Chapter 1

Preliminaries

The basic definitions and results which are used in the course of the thesis are collected and given here.

§ 1.1. Topological spaces

Notation 1.1.1. Throughout the dissertation (Y,ζ), (Z, σ) and (P,μ) indicate topological spaces on which no separation axioms are referenced except as otherwise provided.

Definition 1.1.2. Let (Y,ζ) be a topological space. If D is a non-empty subset of (Y,ζ) then the intersection of all closed sets containing D is called **closure of D** and is denoted by Cl(D).

The union of all open sets contained in D is called **interior of D** and is denoted by int(D).

Definition 1.1.3. Let (Y,ζ) be a topological space. A subset D of (Y,ζ) is called

- **regular closed set** (Stone, 1937) if D = Cl(int(D))
- **semi-closed set** (Levine, 1963) if int(Cl(D)) $\subseteq$ D
- **α-closed set** (Njastad, 1965) if Cl(int(Cl(D)) $\subseteq$ D
- **π-closed set** (Zaitsav, 1968) if it is the finite union of regular closed sets.
- **pre-closed set** (Mashhour et al., 1982) if Cl(int(D)) $\subseteq$ D
- **semi pre-closed set** (Andrijevic, 1986) if int(Cl(int(D))) $\subseteq$ D

The complements of the above mentioned sets are called **regular open, semi-open, α-open, π-open and pre-open, semi pre-open sets** respectively.

The intersection of all **regular closed** (resp. **semi-closed,α-closed, π-closed and pre-closed, semi pre-closed**) subsets of (Y,ζ) containing D is called the **regular closure (resp.semi-closure, α-closure, π-closure and pre-closure, semi pre-closure)** of D and is denoted by rCl(D)(resp.sCl(D),αCl(D), πCl(D) and pCl(D),spCl(D)).

A subset D of (Y,ζ) is called **clopen** if it is both open and closed in (Y,ζ).

Definition 1.1.4. The **δ-interior** (Velicko, 1968) of a subset D of Y is the union of all regular open sets of Y contained in D and is denoted by $\mathrm{int}_\delta(\mathrm{D})$.The subset D is called **δ-open** if $\mathrm{D} = \mathrm{int}_\delta(\mathrm{D})$,i.e. a set is δ-open if it is the union of regular open sets, the complement of δ-open is called **δ-closed**.

Alternatively, a set D $\subseteq$ Y is δ-closed if D = δCl(D), where **δCl (D)** is the intersection of all regular closed sets of (Y,ζ) containing D.

Definition 1.1.5. A subset D of a topological space (Y,ζ) is called **generalized closed** (briefly **g-closed**) (Levine, 1970) if Cl(D) $\subseteq$ M whenever D $\subseteq$ M and M is open in (Y,ζ). The complement of g-closed is a **g-open set**.

Definition 1.1.6. If D is a subset of a space (Y,ζ),

(i) The **generalized closure** (Dunham, 1982) of D is defined as the intersection of all g-closed sets in Y containing D and is denoted by Cl*(D).

(ii) The **generalized interior** (Dunham, 1982) of D is defined as the union of all g-open sets in Y contained in D and is denoted by int*(D).

Definition 1.1.7. Let (Y,ζ) be a topological space. A subset D of (Y,ζ) is called **regular*-open (or r*-open)** (Annalakshmi, 2016) if D = int (Cl*(D)).The complement of a regular*-open set is called a **regular*-closed set(or r*-closed)**. The union of all regular*-open sets of Y contained in D is called **regular*-interior** and is denoted by r*int(D). The intersection of all regular*-closed sets of Y containing D is called **regular*-closure** is denoted by r*Cl(D).

Definition 1.1.8. A subset D of a topological space (Y,ζ) is called

1) **semi generalized closed** (briefly **sg-closed**) (Bhattacharya et.al., 1987) if sCl(D) $\subseteq$ M whenever D $\subseteq$ M and M is semi open in (Y,ζ).
2) **generalized semi-closed** (briefly **gs-closed**) (Arya et al., 1990) if sCl(D) $\subseteq$ M whenever D $\subseteq$ M and M is open in (Y, ζ).
3) **regular generalized closed** (briefly **rg-closed**) (Palaniappan, et.al.,1993) if Cl(D) $\subseteq$ M whenever D $\subseteq$ M and M is regular open in (Y, ζ).

4) **α-generalized closed** (briefly **αg-closed**) (Maki et al., 1994) if $\alpha Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is open in (Y, ζ).
5) **generalized semi pre-closed** (briefly **gsp-closed**) (Dontchev, 1995) if $spCl(D) \subseteq M$ whenever $D \subseteq M$ and M is open in (Y, ζ).
6) **δgeneralized-closed** (briefly **δg-closed**) (Dontchev, 1996) if $\delta Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is open in (Y, ζ).
7) **generalized pre regular -closed** (briefly **gpr-closed**) (Gnanambal, 1998) if $pCl(D) \subseteq M$ whenever $D \subseteq M$ and M is regular open in (Y, ζ).
8) **regular weakly generalized-closed** (briefly **rwg-closed**) (Nagaveni,1999) if $Cl(int(D)) \subseteq M$ whenever $D \subseteq M$ and M is regular open in (Y, ζ).
9) **π-generalized closed** (briefly **πg-closed**) (Dontchev et.al., 2000) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is π-open in (Y, ζ).
10) **generalized δ-closed** (briefly **gδ-closed**) (Dontchev, 2000) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is δ-open in (Y, ζ).
11) **$\delta g^{\ddagger}$-closed** (Dontchev, 2000) if $\delta Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is δ-open in (Y, ζ).
12) **g*-closed** (Veerakumar, 2000) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is g-open in (Y, ζ).
13) **$\hat{g}$ -closed** (Veera Kumar, 2003) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is semi-open in (Y, ζ).
14) **#gs -closed** (Veera Kumar, 2005) if $sCl(D) \subseteq M$ whenever $D \subseteq M$ and U is *g-open in (Y, ζ).
15) ***g -closed** (Veera Kumar, 2006) if $Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is $\hat{g}$-open in (Y, ζ) .
16) **π-generalized semi-closed** (briefly **πgs-closed**) (Aslim et.al., 2006) if $sCl(D) \subseteq M$ whenever $D \subseteq M$ and M is π-open in (Y, ζ).
17) **π-generalized pre-closed** (briefly **πgp-closed**) (Park, 2006) if $pCl(D) \subseteq M$ whenever $D \subseteq M$ and M is π-open in (Y, ζ).
18) **π-generalized α -closed** (briefly **πgα-closed**) (Janaki, 2009) if $\alpha Cl(D) \subseteq M$ whenever $D \subseteq M$ and M is π-open in (Y, ζ).

19) **π-generalized semi pre-closed** (briefly **πgsp-closed**) (Sarsak, 2010) if spCl(D) ⊆ M whenever D ⊆ M and M is π-open in (Y, ζ).
20) **generalized semi pre regular -closed** (briefly **gspr-closed**) (Sarsak et.al., 2010) if spCl(D) ⊆ M whenever D ⊆ M and M is regular open in (Y, ζ).
21) **g^*s-closed** (Pushpalatha et al., 2011) if sCl(D) ⊆ M whenever D ⊆ M and M is gs-open in (Y, ζ).
22) **δ generalized star -closed** (briefly **δg*-closed**) (Sudha, 2014) if δCl(D) ⊆ M whenever D ⊆ M and M is g-open in (Y, ζ).

The complements of the above mentioned sets are called their respective open sets.

Remark 1.1.9. regular closed (open) → δ-closed (open) → δg^*-closed (open) → δg-closed (open) → g-closed (open).

Remark 1.1.10. For every subset D of Y, spCl(D)⊆sCl(D)⊆ δCl(D).

Remark 1.1.11. A topological space (Y, ζ) is said to be a

1. **$T_{1/2}$ -space** (Levine , 1970) if every g-closed subset of (Y, ζ) is closed in (Y, ζ).
2. **semi-T_1-space** (Maheshwari , 1975) if every singleton is semi-closed.
3. **A partition space** (Nieminen , 1977) is a topological space where every open set is closed.
4. **semi-$T_{1/2}$ -space** (Bhattacharya,1987) if every sg-closed subset of (Y,ζ) is semi-closed in (Y,ζ).
5. **T_b-space** (Devi, 1993) if every gs-closed subset of (Y, ζ) is closed in (Y, ζ).
6. **T_d-space** (Devi, 1993) if every gs-closed subset of (Y, ζ) is g-closed in (Y, ζ).
7. **semi-pre-$T_{½}$-space** (Dontchev ,1995) if every generalized semi-preclosed set is semi-preclosed.
8. **$T_{3/4}$ -space** (Dontchev, 1996) if every δg-closed subset of (Y, ζ) is δ-closed in (Y, ζ).
9. **semi-regular space** (Dontchev,1996) if every closed subset of (Y,ζ) is δ-closed in (Y,ζ).
10. **almost weakly Hausdorff space** (Dontchev,1996) if its semi regularization is $T_{1/2}$.
11. **αT_b -space** (Devi, 1998) if every αg-closed subset of (Y,ζ) is closed in (Y,ζ).
12. ***$T_{1/2}$-space**(Veerakumar,2000) if every g-closed subset of (Y,ζ) is g*-closed in (Y,ζ).

13. **$T_{1/2}$*-space** (Veerakumar, 2000) if every g*-closed subset of (Y,ζ) is closed in (Y,ζ).
14. **T_c -space** (Veerakumar,2000) if every gs-closed subset of (Y,ζ) is g^*-closed in (Y,ζ).
15. **T_δ -space** (Dontchev , 2000) if every gδ-closed subset of (Y, ζ) is δ-closed in (Y, ζ).
16. **$_{gs}T_{\delta g^*}$ -space** (Sudha ,2014) if every gs-closed subset of (Y,ζ) is δg^*-closed in (Y,ζ).
17. **$_{\delta g*}T_\delta$ –space** (Sudha ,2014) if every δg^*-closed subset of (Y,ζ) is δ-closed in (Y,ζ).
18. **$_{\delta g}T_{\delta g^*}$-space** (Sudha ,2014) if every δg-closed subset of (Y,ζ) is δg^*-closed in (Y,ζ).
19. **$_{g\delta}T_{\delta g^*}$-space** (Sudha ,2014) if every gδ-closed subset of (Y,ζ) is δg^*-closed in (Y,ζ).
20. **$_gT_{\delta g^*}$-space** (Sudha ,2014) if every g-closed subset of (Y,ζ) is δg^*-closed in (Y,ζ).
21. **$_{g*}T_{\delta g^*}$-space** (Sudha ,2014) if every g*-closed subset of (Y,ζ) is δg^*-closed in (Y,ζ).

Results 1.1.12. Let f : (Y,ζ)→ (Z, σ) be a function. If D and E are subsets of Y and Z respectively then the following results are true.

1. If D$\subseteq$ E then f(D) $\subseteq$ f(E).
2. If D$\subseteq$ E then $f^{-1}(D) \subseteq f^{-1}(E)$.
3. In general, D $\subseteq f^{-1}[f(D)]$. If f is injective then D = $f^{-1}[f(D)]$.
4. In general, f $[f^{-1}(D)] \subseteq$ D. If f is surjective then D = f $[f^{-1}(D)]$.
5. If f is surjective then $[f(D)]^c \subseteq f(D^c)$.
6. If f is bijective then $[f(D)]^c = f(D^c)$.

Result 1.1.13. For a subset D of (Y,ζ), (i) Cl(Y – D) = Y – int(D).
(ii) int(Y – D) = Y – Cl(D).

Definition 1.1.14. A function f : Y ⟶ Z is said to be

- **continuous** (Levine, 1970) if for every closed set U in (Z,σ), $f^{-1}(U)$ is a closed set in (Y,ζ).
- **strongly continuous** (Levine, 1960) if the inverse image of every subset of (Z,σ) is clopen in (Y,ζ).
- **δ-continuous** (Noiri, 1980) if for every δ-closed set U of (Z,σ), $f^{-1}(U)$ is a δ-closed set of (Y,ζ).
- **totally continuous** (Jain, 1980) if the inverse image of every open set of (Z,σ) is clopen in (Y,ζ).

- **super continuous** (Munshi, 1982) if for every closed set U of (Z,σ), $f^{-1}(U)$ is a δ-closed set of (Y,ζ).
- **g-continuous** (Balachandran et al., 1991) if for every closed set U in (Z,σ), $f^{-1}(U)$ is a g-closed set in (Y,ζ).
- **rg-continuous** (Palaniappan, et.al., 1993) if $f^{-1}(U)$ is a rg-closed set in (Y,ζ) for every closed set U in (Z,σ).
- **gs-continuous** (Devi et.al, 1995) if $f^{-1}(U)$ is a gs-closed set in (Y,ζ)for every closed set U in (Z,σ) .
- **contra continuous** (Dontchev,1996) if the inverse image of every closed set of (Z, σ) is an open set in (Y,ζ).
- **δg-continuous** (Dontchev,1996) if $f^{-1}(U)$ is a δg-closed set in (Y,ζ) for every closed set U in (Z,σ).
- **gpr-continuous** (Gnanambal**,**1998) if $f^{-1}(U)$ is a gpr-closed set in (Y,ζ) for every closed set U in (Z,σ) .
- **rwg-continuous** (Nagaveni,1999) if $f^{-1}(U)$ is rwg-closed in (Y,ζ) for every closed subset U in (Z,σ) .
- **gδ-continuous** (Dontchev,2000) if $f^{-1}(U)$ is a gδ-closed set in (Y,ζ) for every closed set U in (Z,σ).
- **$\hat{g}$ -continuous** (Veerakumar,2003) if $f^{-1}(U)$ is $\hat{g}$-open in (Y,ζ) for every open set U in (Z,σ) .
- **πgp-continuous** (Park, 2006) if $f^{-1}(U)$ is a πgp-closed set in (Y,ζ) for every closed set U in (Z,σ) .
- **πgs-continuous** (Aslim, 2006) if $f^{-1}(U)$ is a πgs-closed set in (Y,ζ) for every closed set U in (Z,σ) .
- **πgα-continuous** (Ekici et.al., 2007) if $f^{-1}(U)$ is a πgα-closed set in (Y,ζ) for every closed set U in (Z,σ) .
- **πg-continuous** (Ekici et.al., 2007) if $f^{-1}(U)$ is a πg-closed set in (Y,ζ) for every closed set U in (Z,σ) .
- **πgsp-continuous** (Sarsak, 2010) if $f^{-1}(U)$ is a πgsp-closed set in (Y,ζ) for every closed set U in (Z,σ) .

- **gspr-continuous** (Navalagi, 2010) if $f^{-1}(U)$ is a gspr-closed set in (Y,ζ) for every closed set U in (Z,σ) .
- **g^*s-continuous** (Pushpalatha et.al., 2011) if $f^{-1}(U)$ is a g^*s-closed set in (Y,ζ) for every closed set U in (Z, σ) .
- **δg^*-continuous** (Sudha, 2014) if $f^{-1}(U)$ is a δg^*-closed set in (Y,ζ) for every closed set U in (Z, σ).

Definition 1.1.15. A function $f : (Y,\zeta) \to (Z, \sigma)$ is known as

1. **irresolute** (Crossley, 1972) if for every semi open set V in (Z, σ), $f^{-1}(U)$ is a semi open set in (Y,ζ).
2. **g-irresolute** (Balachandran ,1991) if for every g-open set U in (Z, σ), $f^{-1}(U)$ is a g-open set in (Y,ζ).
3. **gs-irresolute** (Devi,1995) if for every gs-open set U in (Z, σ), $f^{-1}(U)$ is a gs-open set in (Y,ζ).
4. **δg-irresolute** (Dontchev, 1996) if for every δg-open set U in (Z, σ), $f^{-1}(U)$ is a δg-open set in (Y,ζ).
5. **contra irresolute** (Miguel Caldas Cueva, 2000) if for each semi-open U in (Z ,σ), $f^{-1}(U)$ is semi-closed in (Y,ζ).
6. **gδ-irresolute** (Dontchev, 2000) if for every gδ-open set U in (Z, σ), $f^{-1}(U)$ is a gδ-open set in (Y, ζ).
7. **g^*s-irresolute** (Pushpalatha, 2011) if for every g^*s-open set U in (Z, σ), $f^{-1}(U)$ is a g^*s-open set in (Y,ζ).
8. **δg*-irresolute** (Sudha, 2014) if for every δg*-open set U in (Z, σ), $f^{-1}(U)$ is a δg*-open set in (Y,ζ).

Definition 1.1.16. A function $f : (Y,\zeta) \to (Z, \sigma)$ is said to be

- **δ-closed** (resp. δ-open) **function** (Noiri, 1978) if for every closed (resp. open) set U in (Y,ζ), f(U) is δ-closed (resp. δ-open) in (Z, σ).
- **g-closed** (resp. g-open) **function** (Malghen, 1982) if for every closed (resp. open) set U in (Y,ζ), f(U) is g-closed (resp. g-open) in (Z,σ).

- **gδ-closed** (resp. gδ-open) **function** (Dontchev et.al., 2000) if for every closed set U in (Y,ζ), f(U) is gδ-closed in (Z,σ).
- **πg-closed** (resp. πg-open) **function** (Dontchev et.al., 2000) if for every closed set U in (Y,ζ), f(U) is πg-closed in (Z, σ).
- **πgp**-**closed** (resp. πgp-open) **function** (Park 2004) if for every closed set U in (Y,ζ), f(U) is πg-closed in (Z, σ).
- **πgs-closed** (resp. πgs-open) **function** (Aslim, 2006) if for every closed (resp. open) set U in (Y,ζ), f(U) is πgs-closed (resp.πgs-open) in (Z,σ).
- **πgα-closed** (resp. πgα-open) **function** (Janaki et.al., 2009) if for every closed set U in (Y,ζ), f(U) is πg-closed in (Z, σ).
- **πgsp-closed** (resp. πgsp-open) **function** (Sarsak, 2010) if for every closed (resp. open) set U in (Y,ζ), f(U) is πgsp-closed (resp. πgsp-open) in (Z, σ).
- **δg*-closed** (resp. δg*-open)**function** (Sudha, 2014) if for every closed (resp. open) set U in (Y,ζ), f(U) is δg*-closed (resp. δg*-open) in (Z, σ).

Definition 1.1.17. A map f : (Y,ζ) → (Z, σ) is said to be a

- **quotient map** (Munkres,Topology, A first course) if f is continuous and f $^{-1}$(V) being open in (Y,ζ) implies V is open in (Z,σ).
- **ĝ-quotient map** (Subasree ,2013) if f is ĝ-continuous and f $^{-1}$(V) being open in (Y,ζ) implies V is ĝ-open in (Z,σ).

Definition 1.1.18. A bijection function f : (Y,ζ) → (Z, σ) is said to be

1. **g-homeomorphism** (Maki, 1991) if f is both g-open and g-continuous.
2. **rg-homeomorphism** (Palaniappan et.al., 1993) if f is both rg-open and rg-continuous.
3. **gs-homeomorphism** (Devi, 1995) if f is both gs-open and gs-continuous.
4. **rwg-homeomorphism** (Nagaveni,1999) if f is both rwg-open and rwg- continuous.
5. **gδ-homeomorphism** (Dontchev, 2000) if f is both gδ-open and gδ-continuous.

6. **πg-homeomorphism** (Dontchev et.al., 2000) if f is both πg-open and πg-continuous.
7. **πgs-homeomorphism** (Aslim, 2006) if f is both πgs-open and πgs-continuous.
8. **πgp-homeomorphism** (Aslim et.al., 2006) if f is both πgp-open and πgp-continuous.
9. **πgα-homeomorphism** (Janaki, 2009) if f is both πgα-open and πgα-continuous.
10. **πgsp-homeomorphism** (Sarsak,2010) if f is both πgsp-open and πgsp-continuous.
11. **generalized pre-regular homeomorphism** (abbreviated by **gpr-homeomorphism**) (Devamanoharan , 2013) if f is both gpr-continuous and gpr-open.
12. **δg*-homeomorphism** (Sudha ,2014) if f is both δg*-open and δg*-continuous.

§ 1.2. Soft Topological spaces

Definition 1.2.1. (Molodtsov,1999) Let Y be an initial universe and E be a set of parameters. Let P(Y) denote the power set Y and A be a non-empty subset of E. A pair (F,A) is called a **soft set** over Y, where F is a mapping given by F : A→P(Y).

In other words, a **soft set** over Y is a parametrized family of subsets of the universe Y. For e ∈ A, F(e) may be considered as the set of e-approximate elements of the soft set (F,A). Clearly a soft set is not a set.

Definition 1.2.2. (Maji,2003) For two soft sets (F,A) and (G,B) over a common universe Y, we say that (F,A) is a **soft subset** of (G,B) if

(a) A$\tilde{\subseteq}$ B and

(b) for all e∈A, F(e) and G(e) are identical approximations. We write (F,A) $\tilde{\subseteq}$(G,A). (F,A) is said to be a **soft super set** of (G,B), if (G,B) is a **soft subset** of (F,A). We denote it by (F,A) $\tilde{\supseteq}$ (G,B).

Definition 1.2.3. (Maji ,2003) Two soft sets (F,A) and (G,B) over a common universe Y are said to be **soft equal** if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A).

Definition 1.2.4. (Maji,2003) A soft set (F,A) over Y is said to be a **Null soft set** denoted by $\emptyset$ if for all e ∈A, F(e) = $\emptyset$.

Definition 1.2.5. (Maji,2003) The **union of two soft sets** (F,E) and (G,E) over Y is the soft set (H, C),where C = A ∪ B and for all e ∈ C, H(e) = F(e) if e ∈ A – B, G(e) if e ∈ B – A and F(e) ∪ G(e) if e ∈ A ∩ B. We write (F, A) ∪ (G, B) = (H,C).

Definition 1.2.6. (Feng,2008) The **intersection (H, C) of two soft sets** (F,A) and (G, B) over a common universe Y, denoted (F, A) ∩ (G, B), is defined as C = A ∩ B and H(e) = F (e) ∩ G(e) for all e ∈ C.

Definition 1.2.7. (Shabir,2011) The **difference (H, E) of two soft sets (F ,E) and (G, E)** over Y, denoted by (F , E) \ (G, E), is defined as H(e) = F (e) \ G(e) for all e ∈ E.

Definition 1.2.8. (Shabir,2011) Let (F,E) be a soft set over Y and x ∈ Y. We say that **x ∈ (F,E) read as x belongs to the soft set (F,E)** whenever x ∈ F(α) for all α ∈ E. Note that for any x ∈ Y, x ∉(F , E), if x ∉ F(α) for some α ∈ E.

Definition 1.2.9. (Shabir,2011) Let Z be a non-empty subset of Y, then $\tilde{Z}$ **denotes the soft set (Z, E)** over Y for which Z (α) = Z, for all α ∈ E. In particular, **(Y, E) will be denoted by** $\tilde{Y}$**.**

Definition 1.2.10. (Shabir,2011) Let ζ be the collection of soft sets over Y, then ζ is said to be a **soft topology** on Y if

(a) $\emptyset$, $\tilde{Y}$ belong to ζ.
(b) The union of any number of soft sets in ζ belongs to ζ.
(c) The intersection of any two soft sets in ζ belongs to ζ.

The triplet (Y,ζ,E) is called a **soft topological space** over Y.

Definition 1.2.11. (Shabir,2011) Let (Y,ζ,E) be a soft space over Y, then the members of ζ are said to be **soft open sets** in Y.

Definition 1.2.12. (Shabir,2011) The **relative complement** of a soft set (F,E) is denoted by $(F,E)^c$ and is defined by $(F,E)^c = (F^c,E)$ where $F^c : E \rightarrow P(Y)$ is a mapping given by $F^c(e) = Y \setminus F(e)$ for all e ∈ E.Clearly, $((F,E)^c)^c = (F,E)$.

Definition 1.2.13. (Shabir,2011) Let (Y,ζ,E) be a soft space over Y. A soft set (F,E) over Y is said to be a **soft closed set** in Y, if its relative complement $(F,E)^c$ belongs to ζ.

Definition 1.2.14. (Shabir,2011) Let Y be an initial universe set, E be the set of parameters and $\zeta = \{ \emptyset, \tilde{Y} \}$. Then ζ is called the **soft indiscrete topology** on Y and (Y,ζ,E) is said to be a **soft indiscrete space** over Y.

Let Y be an initial universe set, E be the set of parameters and let ζ be the collection of all soft sets which can be defined over Y. Then ζ is called the **soft discrete topology** on Y and (Y,ζ,E) is said to be a **soft discrete space** over Y.

Definition 1.2.15. (Shabir,2011) Let (Y,ζ,E) be a soft topological space over Y and (F, E) be a soft set over Y. Then, the **soft closure** of (F,E), denoted by Cl (F,E) is the intersection of all soft closed supersets of (F,E).Clearly (F,E) is the smallest soft closed set over Y which contains (F,E). (i.e.) $Cl(F,E) = \cap\{(O,E): (O,E)$ is soft closed and $(F,E) \tilde{\subseteq} (O,E)\}$.

Definition 1.2.16. (Shabir,2011) Let (Y,ζ,E) be a soft topological space over Y and (F,E) be a soft set over Y. The **soft interior** of (F,E) denoted by Int(F,E) is the union of all soft open subsets of (F,E). Clearly (F, E) is the largest soft open set over Y which is contained in (F,E). **(**i.e.) $Int(F,E) = \cup\{(O,E): (O,E)$ is soft open and $(O,E) \tilde{\subseteq} (F,E)\}$.

Definition 1.2.17. (Shabir,2011) Let (Y,ζ,E) be a soft topological space over Y, (G,E) be a soft set over Y and $x \in Y$. Then x is said to be a **soft interior point** of (G,E) if there exists a soft open set (F,E) such that $x \in (F,E) \tilde{\subseteq} (G,E)$.

Definition 1.2.18. A soft subset (F,E) of Y is called a

- **soft pre-open** (Arockiarani , 2013) if $(F,E) \tilde{\subseteq} Int(Cl(F,E))$
- **soft semi-open** (Chen, 2013) if $(F,E) \tilde{\subseteq} Int(Cl(F,E))$
- **soft regular open** (Janaki , 2013) if $(F,E) = Int(Cl(F,E))$
- **soft α-open** (Akdag , 2014) if $(F,E) \tilde{\subseteq} Int(Cl(Int(F,E)))$

The complement of the soft regular open , soft semi-open , soft α-open, soft pre-open sets are their respective soft regular closed , soft semi-closed , soft α-closed , soft pre-closed sets.

Definition 1.2.19. A soft subset (F,E) of Y is called a **soft clopen** (Janaki , 2013) is (F,E) is both soft open and soft closed.

Definition 1.2.20. (Janaki , 2013) The **soft regular closure** of (F,E) is the intersection of all soft regular closed sets containing (F,E) and is denoted by srcl(F,E).

The **soft regular interior** of (F,E) is the union of all soft regular open sets contained in (F,E) and is denoted by srint(F,E).

Definition 1.2.21. (Kannan , 2012) A soft set (F, E) is called a **soft generalized closed** (**soft g-closed**) in a soft topological space (Y,ζ,E) if Cl(F, E) $\tilde{\subseteq}$ (G, E) whenever (F,E) $\tilde{\subseteq}$ (G,E) and (G, E) is soft open in Y.The complement of soft g-closed is **soft g-open**.

Definition 1.2.22. (Nandhini ,2014) Let (Y,ζ,E) be a soft topological space. A soft set (F, E) is called a **soft ĝ-closed** set if Cl(F, E) $\tilde{\subseteq}$ (G, E) whenever (F, E) $\tilde{\subseteq}$ (G, E) and (G, E) is soft semi open in (Y,ζ,E) .

Definition 1.2.23. (Guzel ,2014) Let (Y,ζ,E) be a soft topological space. A soft set (F, E) is called **soft generalized preregular closed (in short soft gpr-closed**), if pCl(F, E) $\tilde{\subseteq}$ (G, E) whenever (F, E) $\tilde{\subseteq}$ (G, E) and (G, E) is a soft regular open set.

Definition 1.2.24. (Janaki,2014) Let (Y,ζ,E) be a soft topological space. A soft set (F, E) is called a **soft g*-closed** set if Cl(F, E) $\tilde{\subseteq}$ (G, E) whenever (F, E) $\tilde{\subseteq}$ (G, E) and (G, E) is soft g-open in (Y,ζ,E) .

Definition 1.2.25. (Janaki,2014) Let (Y,ζ,E) be a soft topological space. A subset (F,E) of Y is said to be a **soft rwg-closed set** if Cl(Int(F, E)) $\tilde{\subseteq}$ (G, E) whenever (F, E) $\tilde{\subseteq}$ (G, E) and (G, E) is soft regular open.

Definition 1.2.26. (Bishnupada Debnath ,2017) A soft set (F, E) is called a **soft δ-open set** is the union of soft regular open sets. The complement of soft δ-open set is called a **soft δ-closed** set.

List of Symbols

Throughout the dissertation (Y,ζ), (Z, σ) and (P,μ) indicate topological spaces on which no separation axioms are referenced except as otherwise provided. (Y,ζ,E) denotes soft topological space.

r*O(Y,ζ)	-	The family of all regular*- open sets of (Y,ζ).
δC(Y,ζ)	-	The family of all δ-closed sets of (Y,ζ).
δg*C(Y,ζ)	-	The family of all δg*-closed sets of (Y,ζ).
δgC(Y,ζ)	-	The family of all δg-closed sets of (Y,ζ).
gC(Y,ζ)	-	The family of all g-closed sets of (Y,ζ).
$\delta g^{\ddagger}$C(Y,ζ)	-	The family of all $\delta g^{\ddagger}$-closed sets of (Y,ζ).
sC(Y,ζ)	-	The family of all semi -closed sets of (Y,ζ).
ĝC(Y,ζ)	-	The family of all ĝ-closed sets of (Y,ζ).
gδC(Y,ζ)	-	The family of all gδ-closed sets of (Y,ζ).
rgC(Y,ζ)	-	The family of all rg-closed sets of (Y,ζ).
gprC(Y,ζ)	-	The family of all gpr-closed sets of (Y,ζ).
rwgC(Y,ζ)	-	The family of all rwg-closed sets of (Y,ζ).
gsprC(Y,ζ)	-	The family of all gspr-closed sets of (Y,ζ).
πgC(Y,ζ)	-	The family of all πg-closed sets of (Y,ζ).
πgpC(Y,ζ)	-	The family of all πgp-closed sets of (Y,ζ).
πgsC(Y,ζ)	-	The family of all πgs-closed sets of (Y,ζ).
πgspC(Y,ζ)	-	The family of all πgsp-closed sets of (Y,ζ).
πgαC(Y,ζ)	-	The family of all πgα-closed sets of (Y,ζ).
gsC(Y,ζ)	-	The family of all gs-closed sets of (Y,ζ).
ĝO(Z,σ)	-	The family of all ĝ-open sets of (Z,σ).
gsO(Z,σ)	-	The family of all gs-open sets of (Z,σ).

g*sO(Z,σ)	-	The family of all g*s-open sets of (Z,σ).
δg*O(Z,σ)	-	The family of all δg*-open sets of (Z,σ).
δgO(Z,σ)	-	The family of all δg-open sets of (Z,σ).
gO(Z,σ)	-	The family of all g-open sets of (Z,σ).
gδO(Z,σ)	-	The family of all gδ-open sets of (Z,σ).
SRO(Y,ζ,E)	-	The family of all soft regular open sets of (Y,ζ,E).
SδO(Y,ζ,E)	-	The family of all soft δ-open sets of (Y,ζ,E).
srint(F,E)	-	The union of all soft regular open sets contained in (F,E).
srcl(F,E)	-	The intersection of all soft regular closed sets containing (F,E).

The family of all the above sets for some finite topological sets are given in Appendix.