



Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – June 2021
IV Semester

Class : II M.Sc.
Major : Mathematics

Time: 3 Hours
Max. Marks: 100

17MMAC22 Functional Analysis

Part A
Choose the Correct Answer

10x1=10

- If N is a normed linear space, then the norm $\| \cdot \| : N \rightarrow \mathbb{R}$ is a
 - discontinuous function
 - Constant function
 - continuous function
 - None of the above
- If N is a normed linear space, then for all $x, y \in N$
 - $|\|x\| - \|y\|| \leq \|x - y\|$
 - $|\|x\| - \|y\|| \geq \|x - y\|$
 - $|\|x\| - \|y\|| = \|x - y\|$
 - None of the above
- If a one-to-one linear transformation T of a Banach space B onto itself is continuous, then its inverse T^{-1} is
 - continuous
 - Oscillatory
 - discontinuous
 - Constant
- If T is an operator on a Banach space B , then
 - $(T^*)^{-1} \neq (T^{-1})^*$
 - $(T^*)^{-1} = (T^{-1})^*$
 - $(T^*)^{-1} \leq (T^{-1})^*$
 - $(T^*)^{-1} \geq (T^{-1})^*$
- If M is a linear subspace of a Hilbert space H , then M is closed if and only if
 - $M = M^\perp$
 - $M \neq M^\perp$
 - $M = M^\perp$
 - $M = M^{\perp\perp}$
- If x and y are any two vectors in a Hilbert space H , then
 - $|(x, y)| = \|x\| \|y\|$.
 - $|(x, y)| \geq \|x\| \|y\|$.
 - $|(x, y)| \leq \|x\| \|y\|$.
 - None of the above
- If A_1 and A_2 are self-adjoint operators on the Hilbert space H , then their product $A_1 A_2$ is self-adjoint if
 - $A_1 A_2 = A_2 A_1$
 - $A_1 A_2 \neq A_2 A_1$
 - $A_1 A_2 \geq A_2 A_1$
 - $A_1 A_2 \leq A_2 A_1$
- If N is a normal operator on the Hilbert space H , then
 - $\|N^2\| \neq \|N\|^2$
 - $\|N^2\| \geq \|N\|^2$
 - $\|N^2\| = \|N\|^2$
 - $\|N^2\| \leq \|N\|^2$
- A closed linear subspace M of the Hilbert space H is invariant under an operator if
 - M^\perp is invariant under T^*
 - M^\perp is variant under T^*
 - $M^\perp =$
 - None of the above.
- If the operator T is normal, then the eigen space M_i 's are
 - pairwise linear
 - pairwise orthogonal
 - null
 - pairwise non-orthogonal

Part B
Answer ALL questions

5x6=30

- 11.a. If N is a normed linear space, then establish that
 $|\|x\| - \|y\|| \leq \|x - y\|$ for all $x, y \in N$.
(or)
- 11.b. Prove : If N and N' are normed linear spaces and T is a linear transformation of N into N' , then the following conditions on T are all equivalent to one another :
(1). T is continuous
(2). T is continuous at the origin, in the sense that $x_n \rightarrow 0 \Rightarrow T(x_n) \rightarrow 0$.
- 12.a. If N is a normed linear space, then prove that the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.
(or)
- 12.b. State and prove the closed graph theorem.
- 13.a. Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
(or)
- 13.b. If M is a proper closed linear subspace of a Hilbert space H , then prove that there exists a non-zero vector z_0 in H such that $z_0 \perp M$.
- 14.a. If T is an operator on a Hilbert space H , for which $(Tx, x) = 0$ for all x in H then prove that $T = 0$.
(or)
- 14.b. If T is an operator on H , then prove that T is normal if and only if its real and imaginary parts commute.
- 15.a. If P is the projection on a closed linear subspace M of the Hilbert space H , then prove that M is invariant under an operator $T \Leftrightarrow TP = PTP$.
(or)
- 15.b. If the operator T is normal, then establish that each eigen space M_i reduces T .

Part C
Answer ALL questions

5x12=60

- 16.a. If M is a closed linear subspace of a normed linear space N and the norm of a coset $x + M$ in the quotient space N/M is defined by
 $\|x + M\| = \inf \{ \|x + m\| : m \in M \}$, then prove that
(i). N/M is a normed linear space ;
(ii). if N is a Banach space, then N/M is also a Banach space.
(or)
- 16.b. State and prove the Hahn - Banach Theorem.
- 17.a. State and establish the open mapping theorem.
(or)
- 17.b. State and establish the Uniform Boundedness theorem.
- 18.a. If M is a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^\perp$
(or)
- 18.b. If $\{e_i\}$ is an orthonormal set in a Hilbert space H , then prove that
 $\sum |(x, e_i)|^2 \leq \|x\|^2$, for every vector x in H .
- 19.a. If H is a Hilbert space and f is an arbitrary functional in H^* , then prove that there exists a unique vector y in H such that
 $f(x) = (x, y)$, for every x in H .
(or)
- 19.b. If T is an operator on a Hilbert space H , then prove that the following conditions are equivalent to one another :
 $T^*T = I$;
 $(Tx, Ty) = (x, y)$ for all x and y ;
 $\|Tx\| = \|x\|$ for all x .

20.a. If P_1, P_2, \dots, P_n , are the projections on closed linear subspaces M_1, M_2, \dots, M_n of the Hilbert space H , then prove that $P = P_1 + P_2 + \dots + P_n$, is a projection \Leftrightarrow the P_i 's are pairwise orthogonal and P is the projection on $M = M_1 + M_2 + \dots + M_n$.

(or)

20.b.(i). If T is normal, then prove that x is an eigen vector of T with eigen value λ if and only if x is an eigen vector of T^* with eigen value $\bar{\lambda}$.

(ii). If the operator T is normal, then prove that the eigen spaces M_i 's span the Hilbert space H .
