Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – July 2020
VI Semester

Class : III UG **Time : 2 Hours**
Major : Mathematics **Max.Marks : 50**

15BMAC23- COMPLEX ANALYSIS II

Part A **10 x 1 =10**
Choose the Correct Answer

1. The zeros of an analytic function f(z) = $z^2 - 3z + 2$ is
   a. 1 and 2 b. -1 and 2 c. -1 and -2 d.1 and -2

2. The singular points of 1+ e^z is ___________
   a. 0 b. ∞ c. 1 d. ± nπ , n = 0,1,2,3….

3. The Taylors series of log (1+z) about z = 0 is
   a. $1-\frac{z^2}{2}+\frac{z^3}{3}-\frac{z^4}{4}+.....\ ,|z|<1$ b. $z+\frac{z^2}{2}+\frac{z^3}{3}+\frac{z^4}{4}+.....\ ,|z|<1$
   c. $z-\frac{z^2}{2}+\frac{z^3}{3}-\frac{z^4}{4}+.....\ ,|z|<1$ d. $z-\frac{z^2}{2}+\frac{z^3}{3}-\frac{z^4}{4}+.....\ ,|z|>1$

4. If the principal part of the Laurent's series is zero, then the Laurent's series reduces to
   a. Maclaurin's series b. Cauchy series
   c. Taylor's series d. None of the above

5. The nature of the singularity z = 0 of the function $f(z)=\frac{\sin z - z}{z^3}$ is
   a. pole b.essential c. non isolated d. removable

6. The order of the pole of the function $\frac{e^z}{z^2}$ is
   a. simple b. double c. triple d. pole does not exist

7. If z=a is the removable singularity of a function f(z) then the residue of f(z) at z =a is
   a. a b ia c. 0 d. not exist

8. If z=a is a pole of function f(z) then $\lim_{z\to a} f(z)$ is
   a. 0 b. ∞ c. any complex number d. finite
   b.

9. The residue of f(z)=$\frac{1+e^z}{\sin z + z\cos z}$
   a. 1 b. 2 c. 0 d. -1

10. If C is a simple closed curve encircling the origin then $\int_C \frac{e^z}{z} dz$

a. 4πi b.2πi c. 0 d. 6πi

Part B **3 x 6 = 18**

Answer any **Three** questions

Each answer should not exceed 400 words or two pages

11. State and Prove Maximum modulus theorem.
12. State and prove Liouville's theorem.
13. Find the Taylor's expansion about z = 1 of $f(z) = \frac{1}{z}$.
14. Given a point z = a and a function f(z), suppose f(z) is analytic in the annular domain D in between the largest and smallest concentric circles C_1 and C_2 having their centres at z = a. Then for any z in D, prove that f(z) = $\sum_{n=-\infty}^{\infty} \left[\frac{1}{2\pi i} \int_C \frac{f(Z)}{(Z-a)^{n+1}} dZ \right] (z-a)^n$ where C is any simple closed curve around z = a, lying in D and Z is on C.
15. If z=a is a removable singularity of a function f(z) then prove that there exists a deleted neighbourhood of z=a in which f(z) is bounded.
16. Find the nature of the singularity f(z) = $\frac{\tan z}{z}$
17. If C is the circle $|z - i| = 2$ then evaluate $\int_C \frac{z}{2z^2+5iz-2} dz$.
18. Find the residues of $\frac{z \sin z}{(z-\pi)^3}$.
19. Evaluate $\int_0^{2\pi} \frac{1}{a+b\ sin\theta}\ d\theta$, a > |b| >0
20. Using contour integration $\int_0^{2\pi} \frac{1}{2+\cos\theta} d\theta$

Part C **2 x 11 = 22**

Answer any **Two** questions

Each answer should not exceed 800 words or four pages

21. State and Prove Fundamental theorem of Algebra.
22. Find the Laurent's series expansion for $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in (i) 2< |z| <3 (ii) |z| >3
23. State and prove Taylor's theorem.
24. Find the poles and order of $\frac{1}{z^3(z+4)}$.
25. Find the residues of f(z)=$\frac{1}{(z^2+a^2)^2}$ at its singularities.
26. State and prove Weierstrass theorem.
27. State and prove Cauchy residue theorem.
28. Evaluate $\int_C \frac{Z \sec Z}{1-Z^2}\ dz$ where C is the ellipse $4x^2 + 9y^2 = 9$.
29. Evaluate $\int_0^{\infty} \frac{2x^2-1}{x^4+5x^2+4}\ dx$ using contour integration.
30. By using contour integration evaluate $\int_0^{2\pi} \frac{sin^2\theta}{a+b\cos\theta}\ d\theta$