Mathematics

# Avinashilingam Institute for Home Science and Higher Education for Women

**(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)**
**Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B**
**Coimbatore - 641 043, Tamil Nadu, India**

## Bachelor's Degree Examination – March 2021
## I Semester

**Class : I UG** **Time : 3 Hours**
**Major : Special Education and Mathematics** **Max. Marks: 100**

### 18BSMC02 Trigonometry

**Part A** **10 x 1 = 10**
**Choose the Correct Answer**

1. The coefficient of $\cos^n \theta$ in the expansion of $\cos n\theta$ is
   a. 1 b. n c. $2^{n-1}$ d. $\frac{n(n+1)}{2}$

2. $\frac{\sin 3\theta}{\sin \theta} =$
   a. $3 - 4\sin^2 \theta$ b. $3 - 4\cos^2 \theta$ c. $3\cos^2 \theta \sin \theta$ d. $-\cos \theta \ \sin^2 \theta$

3. If $x = \cos\theta + i \sin\theta$, then $x^n + \frac{1}{x^n} =$
   a. $2\sin n\theta$ b. $2\,i \sin n\theta$ c. $2\cos n\theta$ d. $2\,i \cos n\theta$

4. The series $1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} \cdots \infty$ equals
   a. $\sin\theta$ b. $\cos\theta$ c. $\tan\theta$ d. $\sec\theta$

5. $\cosh^2 x + \sinh^2 x =$
   a.1 b. -1 c. $\sinh 2x$ d. $\cosh 2x$

6. In terms of logarithmic functions, $\tanh^{-1} x =$
   a. $\log\left(x + \sqrt{x^2+1}\right)$ b. $\log\left(x + \sqrt{x^2-1}\right)$ c. $\log\left(x - \sqrt{x^2+1}\right)$ d. $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

7. If $\log(x + iy) = \alpha + i\beta$, then $y =$
   a. $e^\alpha \cos\beta$ b. $e^\alpha \sin\beta$ c. $\alpha^2 + \beta^2$ d. $\tan^{-1}\left(\frac{\beta}{\alpha}\right)$

8. If $\mathrm{Log}(1 - i) = \alpha + i\beta$, then$\alpha =$
   a. 0 b. $\frac{3\pi}{4}$ c. $\log 2$ d. $\frac{1}{2}\log 2$

9. $\cos\alpha + \cos\left(\alpha + \frac{2\pi}{n}\right) + \cos\left(\alpha + \frac{4\pi}{n}\right) + \cdots$to n terms is
   a. 0 b. 1 c. n d. ∞

10. Which of the following is incorrect?
    a. $\sin(i\theta) = i\sinh\theta$ b. $\cos(i\theta) = i\cosh\theta$ c. $\tan(i\theta) = i\tanh\theta$ d. $\tanh(i\theta) = i\tan\theta$

**Part B** **5 x 6 = 30**

**Answer ALL questions**

**Each answer should not exceed 400 words or two pages**

11.a. Express $\cos 8\theta$ in terms of $\sin\theta$.

(or)

11.b. Prove that $\sin\frac{\pi}{5}\cdot\sin\frac{2\pi}{5}\cdot\sin\frac{3\pi}{5}\cdot\sin\frac{4\pi}{5}=\frac{5}{16}$.

12.a. Expand $\sin^4\theta\cos^2\theta$ in a series of cosines of multiples of $\theta$.

(or)

12.b. Evaluate $\lim_{x\to 0}\frac{\tan x-\sin x}{\sin^3 x}$.

13.a. If $\cosh u=\sec\theta$, show that $u=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$.

(or)

13.b. If$\sin(A+iB)=x+iy$, prove that

(i)$\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}=1$ , (ii)$\frac{x^2}{\cosh^2 B}+\frac{y^2}{\sinh^2 B}=1$.

14.a. Find the value of $\mathrm{Log}(1+i)$.

(or)

14.b. Reduce $(\alpha+i\beta)^{x+iy}$ to the form $A+iB$.

15.a. Find the sum of the series $\mathrm{cosec}\,\theta+\mathrm{cosec}\,2\theta+\mathrm{cosec}\,2^2\theta+\cdots+\mathrm{cosec}\,2^{n-1}\theta$.

(or)

15.b. Sum the series $\cosh x+\cosh(x+y)+\cosh(x+2y)+\cdots$to $n$ terms.

**Part C** **5 x 12 = 60**

**Answer ALL questions**

**Each answer should not exceed 800 words or four pages**

16.a. Prove that the equation $\frac{ah}{\cos\theta}-\frac{bk}{\sin\theta}=a^2-b^2$ has four roots and that the sum of the four values of $\theta$ which satisfy it is equal to an odd multiple of $\pi$ radians.

(or)

16.b. Prove that $\tan\frac{\pi}{11}\cdot\tan\frac{2\pi}{11}\cdot\tan\frac{3\pi}{11}\cdot\tan\frac{4\pi}{11}\cdot\tan\frac{5\pi}{11}=\sqrt{11}$.

17.a. Expand $\cos^6\theta$ and $\cos^5\theta$ in series of cosines of multiples of $\theta$.

(or)

17.b. Prove that if $l$ is the length of the chord of a circular arc, $l_1$the chord of two – thirds of the arc and $l_2$ the chord of one – third of the arc, the length of the arc is approximately $\frac{1}{10}(l-9l_1+45l_2)$ and that the approximation is correct to the order of $\theta^6$, where $\theta$ is the angle which the arc subtends at its centre.

18.a. If $\tanh x=\sin\theta$, show that $\sinh x=\tan\theta$ and $\cosh x=\sec\theta$.

(or)

18.b. Separate into real and imaginary parts $\tan^{-1}(x+iy)$.

19.a. If $\log\sin(\theta+i\phi)=L+iB$, prove that $2e^{2L}=\cosh 2\phi-\cos 2\theta$.

(or)

19.b. Find the values of (i) $\mathrm{Log}(1+i)^i$ (ii) $\mathrm{Log}(1-i)^{1+i}$.

20.a. If $S_n$ denotes the sum of the series $\sin\alpha+\sin(\alpha+\beta)+\cdots+\sin(\alpha+\overline{n-1}\beta)$, then prove that $S_n=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha+\frac{n-1}{2}\beta\right)$.

(or)

20.b. Sum the series $\sinh x+\sinh(x+y)+\sinh(x+2y)+\cdots$ to n terms.

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