Avinashilingam Institute for Home Science and Higher Education for Women

Deemed to be University Estd. u/s 3 of UGC Act 1956, Category A by MHRD (now MoE)
Re-accredited with A++ Grade by NAAC. CGPA 3.65/4, Category I by UGC
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – May 2025
II Semester

Class : I P.G. **Time: 3 Hours**
Major : Mathematics **Max. Marks: 100**

23MMAC09 Partial Differential Equations

Course Outcomes:
CO1: Solve linear and non-linear partial differential equations of first order and second order.
CO2: Determine special types of first order equations.
CO3: Find the solution of Hyperbolic equations.
CO4: Apply the Dirichlet and Neumann boundary value problems in scientific fields.
CO5: Solve various real life problems by formulating them into partial differential equations

Part A **10 x 1 = 10**
Choose the Correct Answer

1. Recall that the partial differential equation of the form $Pp + Qq = R$ is referred to as CO1K1
   a. Euler's equation b. Laplace's equation
   c. Maxwell's equation d. Lagrange's equation

2. Infer that, along every characteristic strip of the equation $F(x, y, z, p, q) = 0$, the function $F(x, y, z, p, q)$ is CO1K2
   a. a constant b. not a constant c. zero d. continuous

3. Identify the complete integral of the equation $pq = 1$ among the following. CO2K2
   a. $ax + ay - z = c$ b. $a^2x + y - az = c$
   c. $a^2x + a^2y - z = c$ d. $ab = 1$

4. Observe that the first-order partial differential equation is separable if it can be written in the form CO2K1
   a. $f(x, q) = g(y, p)$ b. $f(x, y) = g(p, q)$
   c. $f(x, p) = g(y, q)$ d. $px = qy$

5. Order in which the linear factors occur is unimportant if the operator $F(D, D')$ is CO3K2
   a. reducible b. irreducible c. differentiable d. integrable

6. Identify the one-dimensional diffusion equation among the following: CO3K2
   a. $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ b. $\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$
   c. $\frac{\partial z}{\partial x} = \frac{1}{k}\frac{\partial z}{\partial t}$ d. $\frac{\partial z}{\partial x} = \frac{1}{c^2}\frac{\partial z}{\partial y}$ - 2

7. Recall that, if the function $\psi(x, y, z)$ is a solution of Laplace's equation, the one-parameter system of surfaces $\psi(x, y, z) = c$ is called a family of _____ surfaces. CO4K1
   a. equivalent b. equipotential c. equal d. equicontinuous

8. Recognise the general form of potential function of surfaces CO4K1
$x^2 + y^2 + z^2 = cx^{2/3}$ from the following.

a. $\psi = A(x^2 + y^2 + z^2)^{1/2} + B$ b. $\psi = A(x^2 + y^2 + z^2)^{3/2} + B$

c. $\psi = A(x^2 + y^2 + z^2)^{-1/2} + B$ d. $\psi = A(x^2 + y^2 + z^2)^{-3/2} + B$

9. Select the correct answer: General solution of the wave equation is given by CO5K1

a. $y = f(x + ct) + g(x - ct)$ b. $y = f(x + c^2t) + g(x - c^2t)$

c. $y = f'^{(x+ct)} + g'(x - ct)$ d. $y = f'(x + c^2t) + g'(x - c^2t)$

10. Observe that the three-dimensional wave equation $\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$ has the solutions of the form _____ provided that $k^2 = l^2 + m^2 + n^2$. CO5K1

a. $e^{i(lx+my+nz+kct)}$ b. $e^{(lx+my+nz+kct)}$

c. $e^{\pm i(lx+my+nz+kct)}$ d. $e^{\pm(lx+my+nz+kct)}$

Part B **5 x 6 = 30**

Answer ALL questions

Each answer should not exceed 400 words or two pages

11.a. Developthe integral surface of the linear partial differential equation

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

which contains the straight line $x + y = 0, z = 1$. CO1K3

(or)

11.b. Write the definitions of complete integral, general integral and singular integral in finding the solution of the equation $F(x, y, z, p, q) = 0$ where the function F is not necessarily linear in p and q. CO1K3

12.a. Show that the equations $xp = yq,\ z(xp + yq) = 2xy$ are compatible and find their solution. CO2K4

(or)

12.b. Find a complete integral of the equation $(p^2 + p^2)y = qz$. CO2K3

13.a. Show that $F(D, D')e^{ax+by} = F(a, b)e^{ax+by}$ CO3K3

(or)

13.b. Explain the method of separation of variables for solving second-order linear partial differential equations. CO3K4

14.a.Write a brief description on equipotential surfaces. CO4K3

(or)

14.b. Show that the surfaces $x^2 + y^2 + z^2 = cx^{2/3}$ can form a family of equipotent surfaces. CO4K4

15.a. Establish d'Alembert's solution of the one-dimensional wave equation. CO5K3

(or)

15.b. A thin membrane of great extent is released from rest in the position $z = f(x, y)$. Determine the displacement at any subsequent time using integral transforms. CO5K3

Part C **5 x 12 = 60**

Answer ALL questions

Each answer should not exceed 800 words or four pages

16. a. Formulate the proof for the following: The general solution of the linear partial differential equation $Pp + Qq = R$ is $F(u, v) = 0$ where F is an arbitrary function and $u(x, y. z) = c_1$ and $v(x, y. z) = c_2$ form a solution of the equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$. CO1K6

(or)

16.b. Estimate the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis. CO1K5

17.a. Conclude that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$. CO2K5

(or)

17. b. Summarize the fundamental idea of Jacobi's method of solving the equation $F(x, y, z, p, q) = 0$ and use this to solve $p^2x + q^2y = z$. CO2K5

18.a. Solve the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$. CO3K6

(or)

18.b. Solve the equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ using the method of separation of variables and extend it to find the solution of $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$. CO3K6

19.a. Convince that the surfaces$(x^2 + y^2)^2 - 2a^2(x^2 - y^2) + a^4 = c$ can form a family of equipotent surfaces, and find the general form of the corresponding potential function. CO4K5

(or)

19.b. A uniform circular wire of radius a charged with electricity of line density e surrounds grounded concentric spherical conductor of radius c. Determine the electrical charge density at any point on the conductor. CO4K6

20.a. Solve $\nabla_1^2 z = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ with boundary condition $z = 0$ on Γ for all t and the initial conditions $z = f(x, y), \frac{\partial z}{\partial t} = g(x, y), t = 0, (x, y) \in S$ using the method of separation of variables. Assume that Γ is a rectangle formed by the lines $x = 0, x = a, y = 0, y = b$. CO5K6

(or)

20.b. The points of trisection of a string are pulled aside through a distance ε on opposite sides of the position of equilibrium, and the string is released from rest. Estimate an expression for the displacement of the string at any subsequent time and show that the mid-point of the string always remains at rest. CO5K5

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