CHAPTER - VII

CHAPTER VII

SOFT $\hat{g}$-CLOSED SETS IN SOFT TOPOLOGICAL SPACES

Definition 7.1

A subset A of a topological space (X, τ) is called

i. a **semi open** set if $A \subseteq Cl(Int(A))$ and a **semi closed** set if $Int(Cl(A)) \subseteq A$.

ii. a **pre open** set if $A \subseteq Int(Cl(A))$ and a **pre closed** set if $Cl(Int(A)) \subseteq A$.

iii. an **α-open** set if $A \subseteq Int(Cl(Int(A)))$ and an **α -closed** set if $Cl(Int(Cl(A))) \subseteq A$.

iv. a **regular open** set if $A = Int(Cl(A))$ and a **regular closed** set if $A = Cl(Int(A))$.

v. a **generalized closed** set(briefly **g-closed**) if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is open in (X, τ).The complement of a g-closed set is called a **g-Open** set.

vi. a **semi generalized closed** set(briefly **sg-closed**) if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is semi open in (X, τ).

vii. a **generalized α -closed** set(briefly **αg-Closed**) if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is α-open in (X, τ).

viii. a **regular generalized closed** set(briefly **rg-Closed**) if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is regular open in (X, τ).

ix. a **$\hat{g}$-Closed** set if $Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is semi open in (X, τ).The complement of a $\hat{g}$-closed set is called a **$\hat{g}$-Open** set.

Definition 7.2

In a Soft Topological Spaces (X, τ, E), a soft set (F, E) over X is called

i. a **soft semi open** if $(F, E) \subseteq Cl\big(Int(F, E)\big)$ and **soft semi closed** if $Int(Cl(F, E)) \subseteq (F, E)$.

ii. a **soft pre open** set if $(F, E) \subseteq Int(Cl(F, E))$ and **soft pre closed** set if $Cl\big(Int(F, E)\big) \subseteq (F, E)$.

iii. a **soft α-open** set if $(F,E) \subseteq \mathrm{Int}\left(\mathrm{Cl}\left(\mathrm{Int}(F,E)\right)\right)$ and **soft α-closed** set if $\mathrm{Cl}\left(\mathrm{Int}(\mathrm{Cl}(F,E))\right) \subseteq (F,E)$.

iv. a **soft regular open** if $(F,E) = \mathrm{Int}\left(\mathrm{Cl}(F,E)\right)$ and **soft regular closed** if $(F,E) = \mathrm{Cl}\left(\mathrm{Int}(F,E)\right)$.

v. a **soft generalized closed** set(briefly **soft g-closed**) if $\mathrm{Cl}(F,E) \subseteq (G,E)$ whenever $(F,E) \subseteq (G,E)$ and (G,E) is soft open in (X,τ,E).The complement of a soft g-closed set is called a **soft g-open** set.

vi. a **soft semi generalized closed** set(briefly **Soft sg Closed**) if $\mathrm{Cl}(F,E) \subseteq (G,E)$ whenever $(F,E) \subseteq (G,E)$ and (G,E) is soft semi open in (X,τ,E).

vii. a **soft generalized α -closed** set(briefly **soft αg-Closed**) if $\mathrm{Cl}(F,E) \subseteq (G,E)$ whenever $(F,E) \subseteq (G,E)$ and (G,E) is soft α-open in (X,τ,E).

viii. a **soft regular generalized closed** set(briefly **Soft rg-Closed**) if $\mathrm{Cl}(F,E) \subseteq (G,E)$ whenever $(F,E) \subseteq (G,E)$ and (G,E) is soft regular open in (X,τ,E).

Definition 7.3

Let (X,τ,E) be a soft topological space. A soft set (F,E) is called a **Soft ĝ-Closed** set if $\mathrm{Cl}(F,E) \subseteq (G,E)$ whenever $(F,E) \subseteq (G,E)$ and (G,E) is soft semi open set in (X,τ,E).

Example 7.4

Let $X = \{a,b,c\}$, $E = \{e_1, e_2\}$ and

$\tau = \{\Phi, \tilde{E}, (G_1,E), (G_2,E), (G_3,E), (G_4,E), (G_5,E), (G_6,E), (G_7,E)\}$ where

$G_1(e_1) = \{a,b\}$ $\quad G(e_2) = \{a,b\}$

$G(e_1) = \{b\}$ $\quad G_2(e_2) = \{a,c\}$

$G_3(e_1) = \{b,c\}$ $\quad G(e_2) = \{a\}$

$G_4(e_1) = \{b\}$ $\quad G_4(e_2) = \{a\}$

$G_5(e_1) = \{a,b\}$ $\quad G_5(e_2) = X$

$G_6(e_1) = X$ $\quad G_6(e_2) = \{a,b\}$

$G_7(e_1) = \{b,c\}$ $\quad G_7(e_2) = \{a,c\}$

Consider the soft set (G,E) over X such that

$F(e_1) = \{a\}$ $\quad F(e_2) = \{c\}$

Clearly (G,E) is soft ĝ-Closed set in (X,τ,E).

Theorem 7.5

Every soft closed set is a soft $\hat{g}$-closed set.

Proof

Let (F, E) be soft closed set in (X,τ, E) and (G, E) be soft semi open set such that $(F, E) \subseteq (G, E)$. Consider $Cl(F, E) = (F, E) \subseteq (G, E)$. Therefore (F, E) is soft $\hat{g}$-closed set in (X,τ, E).

Theorem 7.6

Every soft $\hat{g}$-closed set is a soft g-closed set.

Proof

Let (F, E) be soft $\hat{g}$-closed set in (X,τ, E). Let (G, E) be any soft open in X such that $(F, E) \subseteq (G, E)$. Since every soft open set is soft semi open set. Since (F, E) is soft $\hat{g}$-Closed set then $Cl(F, E) \subseteq (G, E)$. Therefore (F, E) is soft g-closed set.

Therefore the class of $\hat{g}$-closed sets is properly contained in the class of g-closed sets and properly contains the class of closed sets.

Theorem 7.7

Every soft $\hat{g}$-closed set is soft rg-closed set.

Proof

Let (F, E) be soft $\hat{g}$-closed set in (X,τ, E) and (H, E) be soft regular open set such that $(F, E) \subseteq (H, E)$. Then $(F, E) \subseteq Int(Cl((H, E))$. Since $Int(Cl((H, E))$ is soft semi-open set containing the soft $\hat{g}$-closed set, then $Cl(F, E) \subseteq Int(Cl((H, E))$. Therefore (F, E) is soft rg-closed set in (X,τ, E).

Theorem 7.8

Union of any two soft $\hat{g}$-closed sets is a soft $\hat{g}$-closed set.

Proof

Suppose (F, E) and (H, E) are soft $\hat{g}$-closed sets in (X, τ, E). Then $Cl(F, E) \subseteq (G, E)$ and $Cl(H, E) \subseteq (G, E)$ where $(F, E) \subseteq (G, E)$ and $(H, E) \subseteq (G, E)$.

Hence $Cl((F,E) \cup (H,E)) = Cl(F,E) \cup Cl(H,E) \subseteq (G,E)$. That is $Cl((F,E) \cup (H,E)) \subseteq (G,E)$. Therefore $(F,E) \cup (H,E)$ is a soft $\hat{g}$-closed set in (X,τ,E).

In a similar way one can prove.

Intersection of any two soft $\hat{g}$-closed sets is a soft $\hat{g}$-closed set.

Theorem 7.9

Let (F,E) be a soft $\hat{g}$-closed set in (X,τ,E). Then $Cl(F,E) \backslash (F,E)$ does not contain any non-empty soft semi closed set.

Proof

Let (A,E) be a soft semi closed set in (X,τ,E) and $(A,E) \subseteq Cl(F,E) \backslash (F,E)$. Then $(A,E) \subseteq Cl(F,E)$ and $(A,E) \subseteq (F,E)^C$. This implies $(F,E) \subseteq (A,E)^C$. Then $Cl(F,E) \subseteq (A,E)^C$. This implies $(A,E) \subseteq (Cl(F,E))^C$. Therefore$(A,E) \subseteq Cl(F,E) \cap (Cl(F,E))^C$. Hence (A,E) does not contain any non-empty soft semi closed set in (X,τ,E).

Theorem 7.10

If (F,E) is a soft $\hat{g}$-closed set in (X,τ,E) and $(F,E) \subseteq (H,E) \subseteq Cl(F,E)$ then(H,E) is also a soft $\hat{g}$-closed set.

Proof

Let (F,E) be soft $\hat{g}$-Closed set in (X,τ,E) such that $(F,E) \subseteq (H,E) \subseteq Cl(F,E)$. Let (G,E) be a soft semi open set in (X,τ,E) such that $(H,E) \subseteq (G,E)$. Then $(F,E) \subseteq (G,E)$. Since (F,E) is soft $\hat{g}$-closed set, $Cl(F,E) \subseteq (G,E)$. Now $Cl(H,E) \subseteq Cl(Cl(F,E)) = Cl(F,E) \subseteq (G,E)$. That is $Cl(H,E) \subseteq (G,E)$. Therefore (H,E) is soft $\hat{g}$ -Closed set in(X,τ,E).

Theorem 7.11

Let (F,E) be a soft $\hat{g}$-closed set in a soft topological space (X,τ,E). Then

(i) $ssInt(F,E)$ is soft $\hat{g}$-closed.

(ii) $spCl(F,E)$ is soft $\hat{g}$-closed.

(iii) If (F,E) is soft regular open then $spInt(F,E)$ and $ssCl(F,E)$ are also soft $\hat{g}$-closed sets.

Proof

First we note that for a (F, E) of (X,τ, E), $ssCl(F, E) = (F, E) \cup Int(Cl(F, E))$ and $spCl(F, E) = (F, E) \cup Cl(Int(F, E))$. Moreover $ssInt(F, E) = (F, E) \cap Cl(Int(F, E))$ and $spInt(F, E) = (F, E) \cap Int(Cl(F, E))$.

(i) Since $Cl(Int(F, E))$ is a soft closed set, then (F, E) and $Cl(Int(F, E))$ are soft $\hat{g}$-closed sets. Therefore, $(F, E) \cap Cl(Int(F, E))$ is also a soft $\hat{g}$-closed set.

(ii) $spCl(F, E)$ is the union of two soft $\hat{g}$-closed sets (F, E) and $Cl(Int(F, E))$. Again by the theorem 7.8, $spCl(F, E)$ is a soft $\hat{g}$-closed.

(iii) Since (F, E) is soft regular open, then $(F, E) = Int(Cl(F, E)$. Then

$ssInt(F, E) = (F, E) \cup Int\big(Cl(F, E)\big) = (F, E)$. Thus $ssCl(F, E)$ is soft $\hat{g}$-closed. Similarly $spInt(F, E)$ is also a soft $\hat{g}$-closed set.