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Double Acceptance Sampling Based on Truncated Life Tests in Marshall – Olkin Extended Lomax Distribution

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Abstract

In this paper, double acceptance sampling plans for truncated life tests are developed when the lifetimes of test items follows Marshall – Olkin extended Lomax distribution. Probability of acceptance is calculated for different consumer's confidence levels fixing the producer's risk. Probability of acceptance and producer's risk are discussed with the help of tables and examples.

Keywords: Truncated Life Test, Consumer's Risk, Producer's Risk, Marshall – Olkin Extended Lomax distribution, Double Acceptance Sampling Plan.

Introduction

An acceptance sampling plan involves quality contracting on product orders between the producers and the consumers. It is an essential tool in the Statistical Quality Control. In most of the statistical quality control experiment, it is not possible to perform hundred percent inspection, due to various reasons. The acceptance sampling plan was first applied in the US Military for testing the bullets during World War II. For example, if every bullet was tested in advance, no bullets were available for shipment, and on the other hand if no bullets were tested, then disaster might occur in the battle field at the crucial time. Acceptance sampling plan is a `middle path' between hundred percent inspection and no inspection at all. In a time – truncated sampling plan, a random sample is selected from a lot of products and put on the test where the number of failures is recorded until the pre – specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Two risks are always attached to an acceptance sampling. The probability of rejecting a good lot is known as the producer's risk and the probability of accepting a bad lot is called the consumer's risk. An acceptance sampling plan should be designed so that both risks are smaller than the required values. These life tests are discussed by many authors, Goode and Kao (1961) have studied sampling plans based on the distributions, Gupta and Groll (1961) have studied application of Gamma distribution in acceptance sampling based on life tests, Aslam M. and Shabaz M.Q. (2007) have studied Economic reliability test plans using the generalized exponential distribution, Balakrishnan N., Leiva V. and Lopez J. (2007) have studied Acceptance sampling plans from truncated life test based on generalized Birnbaum-Saunders distribution, Muhammad Aslam have studied Double acceptance sampling based on truncated life tests in Rayleigh distribution, Srinivasa Rao (2010) have studied Group acceptance sampling plans based on truncated life tests for Marshall – Olkin extended Lomax distribution, Srinivasa Rao S.(2011) have studied Double acceptance sampling plans based on truncated life tests for the Marshall – Olkin extended exponential distribution. All these authors developed the sampling plans for life tests using single acceptance sampling. We in this paper propose a plan to find the probability of acceptance for the double acceptance sampling assuming the experiment is truncated at pre – assigned time and lifetime follows Marshall – Olkin extended Lomax distribution.

Marshall – Olkin extended Lomax Distribution

The cumulative distribution function of the Marshall – Olkin extended Lomax distribution are given by

$$F(t,\sigma)=\frac{(1+t/\sigma)^{\theta}-1}{(1+t/\sigma)^{\theta}-\bar{\gamma}} \qquad (1)$$

where $\bar{\gamma}=1-\gamma$ and γ is the index parameter and let us assume $\gamma = 2$, σ is scale parameter and θ is shape parameter. If some other parameters are involved, then they are assumed to be known, for an example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/σ.

Design of the Proposed Plan

It is known that the double acceptance sampling plan (DASP) is more efficient than the single sampling plan in terms of the sample size required. Further, a DASP is expected to reduce the producer's risk when specifying the consumer's risk. We propose a following DASP based on a truncated life test:

- Draw the first sample of size n_1 and put them on test during time t_0.
- Accept the lot if there are no more than c_1 failures. Reject the lot and terminate the test if there are more than c_2 failures.
- If the number of failures is between c_1 and c_2, then draw the second sample of size n_2 and put them on test during time t_0.
- Accept the lot if the total number of failures from the first and second samples is no greater than c_2. Otherwise, reject the lot and terminate the test.

The DASP is composed of four parameters of (n_1, n_2, c_1, c_2) if t_0 is specified. Here n_1 and n_2 are sample sizes of the first and the second sample, respectively, whereas c_1 and c_2 are the acceptance numbers associated with the first and the second sample, respectively. The single sampling plan is a special case of DASP when $c_1 = c_2$.

We assume that the lot size is large enough to use the binomial distribution to find the probability of lot acceptance. The probability of acceptance $L(p_1)$ and $L(p_2)$ for the sampling plan (n_1, c_1, a) and (n_2, c_2, a) is calculated using the following equations,

$$L(p_1) = \sum_{i=0}^{c_1=0} \binom{n_1}{i} p^i (1-p)^{n_1-i} \tag{2}$$

$$L(p_2) = \sum_{i=0}^{c_2=2} \binom{n_2}{i} p^i (1-p)^{n_2-i} \tag{3}$$

Where,

$$p = \frac{(1+{}^{t}\!/_{\sigma})^{\theta} - 1}{(1+{}^{t}\!/_{\sigma})^{\theta} - \overline{\gamma}} \tag{4}$$

Then the probability of acceptance for DASP is given by P(A) = P(no failure occur in sample 1) + P(1 failure occur in sample 1 and 0,1 failure occur in sample 2) + P(2 failures occur in sample 1 and 0 failure occurs in sample 2).

$$L(p) = \binom{n_1}{0} p^0 q^{n_1} + \binom{n_1}{1} p^1 q^{n_1-1} \left[\sum_{i=0}^{1} \binom{n_2}{i} p^i q^{n_2-i} \right] + \binom{n_1}{2} p^2 q^{n_1-2} \left[\binom{n_2}{0} p^0 q^{n_2} \right] \tag{5}$$

Notations

n_1 -	First sample size
n_2 -	Second sample size
c_1 -	Acceptance number of sample first
c_2 -	Acceptance number of sample second
d -	Number of defectives
t -	Termination time
γ -	Index parameter
σ -	Scale parameter
θ -	Shape parameter
β -	Consumer's risk
p -	Failure probability
L(p) -	Probability of acceptance
p^* -	Minimum probability
σ_0 -	Specified life

Description of Tables and Examples:

In this study we fixed the consumer's risk which did not exceed 1-p*, where p* = 0.75, 0.90, 0.95, 0.99 and and satisfied the following inequality.

$$\sum_{i=0}^{c}\binom{n}{i}p^i(1-p)^{n-i} \leq 1-p^* \tag{6}$$

If in this experiment no failure occurs when sample first is put on test we accept the lot. Probability of acceptance for sample first using the Marshall – Olkin extended Lomex distribution for γ=2 is placed in Table 1 and the probability of acceptance for DASP is calculated and placed in Table 2 using (2), (3) and (4).

Assume that an experimenter wants to establish that true unknown mean life is at least 1000 hours with confidence 0.95. Let the acceptance numbers for this experiment be $c_1 = 0$ and $c_2 = 2$ with sample sizes $n_1 = 5$ and $n_2 = 12$. The lot is accepted if during 628 hours no failure is observed in a sample of 5. Probability of acceptance for this single sampling from Table 1 is 0.212347. Probability of acceptance for the same measurements using DASP for Table 2 is 0.269302. In DASP scheme as σ/σ_0 increases probability of acceptance for DASP decreases. For the above consideration probability of acceptance is 0.963165 when the ratio of unknown average life to specified life is 12. As the time of experiment increases, the probability of acceptance for DASP decreases.

From Table 2, it is clear that when the time of experiment is 4712 hours, probability of acceptance for ratio $\sigma/\sigma_0 = 2$ is 0.031254. For this same experiment time, as increases, probability of acceptance also increases. We here note that double acceptance sampling scheme minimizes the producer's risk, and also exerts pressure on producer to improve the quality of the product. At 4712 hours, $\sigma/\sigma_0 = 12$, and P* = 0.95, the probability of acceptance is 0.753146. Producer's risk for the sample first and double sampling are placed for P* = 0.95 in Table 3.

Table 1: Operating characteristic values for sample first for the sampling plan (n_1, c_1, t/σ) when c_1= 0 and γ=2 using binomial distribution for Marshall – Olkin extended Lomex distribution

P*	t/σ	n_1	μ/μ_0					
			2	4	6	8	10	12
0.75	0.628	3	0.394664	0.625453	0.730906	0.790356	0.828381	0.854763
	0.912	2	0.399605	0.626664	0.731358	0.790570	0.828498	0.854834
	1.257	2	0.299913	0.537786	0.659432	0.731177	0.778180	0.811277
	1.571	2	0.228058	0.462848	0.595272	0.676672	0.731213	0.770162
	2.356	2	0.121249	0.321930	0.462921	0.558696	0.626542	0.676727
	3.141	2	0.069116	0.228155	0.362667	0.462958	0.537941	0.595368
	3.927	2	0.041800	0.164754	0.286370	0.385139	0.462892	0.524494
	4.712	2	0.026600	0.121249	0.228122	0.321930	0.399456	0.462921
0.90	0.628	4	0.289492	0.534884	0.658389	0.730741	0.777989	0.811199
	0.912	3	0.252607	0.496081	0.625453	0.702927	0.754115	0.790356
	1.257	3	0.164245	0.394379	0.535494	0.625221	0.686468	0.730725
	1.571	2	0.228058	0.462848	0.595272	0.676672	0.731213	0.770162
	2.356	2	0.121249	0.321930	0.462921	0.558696	0.626542	0.676727
	3.141	2	0.069116	0.228155	0.362667	0.462958	0.537941	0.595368
	3.927	2	0.041800	0.164754	0.286370	0.385139	0.462892	0.524494
	4.712	2	0.026600	0.121249	0.228122	0.321930	0.399456	0.462921
0.95	0.628	5	0.212347	0.457430	0.593066	0.675623	0.730663	0.769856
	0.912	4	0.159684	0.392708	0.534884	0.625000	0.686410	0.730741
	1.257	3	0.164245	0.394379	0.535494	0.625221	0.686468	0.730725
	1.571	3	0.108910	0.314889	0.459275	0.556631	0.625268	0.675885
	2.356	2	0.121249	0.321930	0.462921	0.558696	0.626542	0.676727
	3.141	2	0.069116	0.228155	0.362667	0.462958	0.537941	0.595368
	3.927	2	0.041800	0.164754	0.286370	0.385139	0.462892	0.524494
	4.712	2	0.026600	0.121249	0.228122	0.321930	0.399456	0.462921
0.99	0.628	8	0.083866	0.286101	0.433476	0.533983	0.605267	0.658440
	0.912	6	0.063810	0.246096	0.391192	0.494106	0.568689	0.624662
	1.257	5	0.049259	0.212091	0.353122	0.457147	0.534196	0.592820
	1.571	4	0.052010	0.214228	0.354348	0.457885	0.534673	0.593149
	2.356	3	0.042220	0.182659	0.314964	0.417602	0.495935	0.556699
	3.141	3	0.018171	0.108979	0.218405	0.315001	0.394550	0.459387
	3.927	2	0.041800	0.164754	0.286370	0.385139	0.462892	0.524494
	4.712	2	0.026600	0.121249	0.228122	0.321930	0.399456	0.462921

Table 2: Operating characteristic values for the double sampling plan (n_2, c_2, t/σ) when c_1=0, c_2=2 and γ=2 using binomial distribution for Marshall – Olkin extended Lomex distribution.

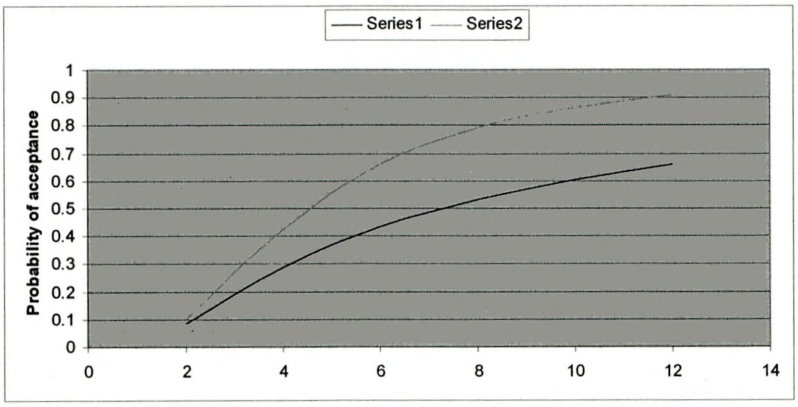
P*	t/σ	n_1	n_2	μ/μ_0					
				2	4	6	8	10	12
0.75	0.628	3	8	0.548579	0.854873	0.939378	0.969486	0.982605	0.989180
	0.912	2	6	0.541533	0.846772	0.934881	0.966921	0.981038	0.988161
	1.257	2	5	0.435197	0.786632	0.904430	0.950066	0.970874	0.981600
	1.571	2	5	0.315025	0.690072	0.848618	0.916991	0.950085	0.967803
	2.356	2	4	0.184133	0.547374	0.753146	0.855703	0.909609	0.940027
	3.141	2	4	0.094327	0.381955	0.612987	0.753191	0.836140	0.886723
	3.927	2	3	0.082441	0.362230	0.594431	0.738535	0.825104	0.878451
	4.712	2	3	0.049055	0.265969	0.490615	0.650897	0.756185	0.825132
0.90	0.628	4	10	0.387714	0.758295	0.889648	0.941709	0.965756	0.978238
	0.912	3	8	0.322745	0.699626	0.854873	0.920884	0.952590	0.969486
	1.257	3	6	0.238709	0.631615	0.814848	0.896877	0.937376	0.959330
	1.571	2	6	0.276064	0.637701	0.812758	0.893860	0.934787	0.957300
	2.356	2	5	0.147495	0.471855	0.690174	0.810457	0.877606	0.917030
	3.141	2	4	0.094327	0.381955	0.612987	0.753191	0.836140	0.886723
	3.927	2	4	0.052321	0.264071	0.486668	0.64716	0.753110	0.822681
	4.712	2	4	0.031254	0.184133	0.381895	0.547374	0.668130	0.753146
0.95	0.628	5	12	0.269302	0.657183	0.830018	0.905980	0.943135	0.963165
	0.912	4	9	0.202283	0.582713	0.781115	0.874995	0.922842	0.949316
	1.257	3	7	0.209992	0.586197	0.782383	0.875517	0.923086	0.949443
	1.571	3	6	0.145618	0.503665	0.723715	0.836651	0.896913	0.931241
	2.356	2	5	0.147495	0.471855	0.690174	0.810457	0.877606	0.917030
	3.141	2	5	0.771090	0.315186	0.538481	0.690225	0.786814	0.848713
	3.927	2	4	0.052321	0.264071	0.486668	0.64716	0.753110	0.822681
	4.712	2	4	0.031254	0.184133	0.381895	0.547374	0.668130	0.753146
0.99	0.628	8	16	0.097636	0.422566	0.659396	0.791437	0.865291	0.908663
	0.912	6	11	0.076519	0.381503	0.624875	0.766441	0.847509	0.895827
	1.257	5	9	0.058351	0.330355	0.575624	0.728301	0.819275	0.874906
	1.571	4	8	0.059025	0.315193	0.555295	0.710356	0.805023	0.863873
	2.356	3	6	0.048180	0.271000	0.503792	0.665999	0.769958	0.836719
	3.141	3	6	0.019211	0.145730	0.334502	0.503855	0.631874	0.723862
	3.927	2	5	0.044403	0.212844	0.412524	0.574250	0.690134	0.770752
	4.712	2	5	0.027521	0.147495	0.315133	0.471855	0.596576	0.690174

Table 3: Producer's risk with respect to time of experiment for double sampling ($p^* = 0.95$)

σ/σ_0	$c_1=0, c_2=2, n_1=5, n_2=12, t/\sigma_0=0.628$	$c_1=0, c_2=2, n_1=2, n_2=4, t/\sigma_0=4.712$
2	0.730698	0.968746
4	0.342817	0.815867
6	0.169982	0.618105
8	0.094020	0.452626
10	0.056865	0.331870
12	0.036835	0.246854

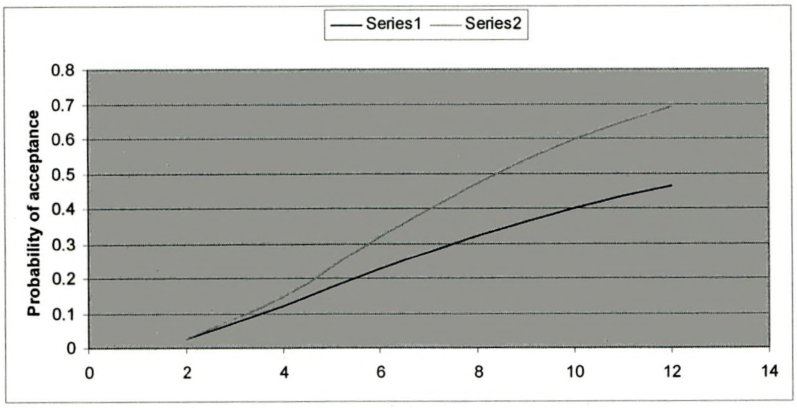
For $\sigma/\sigma_0 = 2$ (if unknown average life is twice of specified average life) producer's risk when time of experiment is 628 hrs and 4712 hrs for $\gamma = 2$ are 0.730698 and 0.968746 respectively.

Producer's risk decreases as the quality level of the product increases with $p^*=0.95$. Table 3. Fig 1 and Fig 2 illustrate our idea.



σ/σ_0

Figure 1: OC Curve with $p^* = 0.99$, $\gamma = 2$ and $t_0 = 628$ hrs



σ/σ_0

Figure 2: OC Curve with $p^* = 0.99$, $\gamma = 2$ and $t_0 = 4712$ hrs

Remarks

We find the acceptance sampling plans for various values of σ/σ_0 and different experiment times assuming that the life test follows the Marshall – Olkin extended Lomex distribution. This distribution provides the high probability for $\sigma/\sigma_0 > 6$.

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