Introduction

INTRODUCTION

Zadeh [40] introduced the notion of fuzzy sets in 1965. Goguen [17] introduced the concept of L-fuzzy sets (where L is a completely distributive lattice) in 1967. Atanassov [3] introduced the notion of intuitionistic fuzzy sets in 1986. Using the concept of fuzzy sets Chang [11] introduced the notion of a fuzzy topology in 1968. Later, in 1976, Lowen [24] redefined it to overcome certain drawbacks in Chang's definition. In 1985, Sostak [33] introduced the notion of a smooth topology as an extension of Chang and Lowen's fuzzy topology. Using the concept of L-fuzzy sets, a theory parallel to that of fuzzy topological spaces was developed in the case of L-fuzzy topological spaces. Using the concept of intuitionistic fuzzy sets Coker [13] introduced the idea of intuitionistic fuzzy topological spaces in 1997. Samanta and Mondal [30] introduced the notion of an intuitionistic gradation of openness as an extension of a smooth topology in Sostak's sense. Like this, the concept of topological spaces has been extended to fuzzy situation by various authors in different ways. Almost all the concepts in general topological spaces have been extended to these topological spaces.

Connectivity is an important notion in general topology. There has been much work on connectedness in fuzzy topological spaces [1, 7, 10, 14, 16, 25, 29, 31, 41, 43] in L-fuzzy topological spaces [4, 5, 6, 9, 12, 23, 35, 37, 38, 39] and in intuitionistic fuzzy topological spaces [21, 27, 28, 36]. In this dissertation we have chosen the following articles for our discussion :

1. Ghosh, B., Semi-connectedness in fuzzy topological spaces [16]
2. Bai, S.Z., P-connectedness in L-topological spaces [5]
3. Li, S.P., Fang, Z., and Zhao, J., P2-connectedness in L-topological spaces [23]
4. Kim, Y.C. and Abbas, S.E., Connectedness in intuitionistic fuzzy topological spaces [21]

In chapter I we have considered semi-connectedness in fuzzy topological spaces. The results are due to Ghosh [16]. He [16] has defined the concept of fuzzy semi-separated sets by using the notion of quasi-coincidence introduced by Pu Pao Ming and Liu Ying Ming [29]. Some interesting properties of fuzzy semi-separated sets are discussed and a characterization theorem for fuzzy semi-separated sets is obtained. Using the notion of fuzzy semi-separated sets, Ghosh [16] has introduced the notion of fuzzy semi-connected sets as a generalization of fuzzy connected sets introduced by Ganguly and Saha [15]. It is seen that every fuzzy semi-connected set is fuzzy connected but the converse is not true in general. An example is discussed to illustrate this. Properties of fuzzy semi-connected sets which are analogous to the corresponding properties in general topological spaces are discussed. Characterizations of fuzzy semi-connected sets are obtained. It has been proved that fuzzy semi-connectedness is preserved by a fuzzy irresolute mapping.

In chapter II we discuss the concepts of P-connectedness and P2-connectedness in L-fuzzy topological spaces introduced by Bai [5] and Li, Fang and Zhao [23] respectively. Both the concepts are defined using the notion of preclosed sets. Some fundamental properties of connectedness in general topological spaces are preserved by P-connectedness as well as by P2-connectedness. It has been proved that the preclosure of a P-connected (resp. P2-connected) set is P-connected (resp. P2-connected) and union of P-connected (resp. P2-connected) sets is P-connected (resp. P2-connected) provided their intersection is a non-null set. Moreover, the image of a P-connected (resp. P2-connected) space under a P-irresolute order-homomorphism is P-connected (resp. P2-connected). Some interesting characterizations of P-connectedness (resp. P2-connectedness) are discussed. K.Fan's theorem has been extended to both P-connectedness and P2-connectedness. Relations among connectedness, P-connectedness and P2-connectedness are discussed. It has been shown that every P2-connected L-fuzzy set is connected as well as P-connected. Interesting examples are given to show that the converse is not true. But if $1 \in M(L)$, then every P-connected L-fuzzy set is also P2-connected. Moreover, it has been proved that if D is a crisp set, then D is P-connected iff it is P2-connected.

In chapter III we discuss the concept of connectedness in intuitionistic fuzzy topological spaces (IFTS, for short) introduced by Kim and Abbas [21]. Using the intuitionistic closure operator $C_{\mathcal{T},\mathcal{T}^*}$ (Theorem 1.4) associated with an IFTS $(X, \mathcal{T}, \mathcal{T}^*)$ they [21] have introduced the notion of (r, s)-separated sets which has led to the concept of (r, s)-connected sets. An example of a (r, s)-connected set is given. Some interesting properties of (r, s)-separated sets, (r, s)-connected sets and (r, s)-components are discussed. It has been proved that

(1) If λ is (r, s)-connected and $\lambda \leq \mu \leq C_{\mathcal{T},\mathcal{T}^*}(\lambda, r, s)$, then μ is (r, s)-connected.

(2) If λ and μ are (r, s)-connected fuzzy sets which are not (r, s)-separated, then $\lambda \vee \mu$ is (r, s)-connected.

(3) Union of (r, s)-connected sets is (r, s)-connected provided their intersection is a non-null set.

(4) (r, s)-connectedness is preserved by an intuitionistic continuous mapping.

Equivalent conditions for a fuzzy set to be (r, s)-connected are obtained. The authors [21] have also obtained a stratification of an IFTS and proved that every (r, s)-component in an IFTS is a (r, s)-component in the stratification of it.