

**NEW APPROACH ON TRANSPORTATION PROBLEMS  
IN LINEAR AND NON LINEAR METHODS**

**VAICIKA V  
(19PMA017)**

THESIS SUBMITTED TO THE  
AVINASHILINGAM INSTUTE FOR HOME SCIENCE AND  
HIGHEREDUCATION FOR WOMEN, COIMBATORE - 641 043

IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR  
THE DEGREE OF  
**MASTER OF SCIENCE IN MATHEMATICS**

**MAY – 2021**

**NEW APPROACH ON TRANSPORTATION PROBLEMS  
IN LINEAR AND NON LINEAR METHODS**

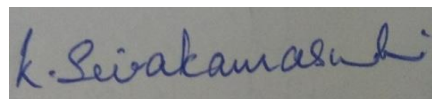
**VAICIKA V  
(19PMA017)**

THESIS SUBMITTED TO THE  
AVINASHILINGAM INSTUTE FOR HOME SCIENCE AND  
HIGHEREDUCATION FOR WOMEN, COIMBATORE - 641 043

IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR  
THE DEGREE OF  
**MASTER OF SCIENCE IN MATHEMATICS**



**Signature of the Supervisor**



**Signature of the Head of the  
Department**

## ACKNOWLEDGEMENT

First and foremost, I extremely thankful to the **LORD ALMIGHTY** for his graces and blessings showered on me.

I am extremely thankful to **Dr.S.P.THYAGARAJAN**, Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for giving me an opportunity to pursue my research in this esteemed institution.

I wish to express my profound gratefulness to **Dr. PREMAVATHY VIJAYAN**, Vice-Chancellor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her worthy encouragement and for providing all the necessary resources.

I like to thank **Dr. S. KOWSALYA**, Registrar, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for granting permission to carry out my research in this institution.

I express my sincere thanks to **Dr. K.UDAYA CHANDRIKA**, Professor of Mathematics, Dean, School of physical sciences and Computational Sciences, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her excellent support, unflinching encouragement and guidance during the course of my study.

I express my heartfelt thanks to **Dr. P. JEYALAKSHMI**, Former Professor and Head, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, for her excellent support, guidance during the course of my study.

I express my heartfelt thanks to **Dr. K.SIVAKAMASUNDARI**, Professor and Head, Department of Mathematics, Avinashilingam Institute for Home Science and

Higher Education for Women, Coimbatore, for her excellent support, guidance during the course of my study.

I express my heartfelt thanks to **Dr. S. PREMA**, Assistant Professor (Temporary), Department of Mathematics and my thesis supervisor, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore , for her support, inspiring guidance, innovation ideas, critical suggestions and constant encouragement throughout the work and guidance during the course of investigation.

I would like to express my sincere thanks to all the **STAFF MEMBERS OF THE DEPARTMENT OF MATHEMATICS** who were responsible for the good finish of this dissertation.

I own my special thanks to my **BELOVED PARENTS, LOVING SISTERS, BROTHERS, FRIENDS AND ALSO THE GRACEFUL RELATIVES**, who helped me by providing full strength, support and encouragement to complete my project successfully

## CONTENTS

Chapter	Title	Page No.
1	INTRODUCTION	
	1.1 Purpose of Work Study	07
2	REVIEW OF LITERATURE	
	2.1 Transportation problem	13
	2.2 Linear programming	15
	2.3 Non-Linear programming	16
3	OFSTF METHOD - AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEM	
	3.1 Transport Problem through OFSTF Method	18
	3.2 OFSTF Origin Method	20
	3.3 OFSTF First Quadrant Method ( +,+)	24
	3.4 OFSTF Second Quadrant Method ( -,+)	29
	3.4 OFSTF Third Quadrant Method ( -,-)	33
4	MDMA METHOD - AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEM	
	4.1 Transport Problem through MDMA Method	40
5	COMPARATIVE STUDY ON MDMA METHOD WITH OFSTF METHOD IN TRANSPORTATION PROBLEM	49
6	Conclusion	58
7	Bibliography	59

# **CHAPTER 1**

## **INTRODUCTION AND DESIGN OF THE STUDY**

## INTRODUCTION

The term operations research was coined in 1940 by McCloskey and Trefe then in a small town of Bawdsey in England. It is a science that came into existence in a military context. During World War II, the military management of UK called in scientists from various disciplines and organized them into teams to assist it in solving strategic and tactical problems relating to air and land defense of the country. They were required to formulate specific proposals and plans for aiding the military commands to arrive at decision an optimal utilization of scarce military resources and efforts and also to implement the decisions effectively. This new approach to the systematic and scientific study of the operations of the system was called operations Research (OR), or operational research. Hence OR can be termed as ‘an art of winning war without actually fighting it.’ Swarup K ., Gupta P.K., Mohan M.,(2006). Operations Research and Taha.H.A.(2004) Operations Research –Introduction are some the text books we are using for studying OR.

Operation Research is a relatively new discipline .The contents and the boundaries of the OR are not yet fixed. Therefore, to give a formal definition of the term operation Research is a difficult task. The OR starts when mathematical and quantitative techniques are used to substantiate the decision making. In our daily life we make the decisions even without noticing them. The decisions are taken simply by common sense, judgment and expertise without using any mathematical or any other model in simple situations. But the decision we are concerned here with are complex and heavily responsible. Example are public transportation network planning in a city having its own layout of factories ,residential blocks or finding the appropriate product mix when there exists a large number of products with difference profit contribution and production requirement etc. Hamdy, A.T.(2007) Operations Research.

Operation Research tools are not from any one discipline Operation Research takes tools from different discipline such as mathematics, statistics, economics, psychology, engineering etc. and combines these tools to make a new set of knowledge for decision making. today , O.R. become a professional discipline

which deals with the application of scientific methods for making decision , and especially to the allocation of scarce resources. The main purpose of O.R. is to provide a rational basis for decisions making in the absence of complete information , because the system composed of human, machine, and procedures in Operations Research and Sharma J.K.(2005) have developed Operations Research for theory and applications.

There is a great scope for economists, statistics, administrators and the technicians working as a team to solve problems of defence by using the OR approach Besides this, OR is useful in the various other important fields like Agriculture, Finance, Industry, Making and Personnel Management, Production management ,Research and development etc. Sharma J.K. (2005), Operation Research and shenoy G.V., Srivastava U.K. and Sharma S.C.(1991) Operation Research Management .

In order to survive in today's competitive market, an industry must be aware of the latest development brought by continuous application of new technology and methods used in production. Also it must continuously strive for improvement in the efficiency of its production and must consistently aim at producing better quality goods at lower price than its competitors. The performance of an industry can be improved by adopping the following two approaches:

1. By improving the process of manufactures that is by adopting technology, by developing better machines and equipment.
2. By improving the operation of existing facilities, both plant and human resources.

The first approach, called new technology development is a long term approach which deals with extensive improvements and involves huge investment in R&D.

The second approach, called work study, aims at achieving higher efficiency and effectiveness of existing facilities through systematic analysis (usually in a relatively short time and with very little or no extra capital expenditure).

We shall discuss the concept, objectives, procedure, tools, techniques, and applications of work study.

## **1.1 PURPOSE OF WORK STUDY**

Hiller F.S. and Hiller M.S,(2004) Introduction to Management [48]. Lapin L.L.,(2004) Cutting your distribution cost. Growing your Business [41] have contributed many papers to the field of OR These work study leads to the following benefits/advantages in industries:

It is a direct means of raising productivity.

- i. It helps to increase the productive efficiency of an operative unit without much capital expenditure.
- ii. It helps to eliminate/reduce waste (waste of capital, material, labour, supervisory effort) and to make better use of resources.
- iii. It helps for establishing standards of performance on which effective production planning and control depends.
- iv. It provides a scientific basis for work improvement through work simplifications.
- v. It provides a better workspace layout and work environment.
- vi. It provides better quality of product to the consumer at a reasonable cost.
- vii. It provides a concept of fair days work to the workers, thus they can product themselves from the overload.
- viii. It provides a basis for negotiations between trade union leaders and the management.

This thesis divided into 5 chapters. The details of various chapters of the thesis are described as follows.

### **CHAPTER 1: INTRODUCTION**

In this first chapter, the basic concept of operation research with transportation problem in linear and non linear methods.

## **CHAPTER 2: REVIEW OF LITERATURE**

In this chapter, A review of literature of recent developments of operation research with transportation problem in linear and non linear methods is given.

## **CHAPTER 3: OFSTF METHODS – AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEM**

In this chapter a different approach OFSTF (Origin, First, Second, Third and Fourth quadrants) Methods is applied for finding a feasible solution for transportation problems directly. The proposed method is a unique, it is gives always feasible (may be an optimal for some extant) solution without disturbance of degeneracy condition. This method takes least iterations to reach optimally. A numerical example is solved to check the validity of the proposed methods, also degeneracy problem is also discussed.

## **CHAPTER 4: MDMA METHODS – AN OPTIMAL SOLUTION FOR TRANSPOTATION PROBLEM**

This chapter deals with MDMA (Maximum Divided Minimum Allotment) methods, is applied for finding the feasible solution for transportation problem. The proposed algorithm is unique way to reach feasible (or) may be an optimal (for some extant) solution without disturbance of degeneracy condition. Also some numerical problem is discussed.

## **CHAPTER 5: COMPARATIVE STUDY ON MDMA METHOD WITH OFSTF METHOD IN TRANSPORTATION PROBLEM AND OPTIMAL SOLUTION OF OFSTF, MDMA METHODS WITH EXISTING METHODS COMPARISON**

In this chapter comparative study of MDMA method discussed in chapter 3 and OFSTF method discussed in chapter 4. MDMA is better than OFSTF for proposed pay off matrix After analysis we have concluded and some numerical problem discussed with conclusion and also the Comparative study on MDMA method with OFSTF method in Transportation Problem discussed in chapter 3

with existing methods so called NWC / LC / MDMA / ORIGIN / FQ / SQ / TQ /  
and FQ analysis made with numerical example is discussed.

# **CHAPTER 2**

## **LITERATURE REVIEW**

## LITERATURE REVIEW

### 2.1. TRANSPORTATION PROBLEM

The transportation model is a special case of linear programming that deals with shipping a commodity from sources to destinations. Transportation problem is a special case for linear programming problems. The objective of transportation model is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limitations. The model assumes that the shipping cost is proportional to the number of units shipped on a given route. The transportation can be extended to the areas of operation, inventory control, employment scheduling and personnel assignment. The transportation model can be solved as a regular linear programming. Its special structure allows the development of a simplex based computational algorithm that makes use of the primal-dual relationships to simplify the computations. Dantzig, G.B. (1951) Application of the Simplex Method to a Transportation Problem. The transportation problems actually are special cases of the minimum cost flow problem and can be used for network representation of the transportation.

The transportation problem constitutes an important part of logistics management. In addition, logistics problems without shipment of commodities may be formulated as transportation problems. Sudhakar. V. J, Arunsankar. N and Karpagam. T(2012) A New Approach for Finding an Optimal Solution for Transportation. Dhose, ED and KR Morrison (1996) have used transportation solutions for a facility location problem. Various methods are available to solve the transportation problem to obtain an optimal solution, Gass, SI (1990) on solving the transportation problem and Abdual Quddoos, Shakeel Javaid, M.M. Khalid.(2012) have discussed —A New Method for Finding an Optimal Solution for Transportation Problems. Arsham, H and Kahn AB (1989) have explained — A simplex-type algorithm for general transportation problems, Charnes A and Cooper WW (1954) have determined —The Stepping Stone method for explaining linear programming calculations in transportation problems, Urashikumari, Patel D, Dhavakumar, Ravi, Bhasvar C (2017) Transportation Problem Using Stepping Stone Method and its Application. Shih, W (1987). Proposed the Modified Stepping-Stone method as a teaching aid for capacitated transportation problems, Ji, P and KF Ghu

(2002). A dual-matrix approach to the transportation problem. Shafaat, A and SK Goyal (2005) proposed a systematic approach for handling the situation of degeneracy in transportation problems. Dantzig.G.B (1951) take the application of the simplex method to a transportation problem, Activity Analysis of production and allocation.

All the optimal solution algorithms for solving transportation problems need an initial basic feasible solution to obtain the optimal solution. Goyal, SK (1991) has note on a heuristic for obtaining an initial solution for the transportation problem. There are various heuristic methods available to get an initial basic feasible solution, Abdual Quddoos, Shakeel Javaid, M.M. Khalid. (2012) have proposed a new method for finding an optimal solution for transportation problems. Joshua RR, Akilandeswari VS, Lakshmi Devi PK and Subashini N (2017) have proposed an Initial Basic Feasible Solution for Transportation Problem with — North- East Corner Method, Ramakrishnan, GS (1988) determined an improvement to Goyal's modified VAM for the unbalanced transportation problem, Shimshak, DG, JA Kaslik and TD Barclay (1981) have proposed a modification of Vogel's approximation method through the use of heuristics, Korukoglu S, Balli S (2011) obtained —An Improved Vogel's Approximation method for the Transportation Problem, Pandian P, Natarajan G (2010) presented a New Approach for Solving Transportation Problem with Mixed Constraints. Reena, Patel G, Bhathawla PH (2014) find out the New Global Approach to Transportation Problem. Goyal, SK (1984) was Improving VAM method for unbalanced transportation problems. Further , Kirca, O and Satir A (2002) developed a heuristic for obtaining an initial solution for the transportation problem.

Recently, Shajma, RRK and KD Sharma (2000) proposed a new heuristic approach for getting good starting solutions for dual based approaches used for solving transportation problems, Quddos A, Javaid S, Khalid MM (2012) have proposed a New Method for finding an Optimal Solution for Transportation Problems and also An Innovative Approach to Optimum Solution of Transportation Problem developed by Reena, Patel G, Bhathawla PH (2016).

## 2.2 LINEAR PROGRAMMING

The development of linear programming has been ranked among the most important scientific advances in the year 1930. Its impact since 1950 has been extraordinary. Till today it has saved many thousands of millions of rupees for most companies or business houses in the various industrialized countries of the world. A major portion of all scientific computations on computers is devoted to the use of linear programming. The linear programming problems in general are concerned with the use of allocation of scarce resources such as labour, materials, machines and capital in the best possible manner so that the costs are minimized and profits are maximized. The term linear programming defines a particular class of programming problems that meet the following conditions. The decision variables involved in the problem are non-negative i.e., positive or zero in nature, the function is normally referred to as the objective function. The operating rule governing the process can be expressed as a set of linear equations or inequalities. This set is referred to as the constraint set, Vipul Parkhi, Pooja Pawar, Archana Surve (2013) have used linear programming for Computer Automation for Malaria Parasite Detection, Sofi N.A, Aquil Ahmed, Mudasir Ahmad, Bilal Ahmad Bhat (2015) have used a Linear programming Approach for Decision Making in Agriculture. Linear Programming and Extensions Dantzig, G.B. (1963), An Introduction to Linear Programming Charnes A., Cooper W.W and Henderson A. (1954).

Linear programming technique is widely used to solve a number of military, economic, industrial and social problems. The primary reasons for its wide use are that a large variety of problems in diverse fields can be represented by it. Any problem whose mathematical model fits the general format of the linear programming model can be solved efficiently by a simplex method. These are some reasons for the tremendous impact of linear programming in recent decades. Balogun O.S, Jolayemi E.T, Akingbade T.J, Muazu, Useof H.G (2012) have used Linear Programming for Optimal Production in a Production Line in Coca-Cola Bottling Company and Kourosch Rajeiyan, Farhang Khalagh, doost Nejadi, Reza Hajati, Hamid Reza Safari, Ebrahim Alizadeh (2013), Using Linear Programming in Solving the Problem of Services Company's Costs. Chongyu (2016) developed an application of linear programming to the refugee migrating problem. Linear programming is a mathematical programming technique to optimize performance

(e.g profit or cost) under a set of resource constraints (e.g. machine-hours, man-hours, money, materials, etc.) as specified by an organization.

### **2.3 NON-LINEAR PROGRAMMING**

Linear programming assumptions or approximations may also lead to appropriate problem representations over the range of decision variables being considered. At other times, though, nonlinearities in the form of either nonlinear objective functions or nonlinear constraints are crucial for representing an application properly as a mathematical program. Mangasarian O.L (1969) explained Non-linear Programming approach. It provides an initial step toward coping with such nonlinearities, first by introducing several characteristics of nonlinear programs. that can be solved using simplex-like pivoting procedures Robinson S.M.(1982) had Generalized the nonlinear programming equations and applications, Constrained Optimization and Lagrange Multiplier Methods by Bertsekas D.P.(1982). Linear and Nonlinear Programming were explained by Luenberger D. G. (1984), the techniques to be discussed are primarily algebra based. Non-Linear programming technique is widely used to solve a number of military economic, industrial and social programs. The primary reasons for its wide use are that a large variety of problems in diverse fields can be represented by it. Non-linear programming is a mathematical programming technique to optimize performance (e.g profit or cost) under a set of resources constraints(e.g. machine-hours, man-hours, money, materials, etc.)as specified by an organization . Non-linear programming (NLP) is an extension of linear programming (LP). The principal differences between NLP and LP is that in NLP, the variables which are either in objective function and/or in the constraints occurs in higher powers such as ( $x^2$ ) or in the multiplication from such as ( $x_1, x_2, \dots$ ) with a greater number of variable, i.e., non-linear.

This thesis “NEW APPROACH ON TRANSPORTATION PROBLEMS IN LINEAR AND NON LINEAR METHODS” is carefully formulated.

# **CHAPTER-3**

## **OFSTF METHOD – AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEM**

## **OFSTF METHOD – AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEM**

In this chapter a different approach OFSTF (origin, first, second, third, and, fourth quadrants) Method is applied for finding a feasible solution for transportation problem directly. The proposed method is a unique, it gives always feasible (may be an optimal for some extant) solution without disturbance of degeneracy condition. This method takes least iterations to reach optimality. A numerical example is solved to check the validity of the proposed method. Also degeneracy problem is also discussed.

### **3.1Transportation problem through OFSTF (origin, first, second, third, and, fourth quadrants) Method**

We now introduce a new method called the transport problem through OFSTF method for finding an feasible solution to a transportation problem. The OFSTF method algorithm proceeds as follows.

#### **STEP 1**

Construct Transportation Table (TT) for the given pay off Matrix(POM).

#### **STEP 2**

Choose the maximum and minimum element in the constructed Transportation Table (TT).

#### **STEP 3**

Find the difference between the maximum and minimum element from step 2.

if the resultant Element (RE) matched with anyone of the element in the POM , then find the difference between each element in the Transportation table (TT)with the resultant Element (RE) That is .

Maximum Element- Minimum Element=R.E

If R.E= An element in TT

Every element in TT-R.E.

If R.E ≠an element in TT

select next minimum element in TT and repeat the step3.3.1.,

Repeat the process until the condition satisfied.

#### **STEP 4**

In the Reduced POM , there will be at least one zero in the TT, select a particular zero based on the maximum deviation element from the given zeros.

#### **STEP 5**

##### **Case 1**

Fix zero as origin and find the maximum deviated element from the selected zero.

##### **Case 2**

Fix zero as origin and find the maximum deviated element in the first quadrant (+,+) from the selected zero.

##### **Case 3**

Fix zero as origin and find the maximum deviated element in the first quadrant (+,-) from the selected zero.

##### **Case 4**

Fix zero as origin and find the maximum deviated element in the first quadrant (-,-) from the selected zero.

##### **Case 5**

Fix zero as origin and find the maximum deviated element in the first quadrant (+,-) from the selected zero.

#### **STEP 6**

Compare and fulfill the demand of the maximum deviated element with the supply in the TT.

#### **STEP 7**

Calculate the total cost for each cases, the feasible solution is obtained in the origin area for all kind of transportation problem.

Hence we say that in transportation problem, calculating the cost from origin will lead to a feasible solution by our OFSTF.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4	6	9	10	40
S <sub>2</sub>	3	4	7	8	12	50
S <sub>3</sub>	13	12	6	5	6	30

S <sub>4</sub>	5	8	12	6	9	30
Demand	30	30	30	40	20	150

3.1 Transportation Table

### 3.2 OFSTF ORIGIN Method

#### STEP 1

Construct the transportation table for the given pay off matrix

#### STEP 2

Choose the maximum element  $a_{31}=13$  and minimum element  $a_{21}=3$  in the constructed Transportation table.

#### STEP 3

The difference between maximum and minimum element is 10 this element is called the resultant element. Maximum element – Minimum element= Resultant element.

#### STEP 4

Subtract this resultant element (10) from each and every element given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4	6	9	10	40
S <sub>2</sub>	3	4	7	8	12 20	50 30
S <sub>3</sub>	13	12	6	5	6	30
S <sub>4</sub>	5	8	12	6	9	30
Demand	30	30	30	40	20	150

Table3.2 OFSTF ORIGIN Method –Allotment-1

The resultant element is (10) subtract these will be at least one zero on the fixed element.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>
S <sub>1</sub>	-5	-6	-4	-1	0
S <sub>2</sub>	-7	-6	-3	-2	2
S <sub>3</sub>	3	2	-4	-5	-4

S <sub>4</sub>	-5	-2	2	-4	-1
----------------	----	----	---	----	----

Table 3.3

**STEP 5**

Fix zero as an origin and take the maximum deviation in all directions, here the elements in all directions are  $\{-1, -2, 2\}$  then maximum deviation is 2 and the minimum demand  $D(20, 50) = 20$  in the corresponding cell (2,5). Cancelling the reduced POM is

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4	6    30	9	10	40    10
S <sub>2</sub>	3	4	7	8	12	50
S <sub>3</sub>	13	12	6	5	6	30
S <sub>4</sub>	5	8	12	6	9	30
Demand	30	30	30	40	20	

Table 3.4 Allotment-2

Take the maximum element  $(a_{31}) = 13$  and the minimum element  $(a_{21}) = 3$  the resultant element is 10. This element is not in the transportation table. Select next minimum element  $(a_{12}) = 4$  the resultant element is 9. Subtract these resultant element (9) from each and every element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
S <sub>1</sub>	-4	-5	-3	0
S <sub>2</sub>	-6	-5	-2	-1
S <sub>3</sub>	4	3	-3	-4
S <sub>4</sub>	-4	-1	3	-3

Table 3.5 Basic for allotment-2

**STEP 6**

Fix zero as an origin and take the maximum deviation in all directions, here the elements in all directions are  $\{-3, -2, -1\}$  then maximum deviation is -3 and the minimum demand  $D(30, 40) = 30$  in the corresponding cell (1,3). Cancelling the reduced POM is

	D <sub>1</sub>	D <sub>2</sub>	D <sub>4</sub>	Supply
S <sub>1</sub>	5	4 <sub>10</sub>	9	10
S <sub>2</sub>	3	4	8	30
S <sub>3</sub>	13	12	5	30
S <sub>4</sub>	5	8	6	30
Demand	30	30 <sub>20</sub>	40	100

Table 3.6 Allotement-3

Take the maximum element ( $a_{31}$ )=13 and minimum element ( $a_{21}$ )=3 the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element ( $a_{12}$ )= 4 the resultant element is 9. Subtract these resultant element (9) from each and element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>4</sub>
S <sub>1</sub>	-4	-5	0
S <sub>2</sub>	-6	-5	-1
S <sub>3</sub>	4	3	-4
S <sub>4</sub>	-4	-1	-3

Table 3.7 Allotement-3

### STEP 7

Fix zero as an origin and take the maximum deviation in all directions, here the element in all directions are  $\{-5,-5,-1\}$  then maximum deviation is -3 and the minimum demand  $D(30,10)=10$  in the corresponding cell (1,2). Cancelling the reduced POM is

	D <sub>1</sub>	D <sub>2</sub>	D <sub>4</sub>	Supply
S <sub>2</sub>	3	4	8	30
S <sub>3</sub>	13	12	5	30
S <sub>4</sub>	5	8	6	30
Demand	30	20	40	90

Table 3.9 Base for Allotment-4

Take the maximum element ( $a_{21}$ )=13 and minimum element ( $a_{11}$ )=3 the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element ( $a_{12}$ )= 4 the resultant element is 9. Subtract these resultant element (8) from each and element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
S <sub>2</sub>	-5	-4	0
S <sub>3</sub>	5	4	-3
S <sub>4</sub>	-3	0	-2

Table 3.9 Allotement-4

Fix zero as an origin and take the maximum deviation in all directions, here the element in all directions are ( 5, 4, 3.2 ) then maximum deviation is 5 and assign the minimum demand  $D(30,30) = 30$  in the corresponding cell (2,1) Cancelling row and column the reduced POM is

**STEP 8**

	<b>D<sub>2</sub></b>	<b>D<sub>4</sub></b>	Supply
S <sub>2</sub>	4	8	30
S <sub>4</sub>	8 20	6	30
Demand	20	40	60

Table3.10 OFSSTF ORRLGIN Method Allotment-5

Take the maximum element  $(a_{12}) = 8$  and minimum element  $(a_{11}) = 4$  the resultant Element is 4 .subtract these element (4) from each every element of given POM.

	<b>D<sub>1</sub></b>	<b>D<sub>4</sub></b>
<b>S<sub>2</sub></b>	<b>0</b>	<b>4</b>
<b>S<sub>2</sub></b>	<b>4</b>	<b>2</b>

Table3.11 base for allotment -5

Fix zero as an origin and take the maximum deviation in all directions, here the element in all directions are ( 4, 4, 2 ) then maximum deviation is 4 and assign the minimum demand  $D(20,30) = 20$  in the corresponding cell (2,1). Cancelling column reduced POM is

	D <sub>4</sub>	Supply
S <sub>2</sub>	8      30	30
S <sub>4</sub>	6      10	10
Demand	40	40

Table3.12 Allotement-6

Allot 30 units in the cell (1,1) and Allot 10 units in the cell (2,1)

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4    10	6    30	9	10	40
S <sub>2</sub>	3	4	7	8    30	12    20	50
S <sub>3</sub>	13    30	12	6	5	6	30
S <sub>4</sub>	5	8    20	12	6    10	9	30
Demand	30	30	30	40	20	150

Optimal Allotment Table for ORIGIN method

Obtain a basic feasible solution is

$$S_1 \rightarrow D_2 \Rightarrow 4 \times 10 = 40 \text{ Units}$$

$$S_1 \rightarrow D_3 \Rightarrow 6 \times 30 = 180 \text{ Units}$$

$$S_2 \rightarrow D_4 \Rightarrow 8 \times 30 = 240 \text{ Units}$$

$$S_2 \rightarrow D_5 \Rightarrow 12 \times 20 = 240 \text{ Units}$$

$$S_3 \rightarrow D_1 \Rightarrow 13 \times 30 = 390 \text{ Units}$$

$$S_4 \rightarrow D_2 \Rightarrow 8 \times 20 = 160 \text{ Units}$$

$$S_4 \rightarrow D_4 \Rightarrow 6 \times 10 = 60 \text{ Units}$$

$$\text{TOTAL} = 1310 \text{ Units}$$

### 3.3 OFSTF First Quadrant Method (+,+)

#### STEP 1

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4	6	9	10    20	40
S <sub>2</sub>	3	4	7	8	12	50
S <sub>3</sub>	13	12	6	5	6	30
S <sub>4</sub>	5	8	12	6	9	30
Demand	30	30	30	40	20	150

Table3.14 Allotment- 1

Choose the maximum  $(a_{31})=13$  and minimum element  $(a_{21})=3$  the resultant element is 10. This element is called the resultant element. The resultant element is 10. Subtract these resultant element (10) from each and every element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>
S <sub>1</sub>	-5	-6	-4	-1	0
S <sub>2</sub>	-7	-6	-3	-2	2
S <sub>3</sub>	3	2	-4	-5	-4
S <sub>4</sub>	-5	-2	2	-4	-1

Table 3.15 Base for Allotment-1

Fix zero as an origin and take the maximum deviation in the quadrant, the maximum deviation is zero itself. since there is no (+,+)elements and the minimum demand  $D(20,40)=20$  in the corresponding cell (1,5). Cancelling the reduced POM is

## STEP 2

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	Supply
S <sub>1</sub>	5	4	6	9 <sub>20</sub>	20
S <sub>2</sub>	3	4	7	8	50
S <sub>3</sub>	13	12	6	5	30
S <sub>4</sub>	5	8	12	6	30
Demand	30	30	30	40 <sub>20</sub>	130

Table 3.16 Allotment-2

Take the maximum element  $(a_{31})=13$  and minimum element  $(a_{21})=3$  the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element  $(a_{12}) = 4$  the resultant element is 9. Subtract these resultant element (9) from each and element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
S <sub>1</sub>	-4	-5	-3	0
S <sub>2</sub>	-6	-5	-2	-1
S <sub>3</sub>	4	3	-3	-4

S <sub>4</sub>	-4	-1	3	-3
----------------	----	----	---	----

Table 3.17 base for allotment-2

Fix zero as an origin and take the maximum deviation in the quadrant, the maximum deviation is zero itself. since there is no (+,+)elements and the minimum demand  $D(40,20)=20$  in the corresponding cell (1,4). Cancelling the reduced POM is

**STEP 3**

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	Supply
S <sub>2</sub>	3	4	7	8    20	50    30
S <sub>3</sub>	13	12	6	5	30
S <sub>4</sub>	5	8	12	6	30
Demand	30	30	30	20	110

Table 3.18 Allotment-3

Take the maximum element  $(a_{31})=13$  and minimum element  $(a_{21})=3$  the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element  $(a_{12})= 4$  the resultant element is 9. Subtract these element  $\neq$  in element  $(a_{31})=5$  the resultant element 8 subtract these resultant element (8) from each and element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>
S <sub>2</sub>	-5	-4	-1	0
S <sub>3</sub>	5	4	-2	-3
S <sub>4</sub>	-3	0	4	-2

Table 3.19 Allotment-3

Fix zero as an origin and take the maximum deviation in the quadrant, the maximum deviation is zero itself. Since there is no (+,+)elements and the minimum demand  $D(20,50)=20$  in the corresponding cell (1,4). Cancelling the reduced POM is

**STEP 4**

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Supply
S <sub>2</sub>	3	4	7	30
S <sub>3</sub>	13	12	6	30
S <sub>4</sub>	5	8	12	30
Demand	30	20	30	90

Table3.20 Allotement-4

Take the maximum element ( $a_{31}$ ) =13 and minimum element ( $a_{21}$ )=3 the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element ( $a_{12}$ ) = 4 the resultant element is 9. Subtract these element  $\neq$  in element ( $a_{31}$ )=5 the resultant element 8 subtract these resultant element (8) from each and element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
S <sub>2</sub>	-5	-4	-1
S <sub>3</sub>	-5	4	-2
S <sub>4</sub>	-3	0	4

Table 3.21 Allotment-4

Fix zero as an origin and take the maximum deviation in the 1<sup>st</sup> quadrant, here the elements in the 1<sup>st</sup> quadrant are {4, 4, -2} then maximum deviation is 4 itself. Assign the minimum demand D (30, 30)= 30 in the corresponding cell (2,2). Cancelling the reduced POM is

**STEP 5**

	D <sub>1</sub>	D <sub>3</sub>	Supply
S <sub>2</sub>	3	7	30
S <sub>4</sub>	5	12	30
Demand	30	30	60

Table 3.22 Allotment-5

Take the maximum element  $(a_{22}) = 12$  and minimum element  $(a_{11}) = 3$  the resultant element is 9. These element  $\neq$  in element in transportation table. Select next minimum element  $(a_{12}) = 4$  the resultant element is 9. Subtract these element  $\neq$  in element  $(a_{12}) = 5$  the resultant element 7 subtract these resultant element (7) from each and element of given POM.

	D <sub>1</sub>	D <sub>3</sub>
S <sub>2</sub>	-4	0
S <sub>4</sub>	-2	5

Table 3.23 Allotment-5

Fix zero as an origin and take the maximum deviation in the 1<sup>st</sup> quadrant and zero itself since there no(+,+)element and Assign the minimum demand D  $(30, 30) = 30$  in the corresponding cell (1,2). Cancelling the reduced POM is

**STEP 6**

	D <sub>1</sub>	D <sub>3</sub>
S <sub>4</sub>	5      30	30
Demand	30	30

Table 3.24 Allotment-6

Allotment 30 units in the cell(1,1)

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4	6	9    20	10   20	40
S <sub>2</sub>	3	4	7    30	8    20	12	50
S <sub>3</sub>	13	12   30	6	5	6	30
S <sub>4</sub>	5    30	8	12	6	9	30
Demand	30	30	30	40	20	150

Table 3.25

Obtain a base feasible solution

$S_1 \rightarrow D_4 \Rightarrow 9 \times 20 = 180$  Units

$S_1 \rightarrow D_5 \Rightarrow 10 \times 20 = 200$  Units  
 $S_2 \rightarrow D_3 \Rightarrow 7 \times 30 = 210$  Units  
 $S_2 \rightarrow D_4 \Rightarrow 8 \times 20 = 160$  Units  
 $S_3 \rightarrow D_2 \Rightarrow 12 \times 30 = 360$  Units  
 $S_4 \rightarrow D_1 \Rightarrow 5 \times 30 = 150$  Units  
 TOTAL = 1260 Unit

### 3.4 OFSTF Second Quadrant Method (-,+)

#### STEP 1

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4	6	9 40	10	40
S <sub>2</sub>	3	4	7	8	12	50
S <sub>3</sub>	13	12	6	5	6	30
S <sub>4</sub>	5	8	12	6	9	30
Demand	30	30	30	40	20	150

Table 3.14 Allotment- 1

Choose the maximum ( $a_{31}$ ) = 13 and minimum element ( $a_{21}$ ) = 3 in the constructed Transportation Table. The different between the maximum and minimum element is 10. This element is called the resultant element. Subtract these resultant element (10) from each and every element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>
S <sub>1</sub>	-5	-6	-4	-1	0
S <sub>2</sub>	-7	-6	-3	-2	2
S <sub>3</sub>	3	2	-4	-5	-4
S <sub>4</sub>	-5	-2	2	-4	-1

Table 3.15 Base for Allotment-1

Fix zero as an origin and take the maximum deviation in the 2<sup>nd</sup> quadrant here the elements in the second quadrant is (-1), the maximum deviation is -1 and assign the minimum demand  $D(40,40) = 40$  in the corresponding cell (1,4). Cancelling row and column the reduced POM is

**STEP 2**

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>5</sub>	Supply
S <sub>2</sub>	3	4	7	12	50
S <sub>3</sub>	13	12	6	6    20	30    10
S <sub>4</sub>	5	8	12	9	30
Demand	30	30	30	20	110

Table 3.28 Allotment-2

Take the maximum element ( $a_{21}$ )=13 and minimum element ( $a_{11}$ )=3 the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element ( $a_{12}$ ) = 4 the resultant element is 9. Subtract these resultant element (9) from each and element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>5</sub>
S <sub>2</sub>	-6	-5	-2	3
S <sub>3</sub>	4	3	-3	-3
S <sub>4</sub>	-4	-1	3	0

Table 3.17 base for allotment-2

Fix zero as an origin and take the maximum deviation in the 2<sup>nd</sup> quadrant, here the element in the second quadrant are {3,-3,-3} the maximum deviation is -3 and assign the minimum demand D (20,30) = 20 in the corresponding cell (2,4). Cancelling the reduced POM is

**STEP 3**

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Supply
S <sub>2</sub>	3	4	7	50
S <sub>3</sub>	13    10	12	6	10
S <sub>4</sub>	5	8	12	30
Demand	30    20	30	30	90

Table 3.30 Allotment-3

Take the maximum element ( $a_{21}$ ) = 13 and minimum element ( $a_{11}$ )=3 the resultant element is 10. These element  $\neq$  in element in transportation table. Select next

minimum element ( $a_{12}$ ) = 4 the resultant element is 9. Select next minimum element ( $a_{31}$ ) = 5 the resultant element (9) from each and element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
S <sub>2</sub>	-5	-4	-1
S <sub>3</sub>	5	4	-2
S <sub>4</sub>	-3	0	4

Table 3.31 Allotment-3

Fix zero as an origin and take the maximum deviation in the 2<sup>nd</sup> quadrant, here the elements in the 2<sup>nd</sup> quadrant are {-3,5,4}the maximum deviation 5 and assign the minimum demand D(30,10)=10 in the corresponding cell (2,1). Cancelling the reduced POM is

**STEP 4**

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	Supply
S <sub>2</sub>	3      20	4	7	50    30
S <sub>4</sub>	5	8	12	30
Demand	20	30	30	80

Table 3.32 Allotment-4

Take the maximum element ( $a_{23}$ ) = 12 and minimum element ( $a_{11}$ )=3 the resultant element is 9. These element  $\neq$  in element in transportation table. Select next minimum element ( $a_{12}$ ) = 4 the resultant element is 8. Select next minimum element ( $a_{12}$ ) = 4 the resultant element (8) from each and element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>
S <sub>2</sub>	-5	-4	-1
S <sub>4</sub>	-3	0	4

Table 3.33 Allotment-4

Fix zero as an origin and take the maximum deviation in the 2<sup>nd</sup> quadrant, here the elements in the 2<sup>nd</sup> quadrant are {-5,-3,4}the maximum deviation -5 and assign the

minimum demand  $D(20,50)=20$  in the corresponding cell (1,1). Cancelling the reduced POM

**STEP 5**

	D <sub>2</sub>	D <sub>3</sub>	Supply
S <sub>2</sub>	4                      30	7	30
S <sub>4</sub>	8	12	30
Demand	30	30	60

Table 3.34 Allotment-5

Take the maximum element  $(a_{21}) = 13$  and minimum element  $(a_{11})=3$  the resultant element is 10. Subtract the resultant element (9) from each and element of given POM.

	D <sub>2</sub>	D <sub>3</sub>
S <sub>2</sub>	-4	-1
S <sub>4</sub>	0	4

Table 3.36 Allotment-6

**STEP 6**

	D <sub>2</sub>	D <sub>3</sub>
S <sub>4</sub>	12                      30	30
Demand	30	30

Table 3.36 Allotment-6

Allot 30 units in the cell (1,1)

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4	6	9                      40	10	40
S <sub>2</sub>	3                      20	4                      30	7	8	12	50
S <sub>3</sub>	13                      10	12	6	5	6                      20	30
S <sub>4</sub>	5	8	12                      30	6	9	30
Demand	30	30	30	40	20	150

Table 3.37 2<sup>nd</sup> quadrant method

Obtain a base feasible solution

$S_1 \rightarrow D_4 \Rightarrow 9 \times 40 = 360$  Units  
 $S_2 \rightarrow D_1 \Rightarrow 3 \times 20 = 60$  Units  
 $S_2 \rightarrow D_2 \Rightarrow 4 \times 30 = 120$  Units  
 $S_3 \rightarrow D_1 \Rightarrow 13 \times 10 = 130$  Units  
 $S_3 \rightarrow D_5 \Rightarrow 6 \times 20 = 120$  Units  
 $S_4 \rightarrow D_5 \Rightarrow 12 \times 30 = 360$  Units  
 TOTAL = 1150 Units

### 3.5 OFSTF Third Quadrant Method (-,-)

#### STEP 1

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	Supply
$S_1$	5	4	6	9	10	40
$S_2$	3	4	7	8	12	50
$S_3$	13	12	6	5	6	30
$S_4$	5	8	12	6	9	30
Demand	30	30	30	40	20	150

Table 3.38 OFSTF TQ Method Allotment - 1

Choose the maximum element  $a_{31} = 13$  and minimum element  $a_{21} = 3$  in the constructed Transportation Table. The difference between the maximum and minimum element is 10. This element is called the resultant element. Subtract these resultant element (10) from each and every element of given POM.

#### STEP 2

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	5	4	6	9	40
			30		10

$S_2$	3	4	7	8	30
$S_3$	13	12	6	5	30
$S_4$	5	8	12	6	30
Demand	30	30	30	40	130

Table 3.40 OFSTF TQ Method Allotment - 2

Take the maximum element ( $a_{21}$ ) = 13 and minimum element ( $a_{11}$ ) = 3 the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element ( $a_{12}$ ) = 4 the resultant element is 9 subtract these resultant element (9) from each and every element of given POM.

	$D_1$	$D_2$	$D_3$	$D_4$
$S_1$	-4	-5	-3	0
$S_2$	-6	-5	-2	-1
$S_3$	4	3	-3	-4
$S_4$	-4	-1	3	-3

Fix zero as an origin and take the elements in the third quadrant are  $\{-3,-2,-1\}$  the maximum deviation is -3 and assign the minimum demand  $D(30,40) = 30$  in the corresponding cell (1,3) Cancelling column the reduced POM is

**STEP 3**

	$D_1$	$D_2$	$D_4$	Supply
$S_1$	5	4	9	10

S <sub>2</sub>	3	4	8	30
S <sub>3</sub>	13	12	5	30
S <sub>4</sub>	5	8	6	30
Demand	30	30	40	100

Table 3.42 OFSTF TQ Method Allotment – 3

Take the maximum element ( $a_{21}$ ) = 13 and minimum element ( $a_{31}$ ) = 5 the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element ( $a_{12}$ ) = 4 the resultant element is 9 subtract these resultant element (9) from each and every element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	Supply
S <sub>1</sub>	-4	-5	0
S <sub>2</sub>	-6	-5	-1
S <sub>3</sub>	4	3	-4
S <sub>4</sub>	-4	-1	-3

Table 3.43 Allotment-3

Fix zero as an origin and take the elements in the third quadrant are {-5,-5,-1} the maximum deviation is -5 and assign the minimum demand  $D(30,10) = 10$  in the corresponding cell (1,2) Cancelling row the reduced POM is

**STEP 4**

	D <sub>1</sub>	D <sub>2</sub>	D <sub>4</sub>	Supply
S <sub>2</sub>	3	4	8	30
S <sub>3</sub>	13	12	5	30
S <sub>4</sub>	5	8	6	30
Demand	30	20	40	90

### 3.44 OFSTF TQ Method Allotment - 4

Take the maximum element  $(a_{21}) = 13$  and minimum element  $(a_{11}) = 3$  the resultant element is 10. These element  $\neq$  in element in transportation table. Select next minimum element  $(a_{12}) = 4$  the resultant element is 9 These element  $\neq$  in element in transportation table. Select next minimum element  $(a_{31}) = 5$  the resultant element is 8 subtract these resultant element (8) from each and every element of given POM.

	D <sub>1</sub>	D <sub>2</sub>	D <sub>4</sub>
S <sub>2</sub>	-5	-4	0
S <sub>3</sub>	5	4	-3
S <sub>4</sub>	-3	0	-2

Table 3.45 Allotment-4

Fix zero as an origin and take the in the third quadrant here the element in the third quadrant is (-3) the maximum deviation is -3 and assign the minimum demand  $D(30,30) = 30$  in the corresponding cell (3,1) Cancelling row and column the reduced POM is

#### STEP 5

	D <sub>2</sub>	D <sub>4</sub>	Supply
S	4	8	30
	20		10
S	12	5	30
Demand	20	40	60

Table 3.46 OFSTF TQ Method Allotment – 5

Take the maximum element  $(a_{21}) = 12$  and minimum element  $(a_{11}) = 4$  the resultant element is 8. subtract these resultant element (8) from each and every element of given POM.

	D <sub>2</sub>	D <sub>4</sub>
S <sub>2</sub>	-4	0
S <sub>3</sub>	4	-3

Table 3.47 Base for Allotment - 5

Fix zero as an origin and take the in the third quadrant here the elements in the third quadrant are  $\{-4,4,-3\}$  the maximum deviation is -4 and assign the minimum demand  $D(20,30) = 20$  in the corresponding cell (1,1). Cancelling column the reduced POM is

**STEP 6**

	D <sub>4</sub>	Supply
S <sub>2</sub>	8 10	10
S <sub>3</sub>	5 30	30
Demand	40	40

Table 3.48 OFSTF TQ Method Allotment – 6

Allot 10 units in the cell (1,1) and 30 units in the cell(2,1)

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	Supply
S <sub>1</sub>	5	4 1 0	6 3 0	9	10	40
S <sub>2</sub>	3	4 2 0	7	8 1 0	12 20	50
S <sub>3</sub>	13	12	6	5 3 0	6	30
S <sub>4</sub>	5 <sub>30</sub>	8	12	6	9	30

Demand	30	30	30	40	20	150
--------	----	----	----	----	----	-----

Table 3.49 OFSTF - Optimal Allotment Table for Third Quadrant Method

Obtain a basic feasible solution is

$$S_1 \rightarrow D \Rightarrow 4 \times 10 = 40 \text{ Units}$$

$$S_1 \rightarrow D \Rightarrow 6 \times 30 = 180 \text{ Units}$$

$$S_2 \rightarrow D \Rightarrow 4 \times 20 = 80 \text{ Units}$$

$$S_2 \rightarrow D \Rightarrow 8 \times 10 = 80 \text{ Units}$$

$$S_2 \rightarrow D \Rightarrow 12 \times 20 = 240 \text{ Units}$$

$$S_3 \rightarrow D \Rightarrow 5 \times 30 = 150 \text{ Units}$$

$$S_4 \rightarrow D \Rightarrow 5 \times 30 = 150 \text{ Units}$$

$$\text{Total} = 890 \text{ Units}$$

Thus the OFSTF method provides an feasible value of the objective function for the transportation problem. The proposed algorithm carries systematic procedure, and very easy to understand. It can be extended to assignment problem and travelling salesman problems to get an optimal solution. The proposed method is important tool for the decision makers when they are handling various types of logistic problems, to make the decision optimally.

# **CHAPTER-4**

## **MDMA METHOD- AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEM**

## MDMA METHOD- AN OPTIMAL SOLUTION FOR TRANSPORTATION PROBLEM

This chapter deals with MDMA (Maximum Divide Minimum Allotment) method, is applied for finding the feasible solution for transportation problem. The proposed algorithm is unique way to reach feasible (or) may be an optimal (for some extant) solution without disturbance of degeneracy condition .Also some numerical problem is discussed.

### 4.1 Transport Problem through MDMA (Maximum Divide Minimum Allotment) Method

We now introduce a new method called the Transport Problem through MDMA method for finding an feasible solution to a transportation problem. The MDMA method algorithm proceeds as follows.

#### STEP 1

Construct the Transportation Table (TT) for the given Pay Off Matrix (POM).

#### STEP 2

Choose the maximum element(ME) from POM and divide all elements by the ME in the Constructed Transportation Table (CTT).

#### STEP 3

Supply the demand for the minimum element newly CTT.

#### STEP 4

Select the next maximum element in CTT and repeat the same procedure for remaining allotments.

Consider the following cost minimizing transportation problems.

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	Supply
$S_1$	9	12	9	6	9	10	5
$S_2$	7	3	7	7	5	5	6
$S_3$	6	5	9	11	3	11	2

$S_4$	6	8	11	2	2	10	9
Demand	4	4	6	2	4	2	22

Table 4.1 Transportation Table

**STEP 1**

Here the maximum element is 12, Divide all elements by Maximum Element = 12

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	Supply
$S_1$	9/12	1	9/12	6/12	9/12	10/12	5
$S_2$	7/12	3/12	7/12	7/12	5/12	5/12	6
$S_3$	6/12	5/12	9/12	11/12	3/12	11/12	2
$S_4$	6/12	8/12	11/12	2/12	2/12	10/12	9 7
Demand	4	4	6	2	4	2	22

Table 4.2 MDMA Method Step-1

Select the minimum element (2/12) allot the minimum demand and allot  $D(2,9) = 2$  unit in the cell(4,3) cancelling column the reduced POM is

	$D_1$	$D_2$	$D_3$	$D_5$	$D_6$	Supply
$S_1$	9/12	1	9/12	9/12	10/12	5
$S_2$	7/12	3/12	7/12	5/12	5/12	6
$S_3$	6/12	5/12	9/12	3/12	11/12	2

$S_4$	6/12	8/12	11/12	2/12	10/12	7
Demand	4	4	6	4	2	20

Table 4.3 Base for Step-1

**STEP 2**

Choose the next maximum element is 11/12, Divide all elements by Maximum element=11/12

	$D_1$	$D_2$	$D_3$	$D_5$	$D_6$	Supply
$S_1$	9/11	12/11	9/11	9/11	10/11	5
$S_2$	7/11	3/11	7/11	5/11	5/11	6
$S_3$	6/11	5/11	9/11	3/11 <span style="border: 1px solid black; padding: 0 2px;">2</span>	11/11	2
$S_4$	6/11	8/11	11/11	12/11	10/11	7
Demand	4	4	6	4	2	20

Table 4.4 MDM Method Step-2

Select the minimum element and assign the minimum demand  $[D(2,2)] = 2$  units in the cell(3,4). Cancelling row and column the reduced POM is

	$D_1$	$D_2$	$D_3$	$D_5$	$D_6$	Supply
$S_1$	9/11	12/11	9/11	9/11	10/11	5

$S_2$	7/11	3/11	7/11	5/11	5/11	6
$S_4$	6/11	8/11	1	12/11	10/11	7
Demand	4	4	6	2	2	18

Table 4.5 Base for Step-2

### STEP 3

Take the next maximum element 12/11 divide all element by 12/11

	$D_1$	$D_2$	$D_3$	$D_5$	$D_6$	Supply
$S_1$	9/12	1	9/12	9/12	10/12	5
$S_2$	7/12	3/12 4	7/12	5/12	5/12	6 2
$S_4$	6/12	8/12	11/12	1	10/12	7
Demand	4	4	6	2	2	18

Table 4.6 MDMA Method Step-3

Select the minimum element and assign the minimum demand  $D(4,6)=4$  in the cell (2,2) cancelling column the reduced POM is

	$D_1$	$D_3$	$D_5$	$D_6$	Supply

$S_1$	9/12	9/12	9/12	10/12	5
$S_2$	7/12	7/12	5/12	5/12	2
$S_4$	6/12	11/12	1	10/12	7
Demand	4	6	2	2	14

Table 4.7 Base for Step-3

**STEP 4**

Take the next maximum element 11/12 and divide all element by 11 /12

	$D_1$	$D_3$	$D_5$	$D_6$	Supply
$S_1$	9/11	9/11	9/11	10/11	5
$S_2$	7/11	7/11	5/11	5/11	2
$S_4$	6/11	1	12/11	10/11	7
Demand	4	6	2	2	14

Table 4.8 MDMA Method Step-4

Select the minimum element 5/11 and assign the minimum element  $D(2,2)=2$  units in the cell (2,3). Cancelling both low and column the reduced POM is

	$D_1$	$D_3$	$D_6$	Supply
$S_1$	9/11	9/11	10/11	5
$S_4$	6/11	1	10/11	7
Demand	4	6	2	12

Table 4.9 Base for Step-4

**STEP 5**

Take the maximum element 10/11 and divide all element by 10/11

	$D_1$	$D_3$	$D_6$	Supply
$S_1$	9/10	9/10	1	5
$S_4$	6/10 4	11/10	1	7 3
Demand	4	6	2	12

Table 4.10 MDMA Method Step-5

Take the minimum element 6/10 and assign the minimum element  $D(4,7)=4$  cancelling column the reduced POM is

	$D_3$	$D_6$	Supply
$S_1$	9/10	1	5
$S_4$	11/10	1	3
Demand	6	2	8

Table 4.11 Base for Step-5

**STEP 6**

Take the maximum element 11/10 and divide all elements by 11/10

	$D_3$	$D_6$	Supply
$S_1$	$\frac{11}{10}$	10/11	5
$S_4$	1	10/11	3
Demand	6 1	2	8

Table 4.12 MDMA Method Step-6

Select the minimum element 9/11 and assign minimum Demand  $D(6,5) = 5$  in the cell (1,1) cancelling the row reduced POM is

	$D_3$	$D_6$	Supply
$S_4$	1 1	$\frac{1}{11}$	3
Demand	1	2	3

Table 4.13 Base for Step-6

Allot 1 unit in the cell (1,1) and 2 unit in the cell (1,2)

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$	Supply
$S_1$	9	12	9 5	6	9	10	5
$S_2$	7	3	7	7	5	5	6

		4			2		
$S_3$	6	5	9	11	3	11	2
$S_4$	6	8	11	2	2	10	9
	4		1	2		2	
Demand	4	4	6	2	4	2	22

Table 4.14 MDMA Method Optimal Allotment Table

Obtain a basic feasible solution is

$$S_1 \rightarrow D_3, 5 \text{ units cost } 9 \times 5=45$$

$$S_2 \rightarrow D_2, 4 \text{ units cost } 3 \times 4=12$$

$$S_2 \rightarrow D_5, 2 \text{ units cost } 5 \times 2=10$$

$$S_3 \rightarrow D_5, 2 \text{ units cost } 3 \times 2=6$$

$$S_4 \rightarrow D_1, 4 \text{ units cost } 6 \times 4=24$$

$$S_4 \rightarrow D_3, 1 \text{ units cost } 11 \times 1=11$$

$$S_4 \rightarrow D_4, 2 \text{ units cost } 2 \times 2=4$$

$$S_4 \rightarrow D_6, 2 \text{ units cost } 10 \times 2=20$$

$$\text{Total cost} = 132$$

Thus the MDMA method provides an feasible value of the objective function for the transportation problem. The proposed algorithm carries systematic procedure, and very easy to understand. It can be extended to assignment problem and travelling salesman problems to get an optimal solution. The explained method is providing tool for the decision makers when they are handling various types of logistic problems in real time.

# **CHAPTER-5**

**COMPARATIVE STUDY ON MDMA METHOD WITH  
OFSTF METHOD IN TRANSPORTATION PROBLEM**

## COMPARATIVE STUDY ON MDMA METHOD WITH OFSTF METHOD IN TRANSPORTATION PROBLEM

In this chapter comparative study of MDMA method discussed in chapter II and OFSTF method discussed in chapter I. MDMA is better than OFSTF for proposed pay off matrix After analysis we have concluded and some numerical problem discussed with conclusion

Consider the following cost minimizing transportation problems.

### 5.1 OFSTF Method

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	6	8	8	5	30
$S_2$	5	11	9	7	40
$S_3$	8	9	7	13	50
Demand	35	28	32	25	120

Table 5. Transportation Table

### 5.2 Origin Method

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	6 <sub>5</sub>	8	8	5 <sub>25</sub>	30
$S_2$	5 <sub>12</sub>	11 <sub>28</sub>	9	7	40
$S_3$	8 <sub>18</sub>	9	7 <sub>32</sub>	13	50

Demand	35	28	32	25	120
--------	----	----	----	----	-----

Table 5.2 OFSTF Origin Method Optimal Allotment Table

Total Cost:

$$S_1 \rightarrow D_1, 5 \text{ units cost } 5 \times 6 = 30$$

$$S_1 \rightarrow D_4, 25 \text{ units cost } 25 \times 5 = 125$$

$$S_2 \rightarrow D_1, 12 \text{ units cost } 12 \times 5 = 60$$

$$S_2 \rightarrow D_2, 28 \text{ units cost } 28 \times 11 = 308$$

$$S_3 \rightarrow D_1, 18 \text{ units cost } 18 \times 8 = 144$$

$$S_3 \rightarrow D_3, 32 \text{ units cost } 32 \times 7 = 224$$

$$\text{Total cost} = 891$$

### 5.3 First Quadrant Method (+,+)

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	6	8 <sub>28</sub>	8	5 <sub>2</sub>	30
$S_2$	5 <sub>35</sub>	11	9	7 <sub>5</sub>	40
$S_3$	8	9	7 <sub>32</sub>	13 <sub>18</sub>	50
Demand	35	28	32	25	120

Table 5.3 1First Quadrant Method Optimal Allotment Table

Total Cost :

$$S_1 \rightarrow D_2, 28 \text{ units cost } 28 \times 8 = 224$$

$$S_1 \rightarrow D_4, 2 \text{ units cost } 2 \times 5 = 10$$

$$S_2 \rightarrow D_1, 35 \text{ units cost } 35 \times 5 = 175$$

$$S_2 \rightarrow D_4, 5 \text{ units cost } 5 \times 7 = 35$$

$$S_3 \rightarrow D_3, 32 \text{ units cost } 32 \times 7 = 224$$

$$S_3 \rightarrow D_4, 18 \text{ units cost } 18 \times 13 = 234$$

$$\text{Total cost} = 902$$

#### 5.4 Second Quadrant Method (-,+)

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	6 <sub>30</sub>	8	8	5	30
$S_2$	5 <sub>10</sub>	11 <sub>10</sub>	9	7 <sub>25</sub>	40
$S_3$	8	9 <sub>18</sub>	7 <sub>32</sub>	13	50
Demand	35	28	32	25	120

Table 5.4 Allotment Table

Table Total Cost :

$$S_1 \rightarrow D_1, 30 \text{ units cost } 30 \times 6 = 180$$

$$S_2 \rightarrow D_1, 5 \text{ units cost } 5 \times 5 = 25$$

$$S_2 \rightarrow D_2, 10 \text{ units cost } 10 \times 11 = 110$$

$$S_2 \rightarrow D_4, 25 \text{ units cost } 25 \times 7 = 175$$

$$S_3 \rightarrow D_2, 18 \text{ units cost } 18 \times 9 = 162$$

$$S_3 \rightarrow D_3, 32 \text{ units cost } 32 \times 7 = 224$$

$$\text{Total cost} = 876$$

#### 5.5 Third Quadrant Method (-,-)

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	6	8	8	5 <sub>25</sub>	30

$S_2$	5 <sub>12</sub>	11 <sub>28</sub>	9	7	40
$S_3$	8 <sub>23</sub>	9	7	13	50
Demand	35	28	32	25	120

Table 5.5 Allotment Table

Total Cost :

$$S_1 \rightarrow D_3, 5 \text{ units cost } 5 \times 8 = 40$$

$$S_1 \rightarrow D_4, 25 \text{ units cost } 25 \times 5 = 125$$

$$S_2 \rightarrow D_1, 12 \text{ units cost } 12 \times 5 = 60$$

$$S_2 \rightarrow D_2, 28 \text{ units cost } 28 \times 11 = 308$$

$$S_3 \rightarrow D_1, 23 \text{ units cost } 23 \times 8 = 184$$

$$S_3 \rightarrow D_3, 27 \text{ units cost } 27 \times 7 = 189$$

Total Cost = 906

### 5.6 Forth Quadrant Method (+,-)

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	6 <sub>5</sub>	8	8	5 <sub>25</sub>	30
$S_2$	5 <sub>12</sub>	11 <sub>28</sub>	9	7	40
$S_3$	8 <sub>18</sub>	9	7	13	50
Demand	35	28	32	25	120

Table 5.6 Allotment Table

$$S_1 \rightarrow D_3, 5 \text{ units cost } 5 \times 6 = 30$$

$$S_1 \rightarrow D_4, 25 \text{ units cost } 25 \times 5 = 125$$

$$S_2 \rightarrow D_1, 12 \text{ units cost } 12 \times 5 = 60$$

$$S_2 \rightarrow D_2, 28 \text{ units cost } 28 \times 11 = 308$$

$$S_3 \rightarrow D_2, 18 \text{ units cost } 18 \times 8 = 144$$

$$S_3 \rightarrow D_3, 32 \text{ units cost } 32 \times 7 = 224$$

Total Cost = 891

## 5.2 MDMA Method

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$S_1$	6	8 <sub>5</sub>	8	5 <sub>25</sub>	30
$S_2$	5 <sub>35</sub>	11 <sub>5</sub>	9	7	40
$S_3$	8	9 <sub>18</sub>	7 <sub>35</sub>	13	50
Demand	35	28	32	25	120

Table 5.7 Allotment Table

Table Total Cost :

$$S_1 \rightarrow D_2, 5 \text{ units cost } 5 \times 8 = 40$$

$$S_1 \rightarrow D_4, 25 \text{ units cost } 25 \times 5 = 125$$

$$S_2 \rightarrow D_1, 35 \text{ units cost } 35 \times 5 = 175$$

$$S_2 \rightarrow D_2, 5 \text{ units cost } 5 \times 11 = 55$$

$$S_3 \rightarrow D_2, 18 \text{ units cost } 18 \times 9 = 162$$

$$S_3 \rightarrow D_3, 32 \text{ units cost } 32 \times 7 = 224$$

Total cost = 781

## 5.3 Comparative Study on OFSTF with MDMA Method

Methods	Total Cost
Origin	891
First Quadrant	902

OFSTF	Second Quadrant	876
	Third Quadrants	906
	Fourth Quadrant	891
MDMA		781

Table 5.8 Comparative Table on OFSTF with MDMA Method

#### 5.4 Comparative study on OFSTF, MDMA with North West Corner Method

Comparative Study on the same problem with other methods-OFSTF Method, MDMA Method, total cost have been reduced to North West Corner Method cost.

Methods		Total Cost	Reduced % with NWC cost
OFSTF	Origin	891	17%
	First Quadrant	902	16%
	Second Quadrant	876	19%
	Third Quadrant	906	16%
	Fourth Quadrant	891	17%
MDMA Method		781	27%
North West Corner Method		1076	

Table 5.9 Comparative Table on OFSTF, MDMA with North West Corner Method

#### 5.5 Optimal Solution of OFSTF, MDMA Methods with Existing Methods Comparison

The Comparative study on MDMA method , OFSTF method with the existing methods North West Corner method and Least Cost method. NCW /LC / MDMA / ORIGIN / FQ / SQ / TQ / and FHQ analysis made with Numerical Example is discussed.

Consider the following cost minimizing transportation problems

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	Supply
$S_1$	3	4	6	8	9	20
$S_2$	2	10	1	5	8	30
$S_3$	7	11	20	40	3	15
$S_4$	2	1	9	14	16	13
Demand	40	6	8	18	6	78

Table 5.10 Transportation Table

Now Using the OFSTF method, TC in origin = 777, TC1Q = 789, TC2Q = 255, TC3Q = 327, TC4Q = 825.

### MDMA Method

Consider the following CMITP.

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	Supply
$S_1$	3	4	6	8	9	20
$S_2$	2	10	1	5	8	30
$S_3$	7	11	20	40	3	15
$S_4$	2	1	9	14	16	13
Demand	40	6	8	18	6	78

Table 5.11 Cost Minimizing Transportation Table

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	Supply
--	-------	-------	-------	-------	-------	--------

$S_1$	3 <sub>11</sub>	4	6	8 <sub>9</sub>	9	20
$S_2$	2 <sub>22</sub>	10	1 <sub>8</sub>	5	8	30
$S_3$	7	11	20	40 <sub>9</sub>	3 <sub>6</sub>	15
$S_4$	2 <sub>7</sub>	1 <sub>6</sub>	9	14	16	13
Demand	40	6	8	18	6	78

Table 5.12 MDMA Method Optimal Allotment Table

$$S \rightarrow D_1 \quad 11 \text{ units} \quad : \quad \text{Cost} \quad 11 \times 3 = 33$$

$$S \rightarrow D_4 \quad 9 \text{ units} \quad : \quad \text{Cost} \quad 9 \times 8 = 72$$

$$S \rightarrow D_1 \quad 22 \text{ units} \quad : \quad \text{Cost} \quad 22 \times 2 = 44$$

$$S \rightarrow D_3 \quad 8 \text{ units} \quad : \quad \text{Cost} \quad 8 \times 1 = 8$$

$$S \rightarrow D_4 \quad 9 \text{ units} \quad : \quad \text{Cost} \quad 9 \times 40 = 360$$

$$S \rightarrow D_5 \quad 6 \text{ units} \quad : \quad \text{Cost} \quad 6 \times 3 = 18$$

$$S \rightarrow D_1 \quad 7 \text{ units} \quad : \quad \text{Cost} \quad 7 \times 2 = 14$$

$$S \rightarrow D_2 \quad 6 \text{ units} \quad : \quad \text{Cost} \quad 6 \times 1 = 6$$

$$\text{Total cost} = 555$$

### Result

North West Corner Method : 918

Least Cost Method : 555

MDMA Method : 555

OFSTF Origin Method : 777

OFSTF FQ Method : 789

OFSTF SQ Method : 255

OFSTF TQ Method : 327

OFSTF FHQ Method : 825

OFSTF SQ Method  $\leq$  OFSTF TQ Method  $\leq$  Least Cost Method  $\leq$  MDMA Method  $\leq$  OFSTF Origin Method  $\leq$  OFSTF FQ Method  $\leq$  OFSTF FHQ Method  $\leq$  North West Corner Method.

Thus the OFSTF method provides an feasible value of the objective function for the transportation problem. The proposed algorithm carries systematic procedure, and very easy to understand. It can be extended to assignment problem and travelling salesman problems to get an optimal solution. The proposed method is important tool for the decision makers when they are handling various types of logistic problems, to make the decision optimally and from the comparison MDMA leads the an optimal solution other than all the methods. The comparative analysis with best solution of the given problem in prescribed method here the compared and conclude the results of the different methods. These methods are important tool for the decision makers when they are handling various types of logistic problems to make the decision optimally.

## CONCLUSION

Thus the OFSTF method, MDMA method, Origin max to max method, Origin max to min method, centralized max-max method centralized max-min method, OFSTF to Lagrange's method provides an feasible value of the objective function for the transportation problem. The proposed algorithm carries systematic procedure, and very easy to understand. It can be extended to assignment problem and travelling sales man problems to get optimal solution. The proposed method is important tool for the decision makers when they are handling various types of logistic problems, to make the decision optimally. The cost analysis method provides the minimum number of negative solution with maximum number of optimal positive solution for Non Liner Programming Problem.

In chapter 3, a different approach OFSTF (Origin, First, Second, Third and Fourth quadrants) methods is applied for finding a feasible solution for transportation problems directly. The proposed method is a unique, it is gives always feasible (may be an optimal for some extant) solution without disturbance of degeneracy condition. This method takes least iterations to reach optimally. A numerical example is solved to check the validity of the proposed methods also degeneracy problem is also discussed.

In chapter 4, deals with MDMA (Maximum Divided Minimum Allotment) methods, is applied for finding the feasible solution for transportation problem. The proposed algorithm is unique way to reach feasible (or) may be an optimal (for some extant) solution without disturbance of degeneracy condition. Also some numerical problem is discussed.

In chapter 5, The comparative study of MDMA method and OFSTF method discussed. MDMA is better than OFSTF for proposed pay off matrix After analysis we have concluded and some numerical problem discussed with conclusion and also the Comparative study on MDMA method with OFSTF method in Transportation Problem discussed with existing methods so called NWC / LC / MDMA / ORIGIN / FQ / SQ / TQ / and FQ analysis made with numerical example is discusse

## BIBLIOGRAPHY

1. Abdual Quddoos, Shakeel Javaid, M.M. Khalid. (2012), A New Method for Finding an Optimal Solution for Transportation Problems, International Journal on Computer Science and Engineering vol. 4, Issue7, 1271-1274, 2012.
2. Arsham, H and AB Kahn., A simplex-type algorithm for general transportation problems: An alternative to Stepping-Stone, Journal of Operational Research Society, 40(6), 581-590, 1989.
3. Balogun O.S., E.T. Jolayemi, T.J. Akingbade, H.G. Muazu, Use of Linear Programming for Optimal Production in a Production Line in Coca –Cola 242 Bottling Company, Ilorin, International Journal of Engineering Research and Applications, 2(5), 2004-2007, 2012.
4. Chongyu Jiang, Xiaorong Li, Yandong Li, Application of Linear Programming Model to Refugee Migrating Problem, Journal of Applied Mathematics and Physics, 4, 967- 977, 2016.
5. Charnes, A and WW Cooper., The Stepping-Stone method for explaining linear programming calculations in transportation problems, Management Science, 1(1), 49-69, 1954.
6. Dantzig, GB., Linear Programming and Extensions, Princeton, NJ: Princeton University Press, 1963.
7. Dantzig G.B., Application of the simplex method to a transportation problem, Activity Analysis of production and allocation. (T.C. Koopmans ed.) Wiley, New York, 359-373, 1951.
8. Gass, SI., On solving the transportation problem, Journal of Operational Research Society, 41(4), 291-297, 1990.
9. Dhose, ED and KR Morrison., Using transportation solutions for a facility location problem, Computers and Industrial Engineering, 31(1/2), 63-66, 1996.
10. Goyal, SK Improving VAM for unbalanced transportation problems, Journal of Operational Research Society, 35(12), 1113-1114, 1984.

11. Goyal, SK., A note on a heuristic for obtaining an initial solution for the transportation problem, *Journal of Operational Research Society*, 42(9), 819-821, 1991.
12. Hamdy, A.T., *Operations Research: An Introduction*. 8th Edition, Pearson Prentice Hall, Upper Saddle River, 2007.
13. Ji, P and KF Ghu., A dual-matrix approach to the transportation problem, *Asia-Pacific Journal of Operations Research*, 19(1), 35-45, 2002.
14. Joshua RR, Akilandeswari VS, Lakshmi Devi PK and Subashini N., North- East Corner Method- An Initial Basic Feasible Solution for Transportation Problem. *International Journal for Applied Science and Engineering technology* 5(5), 123-131, 2017.
15. Kasana, H.S. and Kumar, K.D., *Introductory Operations Research: Theory and Applications*. Springer International Edition, New Delhi, 2005.
16. Kirca, O and A Satir., A heuristic for obtaining an initial solution for the transportation problem, *Journal of Operational Research Society*, 41(9), 865- 871, 2002.
17. Korukoglu S, Balli S., An Improved Vogel's Approximation method for the Transportation Problem. *Association for Scientific research, Mathematical and computational Application* 16(2), 370-381, 2011.
18. Kourosh Rajeiyan, Farhang Khalaghdoost Nejati, Reza Hajati, Hamid Reza Safari, Ebrahim Alizadeh, Using Linear Programming in Solving the Problem of Services Company's Costs, *Singaporean Journal Of Business Economics, and Management Studies*, 1(10),2013.
19. Luenberger, D. G., *Linear and Nonlinear Programming*, Addison- Wesley, Reading, MA, 1984.
20. Mangasarian O. L., *Non-linear Programming*, Wiley, New York (1969).
21. Pandian P, Natarajan G., A New Approach for Solving Transportation Problem with Mixed Constraints, *Journal of Physical Sciences* 14(1), 53-61, 2010.
22. Quddos A , Javaid S, Khalid MM., A New Method for finding an Optimal Solution for Transportation Problems. *International Journal on Computer Science and Engineering* 4(7), 1271-1274, 2012.

23. Ramakrishnan, GS., An improvement to Goyal's modified VAM for the unbalanced transportation problem, *Journal of Operational Research Society*, 39(6), 609-610, 1988.
24. Reena, Patel G, Bhathawla PH., The New Global Approach to Transportation Problem, *International Journal of Engineering Technology, Management and Applied Science* 2(3), 109-113, 2014.
25. Reena, Patel G, Bhathawla PH., An Innovative Approach to Optimum Solution of Transportation Problem. *International Journal of Innovative Research in Science, Engineering Technology*, 5(4), 5695- 5700, 2016.
26. Sharma J.K., *Operations Research –Theory and Applications*, Macmillan India (LTD), New Delhi, 2005.
27. Shafaat, A and SK Goyal., Resolution of degeneracy in transportation problems, *Journal of Operational Research Society*, 39(4), 411-413, 1988  
Sharma J.K., *Operations Research –Theory and Applications*, Macmillan India (LTD), New Delhi, 2005.
28. Shajma, R.R.K and K.D Sharma., A new dual based procedure for the transportation problem, *European Journal of Operational Research*, 122, 611- 624, 2000.
29. Shih, W., Modified Stepping-Stone method as a teaching aid for capacitated transportation problems, *Decision Sciences*, 18, 662-676, 1987.
30. Shimshak, D.G, J.A Kaslik and T.D Barclay., A modification of Vogel's approximation method through the use of heuristics, *INEOR*, 19, 259-263, 1981.
31. Sofi N. A., Aquil Ahmed, Mudasar Ahmad, Bilal Ahmad Bhat, *Decision Making in Agriculture: A Linear Programming Approach*, *International Journal of Modern Mathematical Sciences*, 13(2), 160-169, 2015.
32. Sudhakar. V. J, Arunsankar. N and Karpagam. T., A New Approach for Finding an Optimal Solution for Transportation Problems, *European Journal of Scientific* Vol.68 No.2, 254-257, 2012.
33. Swarup K., Gupta P.K., Mohan M., *Operations Research*, Sultan Chand and Sons, New Delhi, 2006.
34. Taha. H. A., *Operations Research –Introduction*, Prentice Hall Of India (PVT), New Delhi, 2004.

35. Urashikumari, Patel D, Dhavakumar, Ravi, Bhasvar C., Transportation Problem Using Stepping Stone Method and its Application, International Journal of Advanced Research in Electrical, electronics and Instrumentation Engineering 6(1), 46-50, 2017.
36. Vipul Parkhi, Pooja Pawar and Archana Surve, Computer Automation for Malaria Parasite Detection Using Linear Programming, International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering,2(5),2013.