowen, and Sostak. Karatas S. et al. (2016) [36] formally introduced neutrosophic topology, while Salama A. A. & Alblowi S. A. (2012) [67] offered preliminary investigations on neutrosophic set and neutrosophic topology. Interval-valued temporal neutrosophic fuzzy set (Rajesh K. et al., 2024) [62] extend neutrosophic theory by incorporating time-dependent uncertainty with interval values for truth, indeterminacy, and falsity. Khan M. B. et al. (2023) [38, 39] contributed to fuzzy calculus by developing new concepts of λ-convex mappings and fuzzy fractional integral operators with exponential kernels. Further, Khan V. A. et al. [40, 41] explored convergence in modified fuzzy normed spaces, expanding the scope of fuzzy mathematical structures.

On the other hand, to address indiscernibility in knowledge systems, Pawlak Z. (1982) [59] introduced rough sets, defining lower and upper approximations of sets to model incomplete information. Hybridizations abound: Antony Crispin Sweety C. Arockiarani I. (2017) [5] explored rough sets in neutrosophic approximation spaces.

Multi-criteria decision-making (MCDM) is a major application area [25, 34, 49, 54, 72, 79, 88, 91, 93, 99]. In the area of decision-making, Krishankumar R. et al. [43] proposed a framework using hesitant fuzzy linguistic terms, while Mallick et al. (2024) [56, 57] applied neutrosophic and QNN-based strategies to multi-attribute decision-making and real-world problems such as COVID-19 vaccine selection. Broumi S., Ye J., & Smarandache F. (2015) [16] extended TOPSIS to interval neutrosophic environments. Also, Biswas S. et al. applied cosine similarity measures (2015) [11] and TOPSIS in single-valued neutrosophic contexts (2016) [12] to neutrosophic environments. Mondal K. & Pramanik S. (2015) [58] designed a neutrosophic decision model for school choice. Torra V. (2010) [86] proposed hesitant fuzzy sets to capture membership uncertainty. Rodríguez R. M. et al. (2012) [63] extended this to hesitant fuzzy linguistic term sets for decision making. Amiri M. et al. (2023) [4] proposed a fuzzy extension of the Simplified Best-Worst Method (F-SBWM). Chai J. S. et al. (2021) [17] and Tongjuan Wang (2021) [90] developed new similarity measures for single-valued neutrosophic sets with applications in pattern recognition. Shahzadi G. et al. (2017) [73] applied single-valued neutrosophic sets in the field of medical diagnosis to handle uncertainty in patient data. Other recent MCDM advances include Peng X. at el. (2019) [60] for Pythagorean fuzzy CODAS, Roy J. et al. (2019) [65] for interval-valued intuitionistic fuzzy CODAS, Karagoz S. et al. (2020) [35] for intuitionistic fuzzy CODAS, and Yeni F. B. & Özçelik G. (2019) [94] for interval-valued Atanassov intuitionistic fuzzy CODAS.

Building on these literature studies, his research advances and refines prior studies to contribute new insights.