Avinashilingam Institute for Home Science and Higher Education for Women

(Deemed to be University under Category 'A' by MHRD, Estd. u/s 3 of UGC Act 1956)
Re-accredited with 'A+' Grade by NAAC. Recognised by UGC Under Section 12B
Coimbatore - 641 043, Tamil Nadu, India

Bachelor's Degree Examination – June 2021
VI Semester

Class : III UG | **Time : 3 Hours**
Major : Mathematics / Special Education and Mathematics | **Max. Marks: 100**

18BMAC22/ 18BSMC14 Real Analysis - II

Part A | **10 x 1 = 10**
Choose the correct answer

1. The real valued function $f(x) = arctanx$ maps R^1 onto _____ CO2 K2
   a. $(-1,1)$ b. $[-1,1]$ c. $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ d. $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

2. Every open interval in R is CO2 K1
   a. connected b. disconnected
   c. need not be connected d. none of these

3. If $f(x) = x^2$ for x in R, then f is ________ CO3 K3
   a. not uniformly continuous on N b. uniformly continuous on N
   c. not uniformly continuous on R d. uniformly continuous on R

4. The function which is uniformly continuous on S is _____ CO3 K2
   a. continuous on R b. continuous on S
   c. not continuous on S d. removable discontinuity on S

5. The derivative of f at c is CO1 K2
   a. $f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ b. $f'(c) = \lim_{x\to y}\frac{f(x)-f(y)}{x-y}$
   c. $f'(c) = \lim_{x\to 0}\frac{f(x)-f(0)}{x}$ d. $f'(c) = \lim_{x\to 0} f(x) - f(0)$

6. Let $f(x) = \sin\left(\frac{1}{x}\right)$ then $f'(x) =$ _____ CO1 K6
   a. $\frac{1}{x}\cos\left(\frac{1}{x}\right)$ b. $-\frac{1}{x}\cos\left(\frac{1}{x}\right)$
   c. $\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$ d. $-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

7. Assume f has a finite derivative in (a,b) and is continuous on $[a,b]$ with $f(a) = f(b) = 0$. CO4 K1
   Then for every real λ there is some c in (a,b) such that ______
   a. $f'(c) = \lambda$ b. $f'(c) = \lambda f(c)$ c. $f'(c) = f(c)$ d. $f'(c) = c$

8. If f and g are continuous on $[a,b]$ and have equal finite derivative in (a,b) then $f - g$ is ____ on $[a,b]$ CO4 K2
   a. constant b. increasing c. decreasing d. monotonic

9. Let f be of bounded variation on $[a,b]$. Let V be defined on $[a,b]$ as follows: CO5 K4
   $V(x) = V_f(a,x)$ if $a < x \le b, V(a) = 0$. Then V is ______ on $[a,b]$.
   a. constant b. decreasing c. increasing d. zero

10. Let f be continuous on $[a, b]$. Then f is bounded variation on $[a, b]$ iff f can be expressed as the _______ CO5 K1

a. difference of two increasing continuous functions
b. difference of two decreasing continuous functions
c. sum of two increasing continuous functions
d. sum of two decreasing continuous functions

Part B **5 x 6 = 30**

Answer ALL questions
Each answer should not exceed 400 words or two pages

11.a. Let $f: S \to M$ be a function from a metric space S to another metric space M. Let X be a connected subset of S. If f is continuouson X, prove that $f(X)$ is a connected subset of M. CO2 K3

(or)

11.b. State and prove Bolzano's theorem. CO2 K1

12.a. Prove that a contraction f of a complete metric space S has a unique fixed point p. CO2 K2

(or)

12.b. Define jump discontinuity and give three examples of jump discontinuity. CO2 K1

13.a. Define derivatives and give any three examples of derivatives. CO1K1

(or)

13.b. Assume f and g are defined on (a, b) and differentiable at c, then prove that $fg$is differentiable at c. CO1 K2

14.a. Let f be defined on an open interval (a, b) and assume that f has a local maximum or a local minimum at an interior point $c$of (a, b). If f has a derivative at c, then prove that $f'(c)$ must be 0. CO4 K2

(or)

14.b. State and prove generalized mean-value theorem. CO4 K1

15.a. If f is monotonic on $[a, b]$, then prove that the set of discontinuous of $f$is countable. CO5 K3

(or)

15.b. If f is monotonic on $[a, b]$, then prove that f is of bounded variation on $[a, b]$. CO1 K4

Part C **5 x 12 = 60**

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. Prove that every open connected set in R^n is arcwise connected. CO2 K2

(or)

16.b. Let $S \to T$ be a function from one metric space (S, d_s) to another (T, d_T). CO2 K4
Then prove that f is continuous on S if and only if, for every open set $y$in T, the inverse image $f^{-1}(y)$ is open in S.

17.a .Iff is increasing on $[a, b]$, prove that $f(c+)$ and $f(c-)$ both exist for each c in (a, b) and $f(c-) \le f(c) \le f(c+)$and also at the endpoints$f(a) \le f(a+)$ and $f(b-) \le f(b)$. CO2 K3

(or)

17.b. State and prove Heine theorem for uniform continuity. CO3 K1

18.a.(i) If f is defined on (a, b) and differentiable at a point c in (a, b), then prove that there is a function f^* which is continuous at c and which satisfies the equation $f(x) - f(c) = (x - c)f^*(x)$, for all x in (a, b), with $f^*(c) = f'(c)$. Conversely, if there is a function f^*, continuous at c, which satisfies the above equation, then prove that f is differentiable at c and $f'(c) = f^*(c)$. CO1 K3

(ii) If f is differentiable at c, then prove that f is continuous at c. CO1 K2

(or)

18.b. State and prove chain rule for differentiating composite functions. CO4 K1

19.a. Let f and g be two functions having finite n^{th} derivatives $f^{(n)}$and $g^{(n)}$ in an open interval (a, b) and continuous $(n-1)st$ derivatives in the closed interval $[a, b]$. Assume that $c \in [a, b]$. Then prove that, for every $x$in $[a, b], x \ne c$, there exists a point x, interior to the interval joining x and c CO1 K3

such that $\left[f(n) - \sum_{k=0}^{n-1} \frac{f^{(k)}(c)}{K!}(x-c)^k\right] g^{(n)}(x_1) = f^{(n)}(x_1)\left[g(n) - \sum_{K=0}^{n-1} \frac{g^{(K)}(c)}{K!}(x-c)^K\right]$

(or)

19.b. (i) State and prove Rolle's theorem. CO4 K1

(ii) Let f be defined on an open interval (a, b) and assume that for some c in (a, b) we have $f'(c) > 0$or $f'(c) = +\infty$. Prove that there is a 1-ball $B(c) \subseteq (a, b)$ in which $f(x) > f(c)$if $x > c$, and $f(x) < f(c)$ if $x < c$. CO1 K2

20.a. State and prove additive property of total variation. CO5 K1

(or)

20.b. Let f be of bounded variation on $[a, b]$. If $x \in (a, b]$, let $V(x) = V_f(a, x)$ and put $V(a) = 0$. Prove that every point of continuity of f is also a point of continuity of V. Prove that the converse is also true. CO5 K2

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