3

β^{**} Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

3.1 Introduction

In modern mathematics, especially in topology and functional analysis mapping plays an important role. Here in this chapter we have introduced different types of intuitionistic fuzzy continuous mappings namely intuitionistic fuzzy β^{**} generalized continuous mappings, intuitionistic fuzzy contra β^{**} generalized continuous mappings, intuitionistic fuzzy almost β^{**} generalized continuous mappings and intuitionistic fuzzy almost contra β^{**} generalized continuous mappings. Also we have provided the relations between those continuous mappings. Moreover intuitionistic fuzzy β^{**} generalized irresolute mappings are also introduced and studied.

3.2 Intuitionistic Fuzzy β^{**} Generalized Continuous Mappings

In this section we have introduced intuitionistic fuzzy β^{**} generalized continuous mappings and investigated some of their properties. Also we have established the relation between the newly introduced continuous mappings and the already existing continuous mappings.

Definition 3.2.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is called an **intuitionistic fuzzy β^{**} generalized continuous** (IFβ^{**}G continuous) **mapping** if $f^{-1}(V)$ is an IFβ^{**}GCS in (X, τ) for every IFCS V of (Y, σ).

We shall use the notation $A = \langle x, (\mu_a, \mu_b), (\nu_a, \nu_b)\rangle$ instead of $A = \langle x, (a/\mu_a, b/\mu_b), (a/\nu_a, b/\nu_b)\rangle$ in the following examples. Similarly we shall use the notation $B = \langle y, (\mu_u, \mu_v), (\nu_u, \nu_v)\rangle$ instead of $B = \langle y, (u/\mu_u, v/\mu_v), (u/\nu_u, v/\nu_v)\rangle$ in the following examples.

Example 3.2.2 : Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ and IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here the mapping f is an IFβ^{**}G continuous mapping.

Proposition 3.2.3 : Every IF continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 3.2.4 : Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ and IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Here, f is an IF$\beta^{**}$G continuous mapping but not an IF continuous mapping, since G_3^c is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = 1_\sim \neq f^{-1}(G_3^c)$.

Proposition 3.2.5 : Every IFS continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 3.2.6 : Let $X = \{a, b\}$ and $Y = \{u, v\}$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ and IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$.

Here, f is an IFβ^{**}G continuous mapping but not an IFS continuous mapping, since G_3^c is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) = int(1_\sim) = 1_\sim \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.2.7 : Every IFP continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 3.2.8 : Let X = {a, b} and $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ and IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFβ^{**}G continuous mapping but not an IFP continuous mapping, since G_3^c is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3^c))) = G_1^c \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.2.9 : Every IFα continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFα continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X. Since every IFαCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 3.2.10 : Let X = {a, b} and Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ and IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Here, f is an IF$\beta^{**}$G continuous mapping but not an IFα continuous mapping, since G_3^c is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFαCS in X, as $cl(int(cl(f^{-1}(G_3^c)))) = 1_\sim \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.2.11 : Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping and $f^{-1}(A)$ be an IFRCS in X for every IFCS A in Y. Then f is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let A be an IFCS in Y and $f^{-1}(A)$ be an IFRCS in X. Since every IFRCS is an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 3.2.12 : Let X = {a, b} and Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ and IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Here, f is an IF$\beta^{**}$G continuous mapping but not a mapping as defined in Proposition 3.2.11, since G_3^c is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFRCS in X, as $cl(int(f^{-1}(G_3^c))) = G_1^c \neq f^{-1}(G_3^c)$.

Proposition 3.2.13 : Every IFβ continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFβCS in X. Since every IFβCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

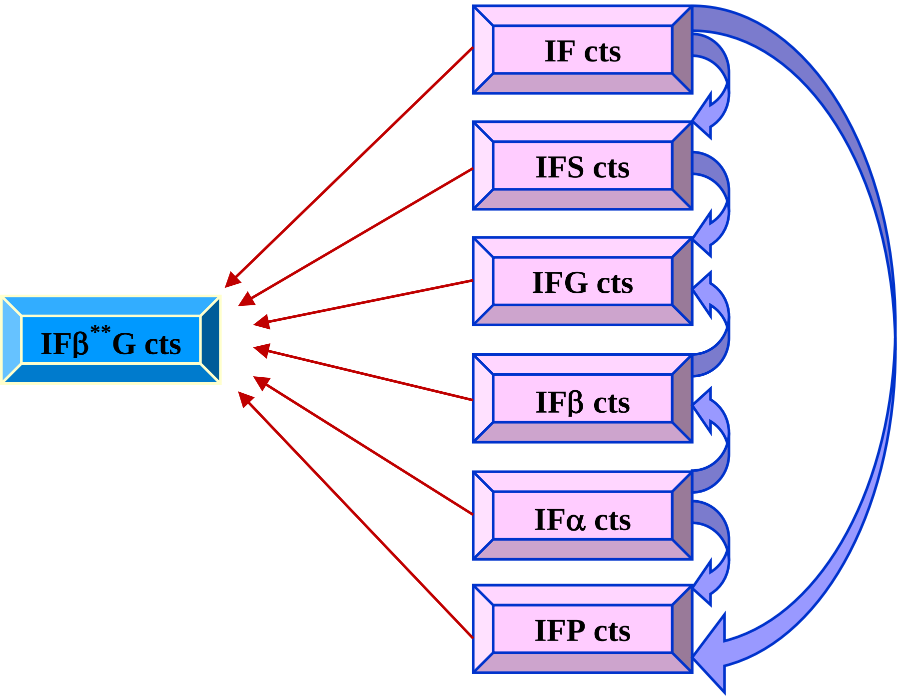
Example 3.2.14 : Let X = {a, b} and Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFβ^{**}G continuous mapping but not an IFβ continuous mapping, since G_3^c is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFβCS in X, as $int(cl(int(f^{-1}(G_3^c)))) = 1_\sim \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.2.15 : Every IFG continuous mapping is an IFβ^{**}G continuous mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IFG continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGCS in X. Since every IFGCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 3.2.16 : Let X = {a, b} and Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ and IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Here, f is an IF$\beta^{**}$G continuous mapping but not an IFG continuous mapping, since G_3^c is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFGCS in X, as $cl(f^{-1}(G_3^c)) = 1_\sim \not\subseteq G_2$, whereas $f^{-1}(G_3^c) \subseteq G_2$.

Relationship between intuitionistic fuzzy continuous mapping with other existing continuous mappings are depicted in the following diagram. In this diagram 'cts' means continuous mapping.



The reverse implications are not true in general in the above diagram.

Proposition 3.2.17 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IFβ^{**}G continuous mapping if and only if the inverse image of each IFOS in Y is an IFβ^{**}GOS in X.

Proof : Necessity : Let A be an IFOS in Y. Then A^c is an IFCS in Y. Since f is an IFβ^{**}G continuous mapping, $f^{-1}(A^c)$ is an IFβ^{**}GCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβ^{**}GOS in X.

Sufficiency : Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFβ^{**}GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Proposition 3.2.18 : If $f : (X, \tau) \to (Y, \sigma)$ is an IFβ^{**}G continuous mapping, then for each IFP $p_{(\alpha,\beta)}$ of X and each $A \in \sigma$ such that $f(p_{(\alpha,\beta)}) \in A$, there exists an IF$\beta^{**}$GOS B of X such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$.

Proof : Let $p_{(\alpha,\beta)}$ be an IFP of X and $A \in \sigma$ such that $f(p_{(\alpha,\beta)}) \in A$. Then $p_{(\alpha,\beta)} \in f^{-1}(A)$, put $B = f^{-1}(A)$. By hypothesis, B is an IFβ^{**}GOS in X such that $p_{(\alpha,\beta)} \in B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Proposition 3.2.19 : If $f : (X, \tau) \to (Y, \sigma)$ is an IFβ^{**}G continuous mapping, then for each IFP $p_{(\alpha,\beta)}$ of X and each $A \in \sigma$ such that $f(p_{(\alpha,\beta)})_q A$, there exists an IF$\beta^{**}$GOS B of X such that $p_{(\alpha,\beta)q} B$ and $f(B) \subseteq A$.

Proof : Let $p_{(\alpha,\beta)}$ be an IFP of X and $A \in \sigma$ such that $f(p_{(\alpha,\beta)})_q A$. Put $B = f^{-1}(A)$. Then by hypothesis, B is an IFβ^{**}GOS in X such that $p_{(\alpha,\beta)q} B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Proposition 3.2.20 : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ^{**}G continuous mapping then f is an IFP continuous mapping if X is an IFβ^{**}p $T_{1/2}$ space.

Proof : Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFβ^{**}GCS in X, by hypothesis. Since X is an IFβ^{**}p$T_{1/2}$ space, $f^{-1}(V)$ is an IFPCS in X. Hence f is an IFP continuous mapping.

Proposition 3.2.21 : Let f : (X, τ) → (Y, σ) be an IFβ^{**}G continuous mapping, then f is an IFG continuous mapping if X is an IFβ^{**}g$T_{1/2}$ space.

Proof : Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFβ^{**}GCS in X, by hypothesis. Since X is an IFβ^{**}g $T_{1/2}$ space, $f^{-1}(V)$ is an IFGCS in X. Hence f is an IFG continuous mapping.

Proposition 3.2.22 : Let f : (X, τ) → (Y, σ) be an IFβ^{**}G continuous mapping and g : (Y, σ) → (Z, δ) is an IF continuous mapping then g o f : (X, τ) → (Z, δ) is an IFβ^{**}G continuous mapping.

Proof : Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y, by hypothesis. Since f is an IFβ^{**}G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GCS in X. Hence g o f is an IFβ^{**}G continuous mapping as $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$.

Proposition 3.2.23 : A mapping f : (X, τ) → (Y, σ) be an IFβ^{**}G continuous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof : Let A be an IFCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. Therefore $f^{-1}(A)$ is an IFαCS and hence it is an IFβ^{**}GCS in X. Then f is an IFβ^{**}G continuous mapping.

Proposition 3.2.24 : Let f : (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X and Y are IFβ^{**}p$T_{1/2}$ spaces :

(i) f is an IFβ^{**}G continuous mapping,

(ii) $f^{-1}(B)$ is an IFβ^{**}GOS in X for each IFOS B in Y,

(iii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFOS B in Y such that $f(p_{(\alpha,\beta)}) \in B$, there exists an IF$\beta^{**}$GOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

Proof : (i) ⇒ (ii) is obvious from the Proposition 3.2.17.

(ii) $\Rightarrow$ (iii) : Let B be any IFOS in Y and let $p_{(\alpha,\beta)} \in X$. Given $f(p_{(\alpha,\beta)}) \in B$. By hypothesis $f^{-1}(B)$ is an IFβ^{**}GOS in X. Take $A = f^{-1}(B)$. Then $p_{(\alpha,\beta)} \in f^{-1}(B) = A$. This implies $p_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) $\Rightarrow$ (i) : Let A be an IFCS in Y. Then its complement, say B is an IFOS in Y. Let $p_{(\alpha,\beta)} \in X$ and $f(p_{(\alpha,\beta)}) \in B$. Then there exists an IFβ^{**}GOS, say C in X such that $p_{(\alpha,\beta)} \in C$ and $f(C) \subseteq B$. Therefore $p_{(\alpha,\beta)} \in C \subseteq f^{-1}(f(C)) \subseteq f^{-1}(B)$ and hence $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} \{p_{(\alpha,\beta)}\} \subseteq \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C$.

Since X is an IFβ^{**}p$T_{1/2}$ space, C is an IFPOS in X and $f^{-1}(B) = \bigcup_{p_{(\alpha,\beta)} \in f^{-1}(B)} C$ is also an IFPOS in X. Hence $f^{-1}(B)$ is an IFβ^{**}GOS in X. That is $f^{-1}(A^c)$ is an IFβ^{**}GOS in X and hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Thus f is an IFβ^{**}G continuous mapping.

Proposition 3.2.25 : Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y that satisfies $f^{-1}(int(B)) = int(cl(f^{-1}(int(B))))$ for every IFS B in Y. Then f is an IFβ^{**}G continuous mapping.

Proof : Let B be an IFOS in Y. Then int(B) = B and by hypothesis $f^{-1}(B) = int(cl(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFROS in X. Therefore it is an IFβ^{**}GOS in X. Hence f is an IFβ^{**}G continuous mapping.

Proposition 3.2.26 : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ^{**}G continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IFG continuous mapping and Y is an IF$T_{1/2}$ space, then $g \text{ o } f : (X, \tau) \to (Z, \delta)$ is an IFβ^{**}G continuous mapping.

Proof : Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFGCS in Y, by hypothesis. Since Y is an IF$T_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GCS in X, by hypothesis. Hence g o f is an IFβ^{**}G continuous mapping.

3.3 Intuitionistic Fuzzy Contra β^{**} Generalized Continuous Mappings

In this section we have introduced intuitionistic fuzzy contra β^{**} generalized continuous mappings and investigate some of their properties.

Definition 3.3.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an **intuitionistic fuzzy contra β^{**} generalized** (IF contra β^{**}G) **continuous mapping** if $f^{-1}(A)$ is an IFβ^{**}GCS in X for every IFOS A in Y.

Example 3.3.2 : Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is an IF contra β^{**}G continuous mapping in (X, τ).

Proposition 3.3.3 : Every IF contra continuous mapping is an IF contra β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF contra continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFCS in X, by hypothesis. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous mapping.

Example 3.3.4 : Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF contra β^{**}G continuous mapping, but not an IF contra continuous mapping, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFCS in X, as $cl(f^{-1}(G_3)) = G_1^c \neq f^{-1}(G_3)$.

Proposition 3.3.5 : Every IF contra semi continuous mapping is an IF contra β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF contra semi continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFSCS in X, by hypothesis. Since every IFSCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous mapping.

Example 3.3.6 : Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF contra β^{**}G continuous mapping but not an IF contra semi continuous mapping, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3))) = G_1 \not\subseteq f^{-1}(G_3)$.

Proposition 3.3.7 : Every IF contra pre continuous mapping is an IF contra β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF contra pre continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFPCS in X, by hypothesis. Since every IFPCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous mapping.

Example 3.3.8 : Let $X = \{a, b\}$ and $Y = \{u, v\}$. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Here f is an IF contra β^{**}G continuous mapping but not an IF contra pre continuous mapping, since $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3))) = G_2^c \not\subseteq f^{-1}(G_2)$.

Proposition 3.3.9 : Every IF contra α continuous mapping is an IF contra β^{**}G continuous mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IF contra α continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFαCS in X, by hypothesis. Since every IFαCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous mapping.

Example 3.3.10 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF contra β^{**}G continuous mapping but not an IF contra α continuous mapping, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFαCS in (X, τ), as $cl(int(cl(f^{-1}(G_3)))) = G_1^c \not\subseteq f^{-1}(G_3)$.

Proposition 3.3.11 : Every IF contra β continuous mapping is an IF contra β^{**}G continuous mapping but not conversely in general.

Proof : Let f : (X, τ) → (Y, σ) be an IF contra β continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFβCS in X, by hypothesis. Since every IFβCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous mapping.

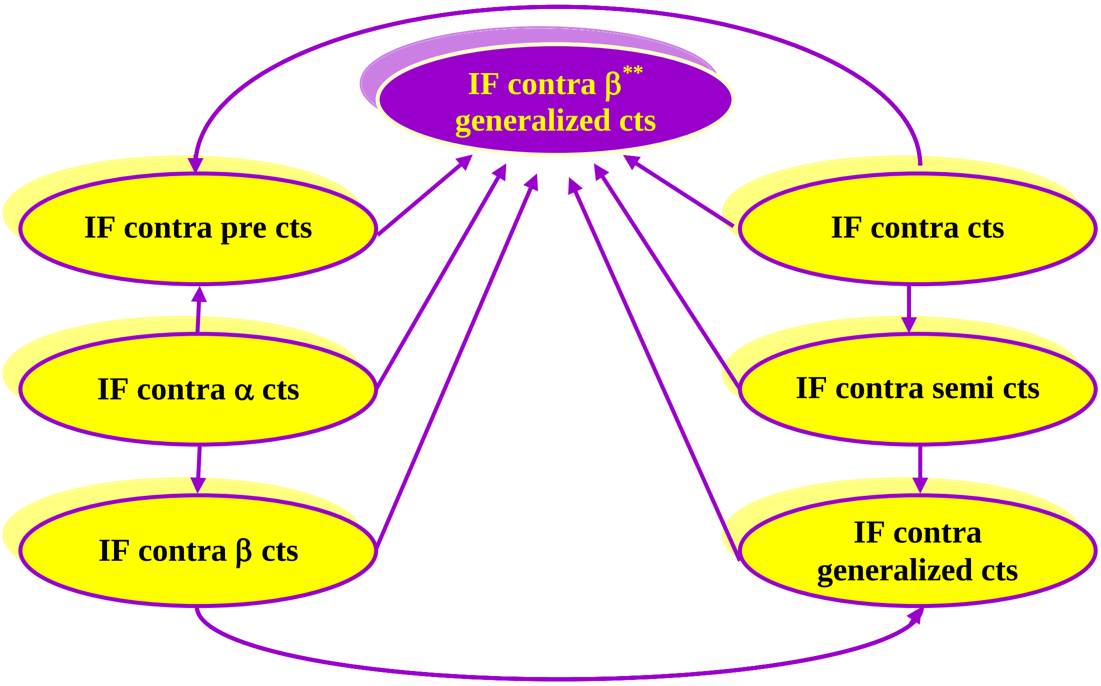
Example 3.3.12 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Here f is an IF contra β^{**}G continuous mapping but not an IF contra β continuous mapping, since $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_2)$ is not an IFβCS in (X, τ), as $int(cl(int(f^{-1}(G_2)))) = 1_\sim \not\subseteq f^{-1}(G_2)$.

Proposition 3.3.13 : Every IF contra generalized continuous mapping is an IF contra β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF contra generalized continuous mapping. Let V be an IFOS in Y. Then $f^{-1}(V)$ is an IFGCS in X, by hypothesis. Since every IFGCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous mapping.

Example 3.3.14 : Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF contra β^{**}G continuous mapping but not an IF contra generalized continuous mapping, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFGCS in X, as $cl(f^{-1}(G_3)) = G_1^c \not\subseteq G_1$, whereas $f^{-1}(G_3) \subseteq G_1$.

Relationship between various types of intuitionistic fuzzy contra continuity are depicted in the following diagram. In this diagram 'cts' means continuous mapping.



The reverse implications are not true in general in the above diagram.

Proposition 3.3.15 : A mapping f : (X, τ) → (Y, σ) is an IF contra β^{**}G continuous mapping if and only if the inverse image of each IFCS in Y is an IFβ^{**}GOS in X.

Proof : Necessity : Let A be an IFCS in Y. This implies A^c is an IFOS in Y. Then $f^{-1}(A^c)$ is an IFβ^{**}GCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβ^{**}GOS in X.

Sufficiency : Let A be an IFOS in Y. Then A^c is an IFCS in Y. By hypothesis $f^{-1}(A^c)$ is IFβ^{**}GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF contra β^{**}G continuous mapping.

Proposition 3.3.16 : Let f : (X, τ) → (Y, σ) be a bijective function, suppose that one of the following properties hold :

(i) $f^{-1}(cl(B)) \subseteq int(\beta cl(f^{-1}(B)))$ for each IFS B in Y,

(ii) $cl(\beta int\,(f^{-1}(B))) \subseteq f^{-1}\,(int\,(B))$ for each IFS B in Y,

(iii) $f(cl(\beta int(A))) \subseteq int(f(A))$ for each IFS A in X,

(iv) $f(cl(A)) \subseteq int(f(A))$ for each IFβOS A in X.

Then f is an IF contra β^{**}G continuous mapping.

Proof : (i) ⇒ (ii) is obvious by taking complement in (i).

(ii) ⇒ (iii) : Let $A \subseteq X$, Then $B = f(A) \subseteq Y$. This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in X. Now $cl(\beta int(A)) = cl(\beta int(f^{-1}(B))) \subseteq f^{-1}(int(B))$ by (ii). Therefore $f(cl(\beta int(A))) \subseteq f(f^{-1}(int(B))) = int(B) = int(f(A))$.

(iii) ⇒ (iv) : Let $A \subseteq X$ be an IFβOS. Then $\beta int(A) = A$. By hypothesis, $f(cl(\beta int(A))) \subseteq int(f(A))$. Therefore $f(cl(A)) = f(cl(\beta int(A))) \subseteq int(f(A))$.

Suppose (iv) holds. Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFS in X and $\beta int(f^{-1}(A))$ is an IFβOS in X. Hence by hypothesis, $f(cl(\beta int(f^{-1}(A)))) \subseteq int(f(\beta int(f^{-1}(A)))) \subseteq int(f(f^{-1}(A))) = int(A) = A$. Therefore $cl(\beta int(f^{-1}(A))) = f^{-1}(f(cl(\beta int(f^{-1}(A))))) \subseteq f^{-1}(A)$. Now $cl(int(f^{-1}(A))) \subseteq cl(\beta int(f^{-1}(A))) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFPCS in X and hence an IFβ^{**}GCS in X. Thus f is an IF contra β^{**}G continuous mapping.

Proposition 3.3.17 : Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping where X is an IFβ^{**}p$T_{1/2}$ space. Suppose that one of the following properties hold :

(i) $f(\beta cl(A)) \subseteq int(f(A))$ for each IFS A in X,

(ii) $\beta cl(f^{-1}(B)) \subseteq f^{-1}$ (int (B) for each IFS B in Y,

(iii) $f^{-1}(cl(B)) \subseteq \beta int(f^{-1}(B))$ for each IFS B in Y.

Then f is an IF contra β^{**}G continuous mapping.

Proof : (i) ⇒ (ii) : Let $B \subseteq Y$, then $f^{-1}(B)$ is an IFS in X. By hypothesis, $f(\beta cl(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. Now $\beta cl(f^{-1}(B)) \subseteq f^{-1}(f(\beta cl(f^{-1}(B)))) \subseteq f^{-1}(int(B))$.

(ii) ⇒ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. Now $f^{-1}(A) = f^{-1}(cl(A)) \subseteq \beta int(f^{-1}(A)) \subseteq f^{-1}(A)$, this implies $f^{-1}(A)$ is an IFβOS in X and hence an IFβ^{**}GOS in X. Therefore f is an IF contra β^{**}G continuous mapping.

Proposition 3.3.18 : Let $f : (X, \tau) \to (Y, \sigma)$ be a bijective mapping. Then f is an IF contra β^{**}G continuous mapping if $cl(f(A)) \subseteq f(\beta int(A))$ for every IFS A in X.

Proof : Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. By hypothesis, $cl(f(f^{-1}(A))) \subseteq f(\beta int(f^{-1}(A)))$. Since f is bijective, $f(f^{-1}(A)) = A$. Therefore $A= cl(A) = cl(f(f^{-1}(A))) \subseteq f(\beta int(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(\beta int(f^{-1}(A)))) =$

$\beta int(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFβOS in X and hence an IFβ^{**}GOS in X. Thus f is an IF contra β^{**}G continuous mapping.

Proposition 3.3.19 : If $f : (X, \tau) \to (Y, \sigma)$ is an IF contra β^{**}G continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IF continuous mapping then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF contra β^{**}G continuous mapping.

Proof : Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFOS in Y, since g is an IF continuous mapping. Since f is an IF contra β^{**}G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GCS in X. Therefore g o f is an IF contra β^{**}G continuous mapping.

Proposition 3.3.20 : If $f : (X, \tau) \to (Y, \sigma)$ is an IF contra β^{**}G continuous mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IF contra continuous mapping, then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IFβ^{**}G continuous mapping.

Proof : Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IF contra continuous mapping. Since f is an IF contra β^{**}G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GOS in X. Therefore g o f is an IFβ^{**}G continuous mapping.

Proposition 3.3.21 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF contra β^{**}G continuous mapping if $f^{-1}(\beta cl(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y.

Proof : Let $B \subseteq Y$ be an IFCS. Since every IFCS is an IFβCS, $\beta cl(B) = B$. Then by hypothesis, $f^{-1}(B) = f^{-1}(\beta cl(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B) = int(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X. Hence f is an IF contra continuous mapping. Then by Proposition 3.3.3, f is an IF contra β^{**}G continuous mapping.

3.4 Intuitionistic Fuzzy Almost β^{**} Generalized Continuous Mappings

In this section we have introduced intuitionistic fuzzy almost β^{**} generalized continuous mappings and investigated some of their properties.

Definition 3.4.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be an **intuitionistic fuzzy almost β^{**} generalized** (IF almost β^{**}G) **continuous mapping** if $f^{-1}(A)$ is an IFβ^{**}GCS in X for every IFRCS A of Y.

Example 3.4.2 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost β^{**}G continuous mapping.

Proposition 3.4.3 : Every IF continuous mapping is an IF almost β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IF continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X, by hypothesis. Since every IFCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF almost β^{**}G continuous mapping.

Example 3.4.4 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost β^{**}G continuous mapping but not an IF continuous mapping, since $G_3^c = \langle y, (0.5_u, 0.6_v), (0.3_u, 0.4_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = 1_\sim \neq f^{-1}(G_3^c)$.

Proposition 3.4.5 : Every IFS continuous mapping is an IF almost β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFS continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X, by hypothesis. Since every IFSCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF almost β^{**}G continuous mapping.

Example 3.4.6 : Let X = {a, b} and Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost β^{**}G continuous mapping but not an IF semi continuous mapping, since $G_3^c = \langle y, (0.5_u, 0.6_v), (0.3_u, 0.4_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) = 1_\sim \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.4.7 : Every IFP continuous mapping is an IF almost β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFP continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X, by hypothesis. Since every IFPCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF almost β^{**}G continuous mapping.

Example 3.4.8 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here, f is an IF almost β^{**}G continuous mapping but not an IFP continuous mapping, since the IFS $G_3^c = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$ is an IFCS in Y, but $f^{-1}(G_3^c) = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)\rangle$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3^c))) = G_2^c \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.4.9 : Every IFα continuous mapping is an IF almost β^{**}G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFα continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X by hypothesis. Since every IFαCS is an IFβ**GCS, $f^{-1}(V)$ is an IFβ**GCS in X. Hence f is an IF almost β**G continuous mapping.

Example 3.4.10 : Let X = {a, b} and Y = {u, v} and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost β**G continuous mapping but not an IFα continuous mapping, since $G_3^c = \langle y, (0.5_u, 0.6_v), (0.3_u, 0.4_v)\rangle$ is an IFCS in Y, but $f^{-1}(G_3^c)$ is not an IFαCS in X, as $cl(int(cl(f^{-1}(G_3^c)))) = 1_\sim \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.4.11 : Every IFβ continuous mapping is an IF almost β**G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFβ continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFβCS in X, by hypothesis. Since every IFβCS is an IFβ**GCS, $f^{-1}(V)$ is an IFβ**GCS in X. Hence f is an IF almost β**G continuous mapping.

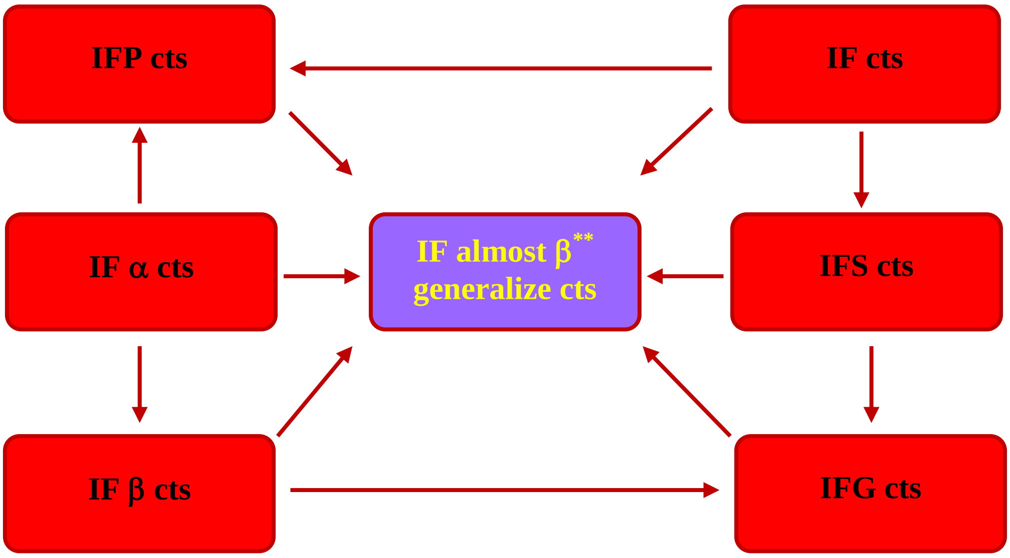
Example 3.4.12 : Let X = {a, b} and Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here, f is an IF almost β**G continuous mapping but not an IFβ continuous mapping, since the IFS $G_3^c = \langle y, (0.7_u, 0.8_v), (0.3_u, 0.2_v)\rangle$ is an IFCS in Y but $f^{-1}(G_3^c) = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$ is not an IFβCS in X, as $int(cl(int(f^{-1}(G_3^c)))) = 1_\sim \not\subseteq f^{-1}(G_3^c)$.

Proposition 3.4.13 : Every IFG continuous mapping is an IF almost β**G continuous mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFG continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGCS in X, by hypothesis. Since every IFGCS is an IFβ^{**}GCS, $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IF almost β^{**}G continuous mapping.

Example 3.4.14 : Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost β^{**}G continuous mapping but not an IFG continuous mapping as G_3^c is an IFCS in Y, but $f^{-1}(G_3^c)$ is not an IFGCS in X, as $cl(f^{-1}(G_3^c)) = 1_\sim \not\subseteq G_2$, whereas $f^{-1}(G_3^c) \subseteq G_2$.

From the above propositions and examples we have the following diagram. However the reverse implications are not true in general. In this diagram 'cts' means continuous mapping.



Proposition 3.4.15 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IF almost β^{**}G continuous mapping if and only if the inverse image of each IFROS in Y is an IFβ^{**}GOS in X.

Proof : Necessity : Let A be an IFROS in Y. Then A^c is an IFRCS in Y. Since f is an IF almost β^{**}G continuous mapping, $f^{-1}(A^c)$ is an IFβ^{**}GCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβ^{**}GOS in X.

Sufficiency : Let A be an IFRCS in Y. Then A^c is an IFROS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFβ^{**}GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF almost β^{**}G continuous mapping.

Proposition 3.4.16 : Let $p_{(\alpha,\beta)}$ be an IFP in X. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF almost β^{**}G continuous mapping if for every IFOS A in Y with $p_{(\alpha,\beta)} \in A$, there exists an IFOS B in X with $p_{(\alpha,\beta)} \in B$ such that $f^{-1}(A)$ is intuitionistic fuzzy dense in B.

Proof : Let A be an IFROS in Y. Then A is an IFOS in Y. Let $p_{(\alpha,\beta)} \in A$, then there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B$ and $cl(f^{-1}(A)) = B$, by hypothesis. Therefore $cl(f^{-1}(A))$ is also an IFOS in X and $int(cl(f^{-1}(A))) = cl(f^{-1}(A))$. Now $f^{-1}(A) \subseteq cl(f^{-1}(A)) = int(cl(f^{-1}(A))) \subseteq cl(int(cl(f^{-1}(A))))$. This implies $f^{-1}(A)$ is an IFβOS in X and hence an IFβ^{**}GOS in X. Thus f is an IF almost β^{**}G continuous mapping.

Proposition 3.4.17 : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. If $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$ for every IFS B in Y, then f is an IF almost β^{**}G continuous mapping.

Proof : Let $B \subseteq Y$ be an IFROS. By hypothesis, $f^{-1}(int(B)) \subseteq int(f^{-1}(B))$. Since B is IFROS, it is an IFOS in Y. Therefore int(B) = B. Hence $f^{-1}(B) = f^{-1}(int(B)) \subseteq int(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFOS in X and hence is an IFβ^{**}GOS in X. Thus f is an IF almost β^{**}G continuous mapping.

Proposition 3.4.18 : Let $f : X \rightarrow Y$ be a mapping. If $\alpha cl(f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B))$ for every IFS B in Y, then f is an IF almost β^{**}G continuous mapping.

Proof : Let B be an IFRCS in Y. Then, it is an IFαCS in Y and $\alpha cl(B) = B$. By hypothesis, $\alpha cl\ (f^{-1}(B)) \subseteq f^{-1}(\alpha cl(B))$. Hence $f^{-1}(B) = f^{-1}(\alpha cl(B)) \supseteq \alpha cl\ (f^{-1}(B)) \supseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFαCS in X and hence $f^{-1}(B)$ is an IFβ^{**}GCS in X. Thus f is an IF almost β^{**}G continuous mapping.

Proposition 3.4.19 : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFβ^{**}G continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is an IF continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is an IF almost β^{**}G continuous mapping.

Proof : Let V be an IFRCS in Z. Since every IFRCS is an IFCS, V is an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y, by hypothesis. Since f is an IFβ^{**}G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GCS in X. Hence g o f is an IF almost β^{**}G continuous mapping.

Definition 3.4.20 : Let A be an IFS in an IFTS (X, τ). Then the β^{**} generalized interior and β^{**} generalized closure of A are defined as

$\beta^{**}g\ int(A) = \cup\ \{G / G$ is an IFβ^{**}GOS in X and $G \subseteq A\}$ and
$\beta^{**}g\ cl(A) = \cap\ \{K / K$ is an IFβ^{**}GCS in X and $A \subseteq K\}$

It is to be noted that for any IFS A in (X, τ), we have $\beta^{**}g\ cl(A^c) = (\beta^{**}g\ int(A))^c$ and $\beta^{**}g\ int(A^c) = (\beta^{**}g\ cl(A))^c$.

Remark 3.4.21 : If an IFS A in an IFTS (X, τ) is an IFβ^{**}GCS in X, then $\beta^{**}g\ cl(A) = A$. But the converse may not be true in general, since intersection of IFβ^{**}GCSs need not be an IFβ^{**}GCS in X.

Remark 3.4.22 : If an IFS A in an IFTS (X, τ) is an IFβ^{**}GOS in X, then $\beta^{**}g\ int(A) = A$. But the converse may not be true in general, since union of IFβ^{**} GOSs need not be an IFβ^{**}GOS in X.

Proposition 3.4.23 : Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. If f is an IF almost β^{**}G continuous mapping, then $\beta^{**}g\ cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every IFβOS A in Y.

Proof : Let A be an IFβOS in Y. Then cl(A) is an IFRCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFβ^{**}GCS in X and $\beta^{**}g\ cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Now $\beta^{**}g\ cl(f^{-1}(A)) \subseteq \beta^{**}g(cl(f^{-1}\ cl(A))) = f^{-1}(cl(A))$.

Proposition 3.4.24 : Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. If f is an IF almost β^{**}G continuous mapping, then $\beta^{**}g\ cl(f^{-1}(cl(A))) = f^{-1}(cl(\beta^{**}g\ int(A)))$ for every IFβOS A in Y.

Proof : Let A be an IFβOS in Y. Then A is an IFβ^{**}GOS in Y and β^{**}g int(A) = A and cl(A) is an IFRCS in Y. By hypothesis, f^{-1}(cl(A)) is an IFβ^{**}GCS in X. Then β^{**}g cl(f^{-1}(cl(A))) = f^{-1}(cl(A)) = f^{-1}(cl(β^{**}g int(A))).

Proposition 3.4.25 : Let f : (X, τ) → (Y, σ) be a mapping where X is an IFβ^{**}p $T_{1/2}$ space. If f is an IF almost β^{**}G continuous mapping, then int(cl(int(f^{-1}(B)))) $\subseteq$ f^{-1}(β cl(B)) for every IFRCS B in Y.

Proof : Let B $\subseteq$ Y be an IFRCS. By hypothesis, f^{-1}(B) is an IFβ^{**}GCS in X. Since X is an IFβ^{**}p $T_{1/2}$ space, f^{-1}(B) is an IFPCS in X. Since every IFPCS is an IFβCS, we have f^{-1}(B) is an IFβCS in X. Therefore βcl(f^{-1}(B)) = f^{-1}(B). Now int(cl(int(f^{-1}(B)))) $\subseteq$ f^{-1}(B) $\cup$ int(cl(int(f^{-1}(B)))) $\subseteq$ βcl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(βcl(B)). Hence int(cl(int(f^{-1}(B))) $\subseteq$ f^{-1}(βcl(B)).

Proposition 3.4.26 : Let f : (X, τ) → (Y, σ) be a mapping where X is an IFβ^{**}p $T_{1/2}$ space. If f is an IF almost β^{**}G continuous mapping, then f^{-1}(βint(B)) $\subseteq$ cl(int(cl(f^{-1}(B)))) for every IFROS B in Y.

Proof : This proposition can be easily proved by taking complement in Proposition 3.4.25.

Proposition 3.4.27 : A mapping f : (X, τ) → (Y, σ) is an IF almost β^{**}G continuous mapping then f^{-1}(A) $\subseteq$ β^{**}g int(f^{-1}(int(cl(A)))) for an IFPOS A of Y.

Proof : Let A be an IFPOS in Y. Then A $\subseteq$ int(cl(A)) and int(cl(A)) is an IFROS in Y. Since f is an IF almost β^{**}G continuous mapping, f^{-1}(int(cl(A))) is an IFβ^{**}GOS in X and hence we obtain that f^{-1}(A) $\subseteq$ f^{-1}(int(cl(A))) = β^{**}g int(f^{-1}(int(cl(A)))).

3.5 Intuitionistic Fuzzy Almost Contra β^{**} Generalized Continuous Mappings

In this section we have introduced intuitionistic fuzzy almost contra β^{**} generalized continuous mappings and studied some of their properties.

Definition 3.5.1 : A mapping f : (X, τ) $\to$ (Y, σ) is said to be an **intuitionistic fuzzy almost contra β^{**} generalized** (IF almost contra β^{**}G) **continuous mapping** if $f^{-1}(A)$ is an IFβ^{**}GCS in X for every IFROS A in Y.

Example 3.5.2 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IF almost contra β^{**}G continuous mapping in (X, τ).

Proposition 3.5.3 : Every IF contra continuous mapping is an IF almost contra β^{**}G continuous mapping but not conversely in general.

Proof : Let A $\subseteq$ Y be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous mapping.

Example 3.5.4 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost contra β^{**}G continuous mapping, but not an IF contra continuous mapping, since G_3 is an IFOS in Y but $f^{-1}(G_3)$ is not an IFCS in X, as $cl(f^{-1}(G_3)) = G_1^c \neq f^{-1}(G_3)$.

Proposition 3.5.5 : Every IF contra semi continuous mapping is an IF almost contra β^{**}G continuous mapping but not conversely in general.

Proof : Let A $\subseteq$ Y be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFSCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous mapping.

Example 3.5.6 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost contra β^{**}G continuous mapping but not an IF contra semi continuous mapping, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is an IFOS in Y but $f^{-1}(G_3)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3))) = G_1 \not\subseteq f^{-1}(G_3)$.

Proposition 3.5.7 : Every IF contra pre continuous mapping is an IF almost contra β^{**}G continuous mapping but not conversely in general.

Proof : Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFPCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous mapping.

Example 3.5.8 : Let X = {a, b} and Y = {u, v}. Let $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)\rangle$ and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IF almost contra β^{**}G continuous mapping but not an IF contra pre continuous mapping, since $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3))) = G_2^c \not\subseteq f^{-1}(G_2)$.

Proposition 3.5.9 : Every IF contra α continuous mapping is an IF almost contra β^{**}G continuous mapping but not conversely in general.

Proof : Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFαCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous mapping.

Example 3.5.10 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then

$\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost contra β^{**}G continuous mapping but not an IF contra α continuous mapping, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFαCS in (X, τ), as $cl(int(cl(f^{-1}(G_3))) = G_1^c \not\subseteq f^{-1}(G_3)$.

Proposition 3.5.11: Every IF contra β continuous mapping is an IF almost contra β^{**}G continuous mapping but not conversely in general.

Proof : Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFβCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous mapping.

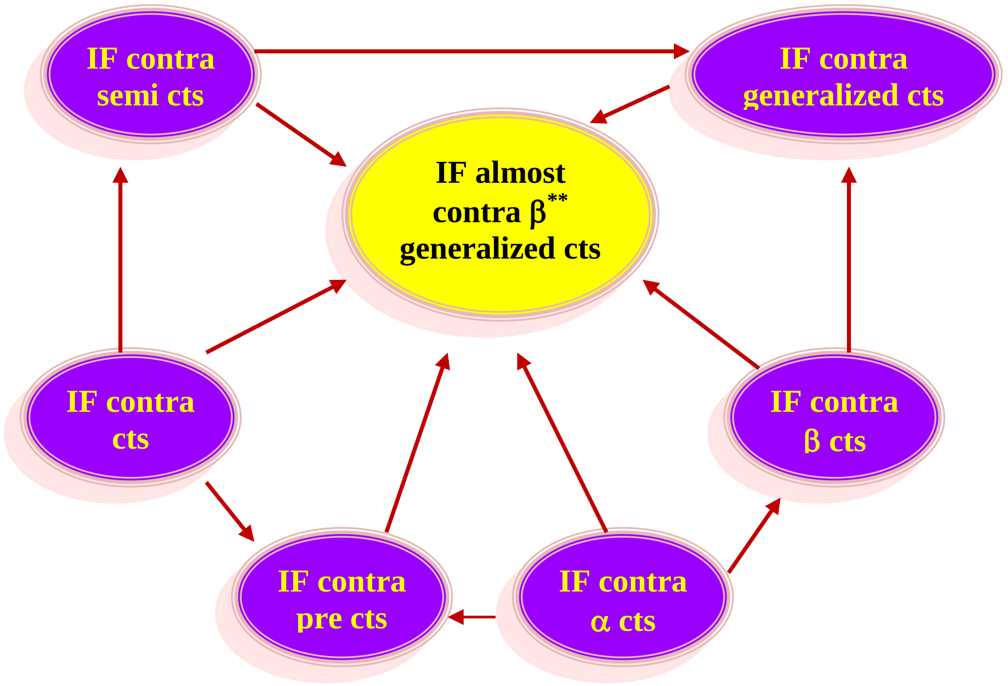
Example 3.5.12 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Here f is an IF almost contra β^{**}G continuous mapping but not an IF contra β continuous mapping, since $G_2 = \langle y, (0.6_u, 0.7_v), (0.4_u, 0.3_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_2)$ is not an IFβCS in (X, τ), as $int(cl(int(f^{-1}(G_2)))) = 1_\sim \not\subseteq f^{-1}(G_2)$.

Proposition 3.5.13 : Every IF contra generalized continuous mapping is an IF almost contra β^{**}G continuous mapping but not conversely in general.

Proof : Let $A \subseteq Y$ be an IFROS. Since every IFROS is an IFOS, A is an IFOS in Y. Then $f^{-1}(A)$ is an IFGCS in X, by hypothesis. Hence $f^{-1}(A)$ is an IFβ^{**}GCS in X. Therefore f is an IF almost contra β^{**}G continuous mapping.

Example 3.5.14 : Let X = {a, b}, Y = {u, v}, $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Here, f is an IF almost contra β^{**}G continuous mapping but not an IF contra generalized continuous mapping, since $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$ is an IFOS in Y, but $f^{-1}(G_3)$ is not an IFGCS in (X, τ), as $cl(f^{-1}(G_3)) = G_1^c \not\subseteq G_1$, whereas $f^{-1}(G_3) \subseteq G$.

The interrelation between intuitionistic fuzzy almost contra β^{**}G continuous mapping with other intuitionistic fuzzy contra continuity is depicted below. In this diagram ‘cts’ means continuous mapping.



The reverse implications are not true in general in the above diagram.

Proposition 3.5.15 : If $f : (X, \tau) \to (Y, \sigma)$ is a mapping, then the following are equivalent :

(i) f is an IF almost contra β^{**}G continuous mapping,

(ii) $f^{-1}(A) \in IF\beta^{**}GO(X)$ for every $A \in IFRC(Y)$.

Proof : (i) $\Rightarrow$ (ii) : Let A be an IFRCS in Y. Then A^c is an IFROS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFβ^{**}GCS in X. Therefore $f^{-1}(A)$ is an IFβ^{**}GOS in X as $f^{-1}(A^c) = f^{-1}(A)^c$.

(iii) $\Rightarrow$ (i) : Let A be an IFROS in Y. Then A^c is an IFRCS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFβ^{**}GOS in X. Therefore $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF almost contra β^{**}G continuous mapping.

Proposition 3.5.16 : If $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFβ^{**}G continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \delta)$ is an IF almost contra continuous mapping, then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ is an IF almost contra β^{**}G continuous mapping.

Proof : Let A be an IFROS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IFβ^{**}G continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFβ^{**}GCS in X. Hence $g \circ f$ is an IF almost contra β^{**}G continuous mapping.

Proposition 3.5.17 : If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a mapping, then the following are equivalent :

(i) f is an IF almost contra β^{**}G continuous mapping,

(ii) $f^{-1}(A) \in$ IFβ^{**}GC(X) for every $A \in$ IFRO(Y),

(iii) $f^{-1}(int(cl(G))) \in$ IFβ^{**}GC(X) for every IFOS $G \subseteq Y$.

Proof : (i) $\Leftrightarrow$ (ii) is obvious from the Definition 3.5.1.

(ii) $\Rightarrow$ (iii) Let G be any IFOS in Y. Then int(cl(G)) is an IFROS in Y. By hypothesis, $f^{-1}(int(cl(G)))$ is an IFβ^{**}GCS in X. Hence $f^{-1}(int(cl(G))) \in$ IFβ^{**}GC(X).

(iii) $\Rightarrow$ (i) Let A be any IFROS in Y. Then A is an IFOS in Y. By hypothesis, we have $f^{-1}(int(cl(A))) \in$ IFβ^{**}GC(X). That is $f^{-1}(A) \in$ IFβ^{**}GC(X), since int(cl(A)) = A. Hence f is an IF almost contra β^{**}G continuous mapping.

Proposition 3.5.18 : Let f : (X, τ) → (Y, σ) be a mapping and let $f^{-1}(A)$ be an IFRCS in X for every IFROS A in Y. Then f is an IF almost contra β^{**}G continuous mapping.

Proof : Let A be an IFROS in Y. By hypothesis, $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IFβ^{**}GCS, $f^{-1}(A)$ is an IFβ^{**}GCS in X. Hence f is an IF almost contra β^{**}G continuous mapping.

Proposition 3.5.19 : Let f : (X, τ) → (Y, σ) be an IF almost contra β^{**}G continuous mapping and X, an IFβ^{**}p$T_{1/2}$ space. Then f is an IF contra pre continuous mapping.

Proof : Let B be an IFOS in Y. By hypothesis, $f^{-1}(B)$ is an IFβ^{**}GCS in X. Since X is an IFβ^{**}p$T_{1/2}$ space, $f^{-1}(B)$ is an IFPCS in X. Hence f is an IF contra pre continuous mapping.

3.6 Intuitionistic Fuzzy β^{**} Generalized Irresolute Mappings

In this section we have introduced intuitionistic fuzzy β^{**} generalized irresolute mappings and studied some of their properties.

Definition 3.6.1 : A mapping f : (X, τ) → (Y, σ) is an **intuitionistic fuzzy β^{**} generalized** (IFβ^{**}G) **irresolute mapping** if $f^{-1}(V)$ is an IFβ^{**}GCS in (X, τ) for every IFβ^{**}GCS V of (Y, σ).

Example 3.6.2 : Let X = {a, b} and Y = {u, v}. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively, where $G_1 = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)\rangle$, $G_2 = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b)\rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v)\rangle$. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFβ^{**}G irresolute mapping.

Proposition 3.6.3 : Let f : (X, τ) → (Y, σ) be an IFβ^{**}G irresolute mapping, then f is an IFβ^{**}G continuous mapping but not conversely.

Proof : Let V be any IFCS in Y. Then V is an IFβ^{**}GCS and by hypothesis $f^{-1}(V)$ is an IFβ^{**}GCS in X. Hence f is an IFβ^{**}G continuous mapping.

Example 3.6.4 : Let X = {a, b} and Y = {u, v} and $G_1 = \langle x, (0.6_a, 0.8_b), (0.2_a, 0.1_b)\rangle$, $G_2 = \langle x, (0.3_a, 0.3_b), (0.2_a, 0.2_b)\rangle$ and $G_3 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v.

Then f is an IFβ^{**}G continuous mapping but not an IFβ^{**}G irresolute mapping, since the IFS $A = \langle y, (0.5_u, 0.3_v), (0.2_u, 0.1_v)\rangle$ is an IFβ^{**}GCS in Y and its inverse $f^{-1}(A)$ is not an IFβ^{**}GCS in X, as $f^{-1}(A) = \langle x, (0.5_a, 0.3_b), (0.2_a, 0.1_b)\rangle \subseteq G_1$, but $\text{int}(\text{cl}(f^{-1}(A))) \cap \text{cl}(\text{int}(f^{-1}(A))) = 1_\sim \not\subseteq G_1$.

Proposition 3.6.5 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IFβ^{**}G irresolute mapping if and only if the inverse image of each IFβ^{**}GOS in Y is an IFβ^{**}GOS in X.

Proof : Straight forward.

Proposition 3.6.6 : The composition of two IFβ^{**}G irresolute mappings is an IFβ^{**}G irresolute mapping.

Proof : Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be any two IFβ^{**}G irresolute mappings. Let V be an IFβ^{**}GCS in Z. Then $g^{-1}(V)$ is an IFβ^{**}GCS in Y, by hypothesis. Since f is an IFβ^{**}G irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GCS in X. Hence g o f is an IFβ^{**}G irresolute mapping.

Proposition 3.6.7 : Let $f : (X, \tau) \to (Y, \sigma)$ be an IFβ^{**}G irresolute mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IFβ^{**}G continuous mapping, then $g \text{ o } f : (X, \tau) \to (Z, \delta)$ is an IFβ^{**}G continuous mapping.

Proof : Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFβ^{**}GCS in Y. Since f is an IFβ^{**}G irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GCS in X. Hence g o f is an IFβ^{**}G continuous mapping.

Proposition 3.6.8 : Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IFβ^{**}p$T_{1/2}$ spaces:

(i) f is an IFβ^{**}G irresolute mapping,

(ii) $f^{-1}(B)$ is an IFβ^{**}GOS in X for each IFβ^{**}GOS B in Y,

(iii) $f^{-1}(\mathrm{pint}(B)) \subseteq \mathrm{pint}(f^{-1}(B))$ for each IFS B of Y,

(iv) $\mathrm{pcl}((f^{-1}(B)) \subseteq f^{-1}((\mathrm{pcl}(B))$ for each IFS B of Y.

Proof : (i) ⇔ (ii) is obvious, since $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) ⇒ (iii) Let B be any IFS in Y and $\mathrm{pint}(B) \subseteq B$. Also $f^{-1}(\mathrm{pint}(B)) \subseteq f^{-1}(B)$. Since pint(B) is an IFPOS in Y, it is an IFβ^{**}GOS in Y. Therefore $f^{-1}(\mathrm{pint}(B))$ is an IFβ^{**}GOS in X, by hypothesis. Since X is an IFβ^{**}p$T_{1/2}$ space, $f^{-1}(\mathrm{pint}(B))$ is an IFPOS in X. Hence $f^{-1}(\mathrm{pint}(B)) = \mathrm{pint}(f^{-1}(\mathrm{pint}(B))) \subseteq \mathrm{pint}(f^{-1}(B))$.

(iii) ⇒ (iv) is obvious by taking complement in (iii).

(iv) ⇒ (i) Let B be an IFβ^{**}GCS in Y. Since Y is an IFβ^{**}p$T_{1/2}$ space, B is an IFPCS in Y and pcl(B) = B. Hence $f^{-1}(B) = f^{-1}(\mathrm{pcl}(B)) \supseteq \mathrm{pcl}(f^{-1}(B))$, by hypothesis. But $f^{-1}(B) \subseteq \mathrm{pcl}(f^{-1}(B))$. Therefore $\mathrm{pcl}(f^{v1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFPCS and hence it is an IFβ^{**}GCS in X.Thus f is an IFβ^{**}G irresolute mapping.

Proposition 3.6.9 : Let f : (X, τ) → (Y, σ) be an IFβ^{**}G irresolute mapping. Then $f^{-1}(B) \subseteq \mathrm{pint}(f^{-1}(\mathrm{int}(\mathrm{cl}(B)))$ for every IFβ^{**}GOS B in Y, if X and Y are IFβ^{**}p$T_{1/2}$ spaces.

Proof : Let B be an IFβ^{**}GOS in Y. Then by hypothesis, $f^{-1}(B)$ is an IFβ^{**}GOS in X. Since X is an IFβ^{**}p$T_{1/2}$ space, $f^{-1}(B)$ is an IFPOS in X. Therefore

$\mathrm{pint}(f^{-1}(B)) = f^{-1}(B)$. Since Y is an IF$\beta^{**}pT_{1/2}$ space, B is an IFPOS in Y and $B \subseteq \mathrm{int}(\mathrm{cl}(B))$. Now $f^{-1}(B) = \mathrm{pint}(f^{-1}(B)) \subseteq \mathrm{pint}(f^{-1}(\mathrm{int}(\mathrm{cl}(B))))$.

Proposition 3.6.10 : If $f : (X, \tau) \to (Y, \sigma)$ is an IFβ^{**}G irresolute mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IF contra continuous mapping, then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF contra β^{**}G continuous mapping.

Proof : Let V be an IFOS in Z. Then $g^{-1}(V)$ is an IFCS in Y, since g is an IF contra continuous mapping. As every IFCS is an IFβ^{**}GCS, $g^{-1}(V)$ is an IFβ^{**}GCS in Y. Since f is an IFβ^{**}G irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GCS in X. Therefore $g \circ f$ is an IF contra β^{**}G continuous mapping.

Proposition 3.6.11 : If $f : (X, \tau) \to (Y, \sigma)$ is an IFβ^{**}G irresolute mapping and $g : (Y, \sigma) \to (Z, \delta)$ is an IFβ^{**}G continuous mapping, then $g \circ f : (X, \tau) \to (Z, \delta)$ is an IF almost β^{**}G continuous mapping.

Proof : Let V be an IFRCS in Z. Since every IFRCS is an IFCS, V is an IFCS in Z. Therefore $g^{-1}(V)$ is an IFβ^{**}GCS in Y, by hypothesis. Since f is an IFβ^{**}G irresolute mapping, $f^{-1}(g^{-1}(V))$ is an IFβ^{**}GCS in X. Hence $g \circ f$ is an IF almost β^{**}G continuous mapping.