Chapter II

CHAPTER II

BIGRAPHS AND NEUTROSOPHIC BIGRAPHS.

In this chapter two new notions of graphs namely bigraphs and neutrosophic bigaphs are introduced which plays the role of representing fuzzy models and neutrosophic models.

SECTION 2.1

BIGRAPHS

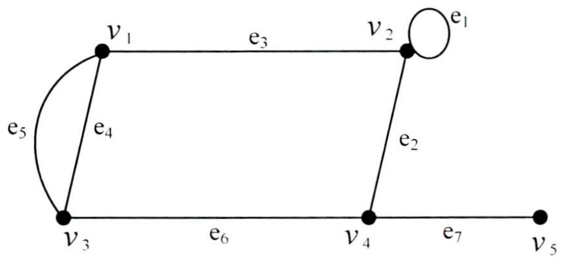
Definition 2.1.1

A linear graph (or simply a **graph**) G = (V,E) consists of a set of objects V = $\{v_1, v_2, \ldots\}$ called vertices and another set E = $\{e_1, e_2, \ldots\}$ whose elements are called edges such that each edge e_k is identified with an un-ordered pair of (V_i, V_j) vertices.

The vertices (V_i, V_j) associated with the edge e_k are called the end vertices of e_k.

Example 2.1.2

Consider the graph G =(V,E) where V = $\{v_1, v_2, \ldots, v_5\}$ and E = $\{e_1, e_2, \ldots, e_7\}$



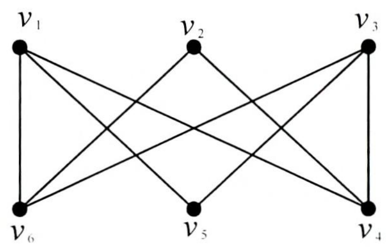
Definition 2.1.3

G = $G_1 \cup G_2$ is said to be a **bigraph** if G_1 and G_2 are two graphs such that G_1 is not a subgraph of G_2 or G_2 is not a subgraph of G_1, i.e., they have either distinct vertices or edges.

Example 2.1.4

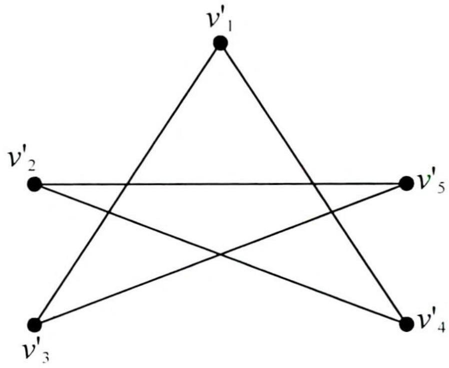
Let G = $G_1 \cup G_2$ where

G_1 =



and

G_2 =



$G = G_1 \cup G_2$ is a bigraph. G can also be represented as

$G = \{v_1, v_2, v_3, v_4, v_5, v_6\} \cup \{v'_1, v'_2, v'_3, v'_4, v'_5\}$;

the vertices of the two graphs G_1 and G_2 respectively.

Note:2.1.5

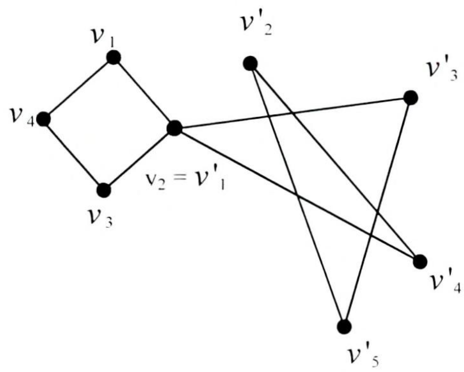
It is very important to note that the vertices of the bigraphs i.e.,$G = G_1 \cup G_2$ in general need not form disjoint subsets of G such that $G_1 \cap G_2 = \varphi$.

In the above example we have $G_1 \cap G_2$ is empty.

Now we can have bigraphs given by the following example.

Example 2.1.6

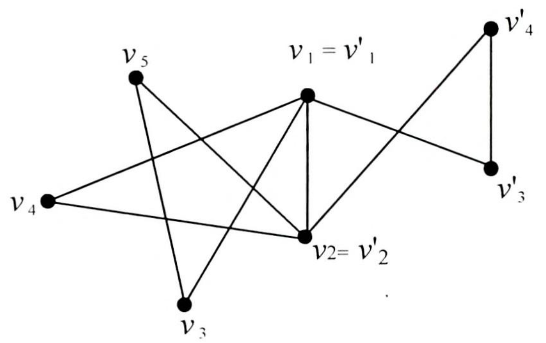
Let $G = G_1 \cup G_2$ be the bigraph given by the following figure.



Here $G = G_1 \cup G_2$, { v_1, $v_2 = v'_1$, v_3, v_4} $\cup$ { v'_1, v'_2, v'_3, v'_4, v'_5} we see that this bigraph is very special in its own way for it has only one point in common, viz. $v_2 = v'_1$.

Example 2.1.7

Let $G = G_1 \cup G_2$ be a bigraph given by the following figure.



It is interesting to observe that the graphs have only two points in common viz. $v_1 = v'_1$ and v_2 and v'_2 .

Thus

$$G = \{v_1, v_2, v_3, v_4, v_5\} \cup \{ v'_1, v'_2, v'_3, v'_4\}.$$

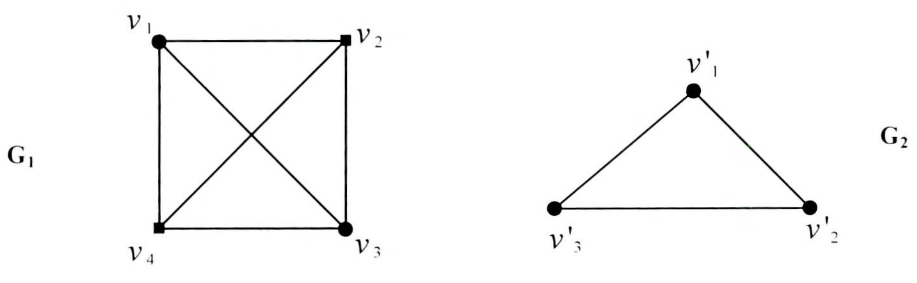
Here $v_1 = v'_1$ and $v_2 = v'_2$.

Definition 2.1.8

Let $G = G_1 \cup G_2$ be a bigraph we say G is said to be a **disjoint bigraph** if $G = G_1 \cup G_2$ are such that $G_1 \cap G_2 = \varphi$

Example 2.1.9

Let $G = G_1 \cup G_2$ be given by the following figure.



The bigraph is such that $G_1 \cap G_2 = \varphi$. This is a disjoint bigraph.

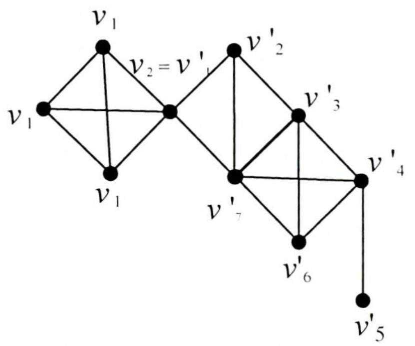
Definition 2.1.10

Let $G = G_1 \cup G_2$ be the bigraph. If G_1 and G_2 are graphs such that they have a single point in common i.e., $G_1 \cap G_2$ = {single vertex}, then we say the bigraph is a pair of graphs glued at a point i.e. **single point glued bigraph** or **vertex glued bigraph**.

Example 2.1.11

The graph $G = G_1 \cup G_2$ where

$G = \{v_1, v_2, v_3, v_4\} \cup \{v'_1, v'_2, v'_3, v'_4, v'_5, v'_6, v'_7\}$ is given by the following figure



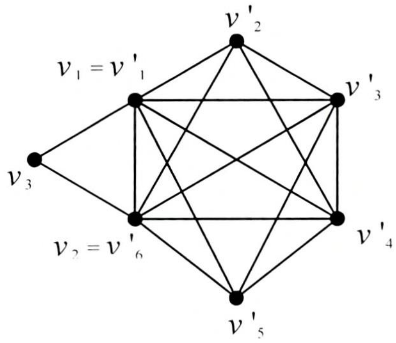
This is a single point glued bigraph glued at the vertex $v_2 = v'_1$.

Definition 2.1.12

$G = G_1 \cup G_2$ be a bigraph. If the bigraph is such that the graphs G_1 and G_2 have a common edge then we call the bigraph to be a **edge glued bigraph** or **a line glued bigraph**.

Example 2.1.13

Let $G = G_1 \cup G_2$ be a bigraph given by the following figure.



Theorem 2.1.14

Let $G = G_1 \cup G_2$ be a bigraph if G is a edge glued bigraph then G is a vertex glued bigraph. A vertex glued bigraph in general need not be edge glued bigraph.

Proof:

Let $G = G_1 \cup G_2$ be a bigraph. Suppose G is a edge glued bigraph then certainly G is a vertex glued bigraph for edge will certainly include at least two vertices. So always a edge glued bigraph will be a vertex glued bigraph.

Theorem 2.1.15

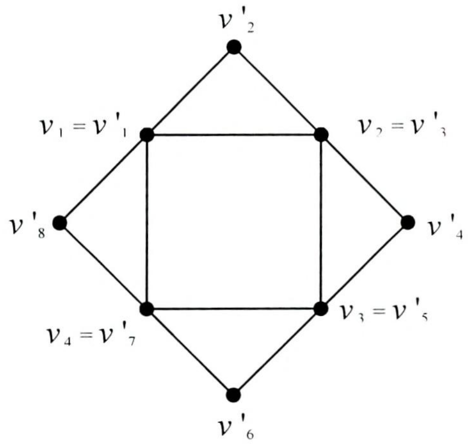
A bigraph glued by even more than two vertices need not in general be glued by an edge.

Proof :

This result is proved by the following example.

Example 2.1.16

Consider the bigraph $G = G_1 \cup G_2$ given by the following figure.



That is

$G_1 = \{v_1, v_2, v_3, v_4\}$ and $G_2 = \{v'_1, v'_2, v'_3, v'_4, v'_5, v'_6, v'_7, v'_8\}$.

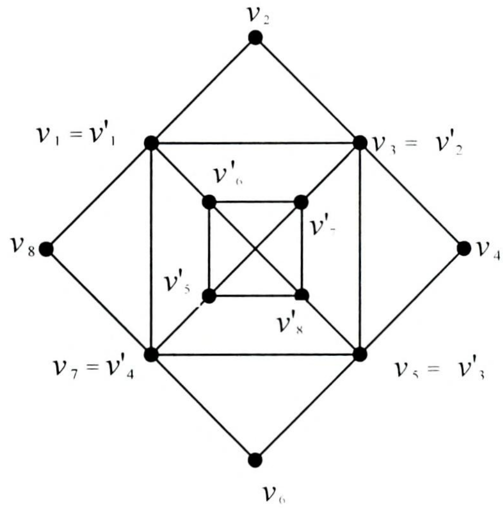
This bigraph has four vertex in common but no edge. There are bigraphs glued by more than one edge or by a subgraph which happen to be the subgraph of both the graphs. It is defined as follows,

Definition 2.1.17

Let $G = G_1 \cup G_2$ be a bigraph. Suppose the bigraphs is glued such that it has a subgraph (with more than one vertex and more than one edge) in common then we define this bigraph as a bigraph glued by a **strong subgraph** or a **strong subgraph glued bigraph**.

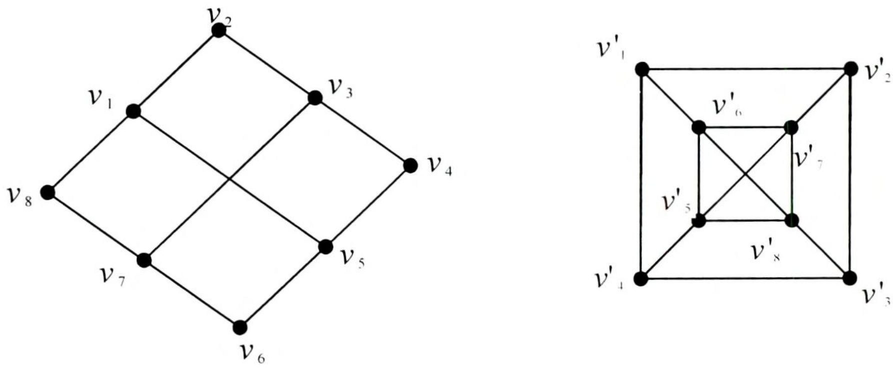
Example 2.1.18

$G = G_1 \cup G_2$ be a bigraph given by the following figure.

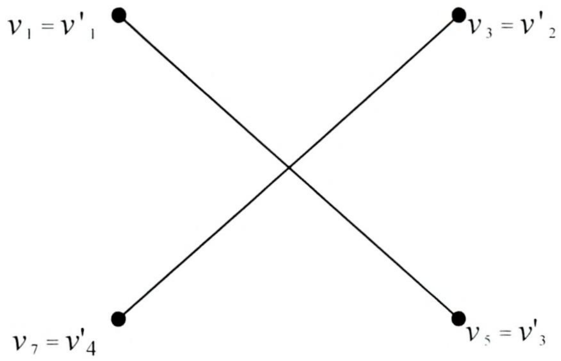


Here $G = G_1 \cup G_2 = \{v_1, v_2, v_3, \ldots, v_8\} \cup \{v'_1, v'_2, v'_3, \ldots, v'_8\}$.

The graphs associated with G_1 and G_2 are given by the following figure.



This bigraph has a subgraph in common given by the following figure.



Theorem 2.1.19

Let G = $G_1 \cup G_2$ be a bigraph which is strong subgraph glued bigraph, then G is a vertex glued graph and a edge glued graph.

Proof :

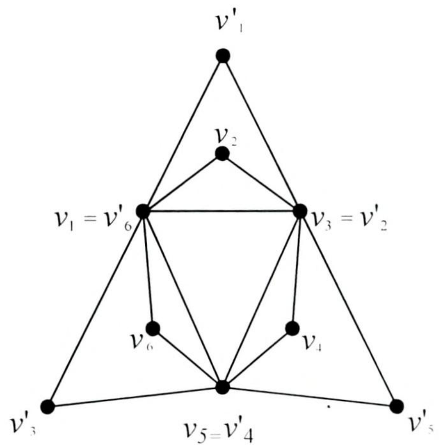
Since single point or an edge is subgraph we see vertex glued bigraph and a edge glued bigraph are also subgraph glued bigraphs, but cannot be called as strong subgraph glued bigraph. But clearly in case of strong subgraph glued bigraphs we see it is both a vertex glued bigraph and edge glued bigraph.

Definition 2.1.20

Let G = $G_1 \cup G_2$ be a bigraph. A non empty subset H of G is said to be a **subbigraph of G** if H itself is a bigraph of G.

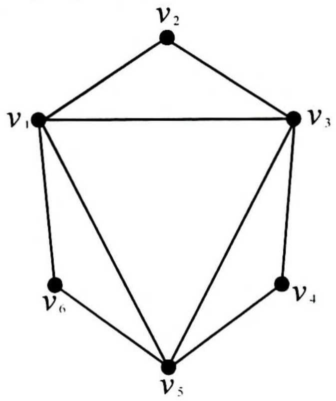
Example 2.1.21

Let G = $G_1 \cup G_2$ be a bigraph given by the following figure.

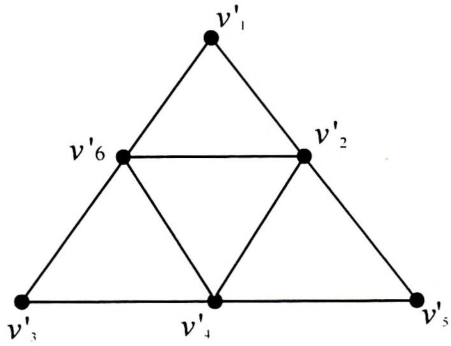


where $G_1 = \{v_1, v_2, v_3, \ldots, v_6\}$ and $G_2 = \{v'_1, v'_2, v'_3, \ldots, v'_6\}$. The graphs of G_1 and G_2 are given by following figures.

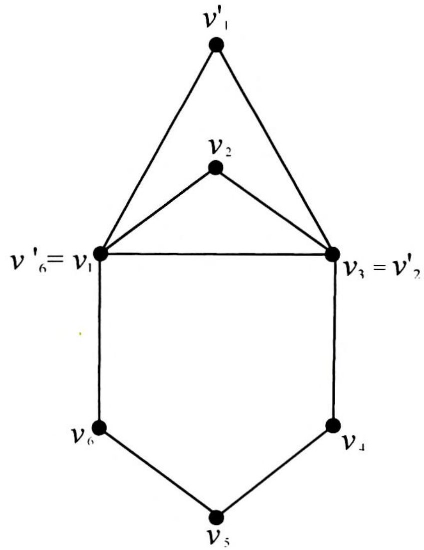
Graph of G_1



Graph of G_2



The subbigraph H is



Here $H = \{v_1, \ldots, v_6\} \cup \{v'_1, v'_2, \ldots, v'_6\}$

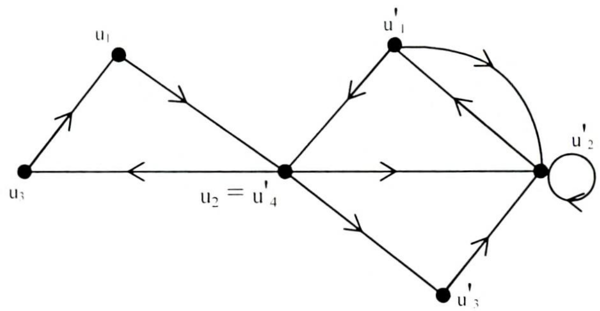
$H_1 = \{v_1, \ldots, v_6\}$ and $H_2 = \{v'_1, v'_2, \ldots, v'_6\}$.

Definition 2.1.22

A directed bigraph $G = G_1 \cup G_2$ is a pair of ordered triple $\{(V(G_1), A(G_1), I_{G_1}), (V(G_2), A(G_2), I_{G_2})\}$ where $V(G_1)$ and $V(G_2)$ are non empty proper sets of $V(G)$ called the **set of vertices** of $G = G_1 \cup G_2$. $A(G_1)$ and $A(G_2)$ is a set disjoint from $V(G_1)$ and $V(G_2)$ respectively called the **set of arcs** of G_1 and G_2 and I_{G_1} and I_{G_2} are incidence map that associates with each arc of G_1 and G_2 an ordered pair of vertices of G_1 and G_2 respectively. A directed bigraph is called the **dibigraph**.

Example 2.1.23

Let $G = G_1 \cup G_2$ be a bigraph given in Figure



Definition 2.1.24

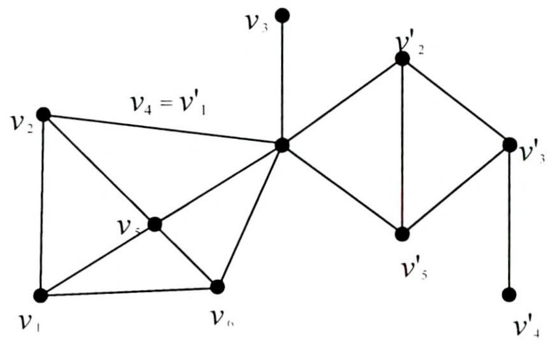
The bigraph of the m×n matrix $A = [a_{ij}]$ is the simple graph with vertex set $U \cup V$,where $U = \{1,2, \ldots ,m\}$ and $V = \{1', 2', \ldots n'\}$ and edge set $\{\{i, j'\}: a_{ij} \neq 0\}$. If A is non negative integer matrix, then the multi-bigraph of A has vertex set $U \cup V$ and edge $\{i, j'\}$ of multiplicity a_{ij} . If A is a genral matrix, then the **weighted bigraph** of A has vertex set $U \cup V$ and edge $\{i, j'\}$ of weight a_{ij} .

MATRIX REPRESENTATION OF BIGRAPHS :

First we give the simple bigraph and the related adjacency bimatrix.

Example 2.1.25

Let $G = G_1 \cup G_2$ be the bigraph $G = G_1 \cup G_2$ in which both G_1 and G_2 are simple. The bigraph $G = G_1 \cup G_2$ is given by the figure



The adjacency bimatrix of the bigraph is a mixed square bimatrix given as $X = X_1 \cup X_2$ where

$$X_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

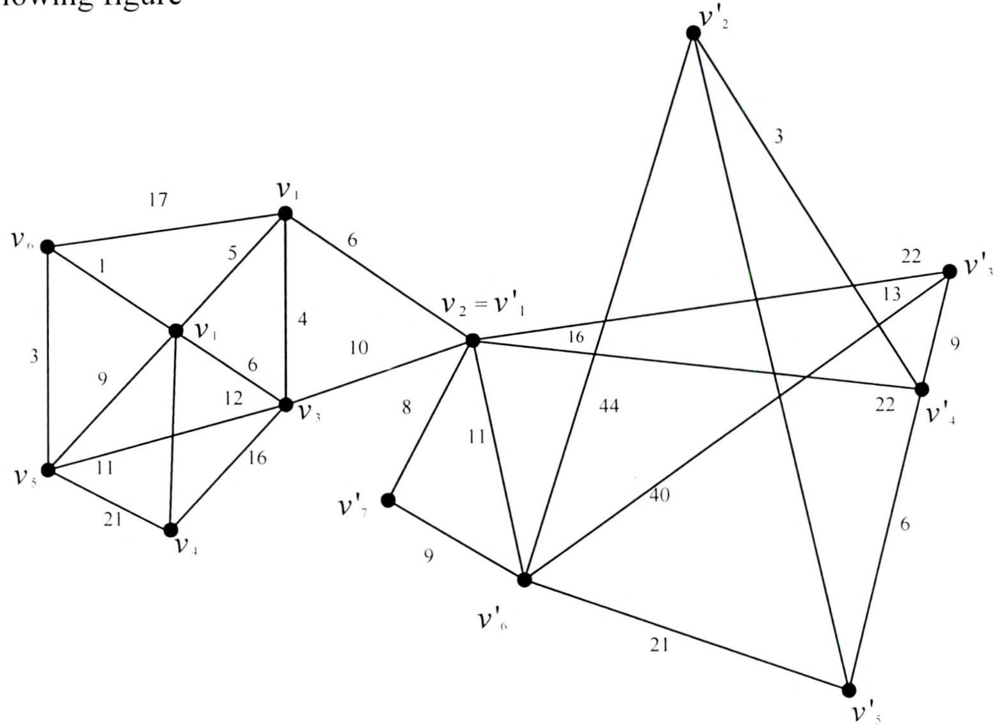
$$X_2 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Thus $X = X_1 \cup X_2$ is the adjacency bimatrix.

Next is the weighted bigraph of the bigraph $G = G_1 \cup G_2$ by the following example.

Example 2.1.26

Let $G = G_1 \cup G_2$ be a bigraph which is a weighted bigraph given by the following figure



We give below the weighted matrix related with the bigraph $G = G_1 \cup G_2$. The bimatrix of a weighted bigraph is always a square bimatrix. Let $W = W_1 \cup W_2$ be the weighted bimatrix of $G = G_1 \cup G_2$.

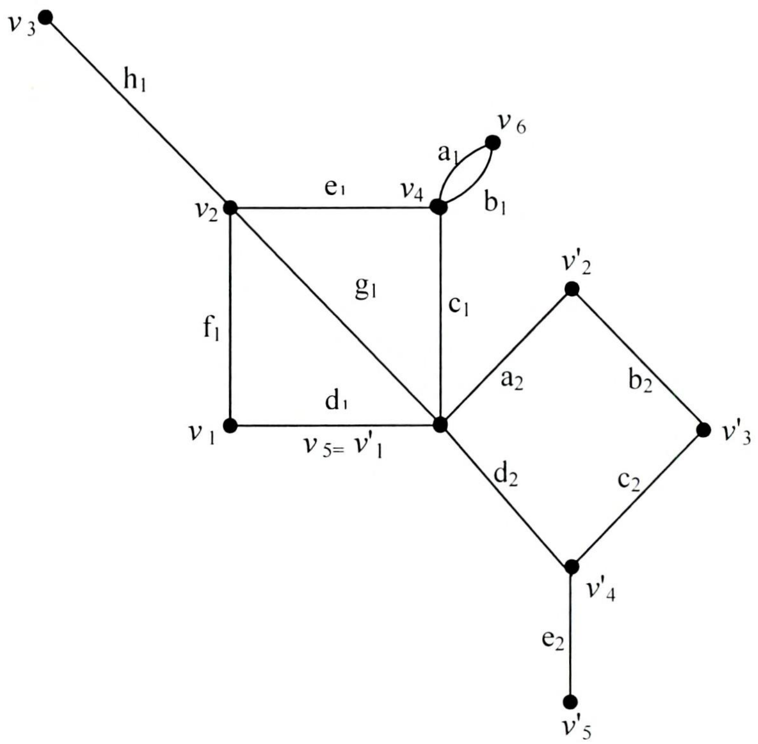
$$W_1 = \begin{bmatrix} \infty & 6 & 4 & \infty & \infty & 17 & 5 \\ 6 & \infty & 10 & \infty & \infty & \infty & \infty \\ 4 & 10 & \infty & 16 & 11 & \infty & 6 \\ \infty & \infty & 16 & \infty & 21 & \infty & 12 \\ \infty & \infty & 11 & 21 & \infty & 3 & 9 \\ 17 & \infty & \infty & \infty & 3 & \infty & 1 \\ 5 & \infty & 6 & 12 & 9 & 1 & \infty \end{bmatrix} \cup \begin{bmatrix} \infty & \infty & 22 & 16 & \infty & 11 & 8 \\ \infty & \infty & \infty & 9 & \infty & 13 & \infty \\ 22 & \infty & \infty & 9 & \infty & 13 & \infty \\ 16 & 3 & 9 & \infty & 6 & \infty & \infty \\ \infty & 22 & \infty & 6 & \infty & 21 & \infty \\ 11 & 4 & 40 & \infty & 21 & \infty & 9 \\ 8 & \infty & \infty & \infty & \infty & 9 & \infty \end{bmatrix} = W_2$$

'∞' symbol denotes when the vertices are non adjacent.

The incidence bimatrix associated with the dibigraph is given by the following example.

Example 2.1.27

Let $G = G_1 \cup G_2$ be a bigraph given by the following figure



The incidence bimatrix of the bigraph.

$$
\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} a_1 & b_1 & c_1 & d_1 & e_1 & f_1 & g_1 & h_1 \end{array} \\
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
\cup
\begin{array}{c} \\ v'_1 \\ v'_2 \\ v'_3 \\ v'_4 \\ v'_5 \end{array}
\begin{array}{c}
\begin{array}{ccccc} a_2 & b_2 & c_2 & d_2 & e_2 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

Theorem 2.1.28

Let $G = G_1 \cup G_2$ which is a single vertex glued bigraph G is separable.

Proof :

Given $G = G_1 \cup G_2$ is a big

raph which is a single vertex glued bigraph say let them be glued by the vertex $v_j = v'_1$ by removing that vertex, the bigraph becomes the separable bigraph.

SECTION 2.2

NEUTROSOPHIC BIGRAPHS

Definition 2.2.1

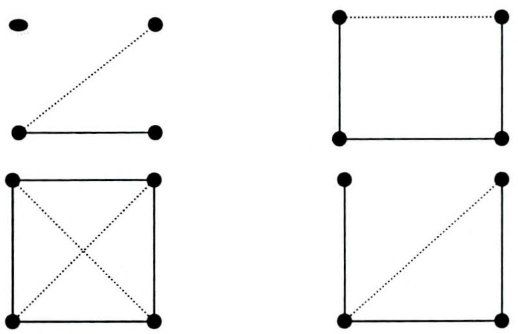
A **neutrosophic graph** is a graph in which at least one edge is an indeterminacy denoted by dotted lines.

Notation 2.2.2

The indeterminacy of an edge between two vertices will always be denoted by dotted lines.

Example 2.2.3

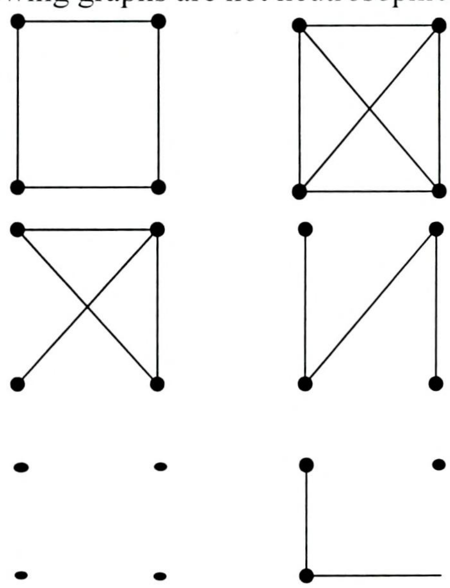
The following are neutrosophic graphs:



All graphs in general are not neutrosophic graphs given in figure.

Example 2.2.4

The following graphs are not neutrosophic graphs given in figure



Definition 2.2.5

A **neutrosophic directed graph** is a directed graph which has at least one edge to be an indeterminacy.

Definition 2.2.6

A **neutrosophic oriented graph** is a neutrosophic directed graph having no symmetric pair of directed indeterminacy lines.

Definition 2.2.7

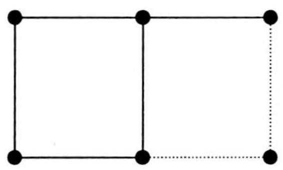
A **neutrosophic subgraph** H of a neutrosophic graph G is a subgraph H which is itself a neutrosophic graph.

Theorem 2.2.8

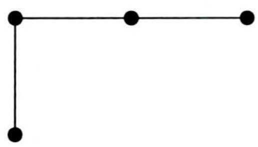
Let G be a neutrosophic graph. All subgraphs of G are not neutrosophic subgraphs of G.

Proof:

By an example. Consider the neutrosophic graph given in figure.



This has a subgraph given by figure.



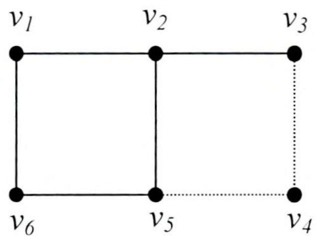
Which is not a neutrosophic subgraph of G.

Theorem 2.2.9

Let G be a neutrosophic graph. In general the removal of a point from G need not be a neutrosophic subgraph.

Proof :

Consider the graph G given in figure



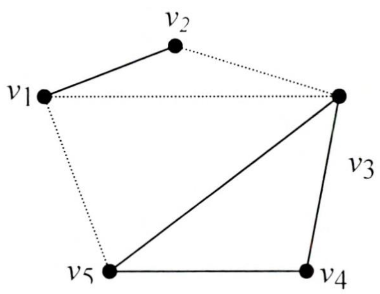
G \ v4 is only a subgraph of G but is not a neutrosophic subgraph of G.

Definition 2.2.10

Let G be a neutrosophic graph. The adjacency matrix of G with entries from the set (I, 0, 1) is called the **neutrosophic adjacency matrix** of the graph.

Example 2.2.11

Neutrosophic adjacency matrix related to a neutrosophic graph G given by figure



The neutrosophic adjacency matrix is N(A)

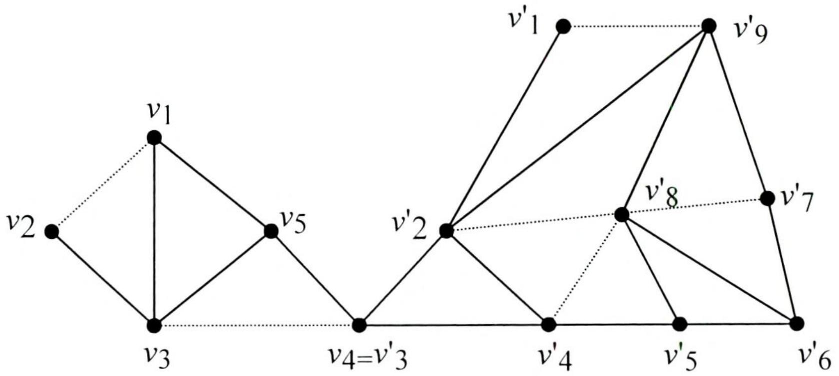
$$N(A) = \begin{bmatrix} 0 & 1 & I & 0 & I \\ 1 & 0 & I & 0 & 0 \\ I & I & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ I & 0 & 1 & 1 & 0 \end{bmatrix}$$

Definition 2.2.12

A neutrosophic graph $G_N = G_1 \cup G_2$ is said to be a **neutrosophic bigraph** if both G_1 and G_2 are neutrosophic graphs that the set of vertices of G_1 and G_2 are different atleast by one coordinate i.e. $V(G_1) \not\subset V(G_2)$ or $V(G_2) \not\subset V(G_1)$ i.e., $V(G_1) \cap V(G_2) = \varphi$ is also possible but is not a condition i.e. the vertex set of G_1 is not a proper subset of the vertex set of G_2 or vice versa or atleast one edge is different in the graphs G_1 and G_2. 'or' is not used in the mutually exclusive sense.

Example 2.2.13

Let $G = G_1 \cup G_2$ be the neutrosophic bigraph given by the following figure



$V(G_1) = (v_1, v_2, v_3, v_4, v_5)$

$V(G_2) = (v'_1, v'_2, v'_3, v'_4, v'_5, v'_6, v'_7, v'_8, v'_9)$

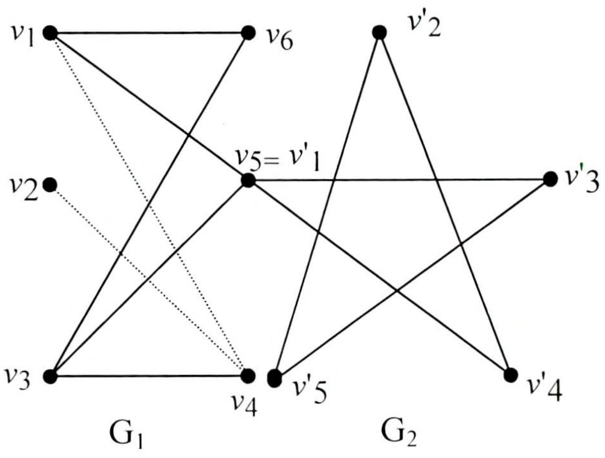
dotted edges are the neutrosophic edges. Thus $G_N = G_1 \cup G_2$ is a neutrosophic bigraph.

Definition 2.2.14

A **neutrosophic weak bigraph** $G = G_1 \cup G_2$ is a bigraph in which at least one of G_1 or G_2 is a neutrosophic graph and the other need not be a neutrosophic graph.

Example 2.2.15

Let $G = G_1 \cup G_2$ be a bigraph given by the Figure



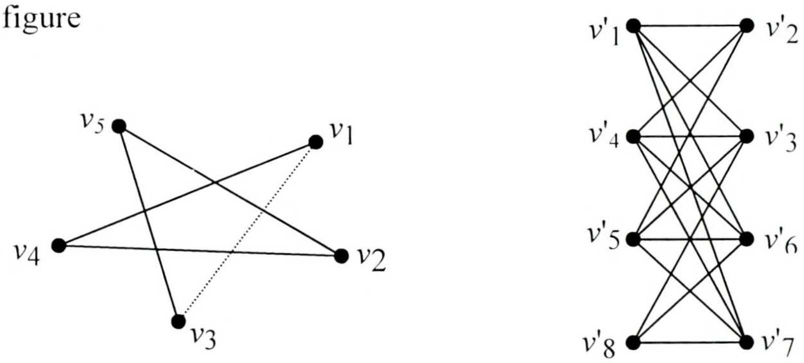
Clearly G_1 is a neutrosophic graph but G_2 is not a neutrosophic graph. So $G = G_1 \cup G_2$ is only a weak neutrosophic bigraph.

Theorem 2.2.16

All neutrosophic bigraph are neutrosophic weak bigraph but a neutrosophic weak bigraph in general is not a neutrosophic bigraph.

Proof:

By the very definition we see all neutrosophic bigraphs are weak neutrosophic bigraphs. To show a weak neutrosophic bigraph in general is not a neutrosophic bigraph. Consider the weak neutrosophic bigraph $G = G_1 \cup G_2$ given by the following figure



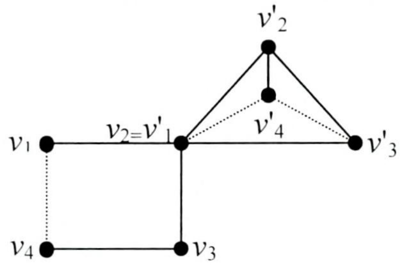
$G = G_1 \cup G_2$

Clearly G_1 is a neutrosophic graph but G_2 is not a neutrosophic graph. So $G = G_1 \cup G_2$ is not a neutrosophic bigraph but only a weak neutrosophic bigraph.

Definition 2.2.17

A neutrosophic bigraph G is **vertex connected** or **vertex glued neutrosophic bigraph** if the bigraph $G = G_1 \cup G_2$ is a vertex glued bigraph.

Example 2.2.18



Clearly $G = G_1 \cup G_2$ where

$V(G_1) = (v_1, v_2, v_3, v_4)$

$V(G_2) = (v'_1, v'_2, v'_3, v'_4)$

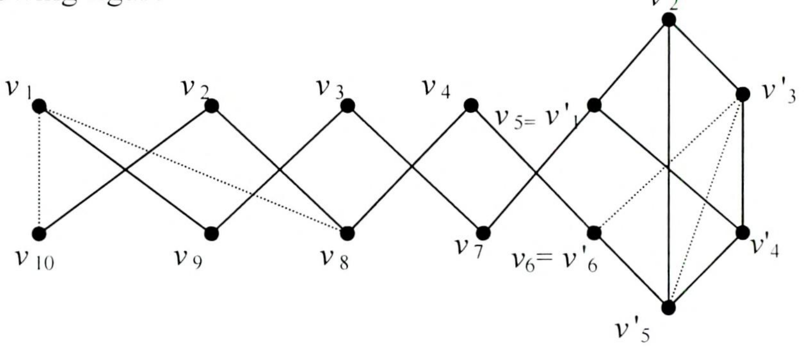
is a vertex glued neutrosophic bigraph given in figure

Definition 2.2.19

Let $G = G_1 \cup G_2$ be a neutrosophic bigraph. G is said to be a **edge glued neutrosophic bigraph** if G as a bigraph is a edge glued bigraph.

Example 2.2.20

Let $G = G_1 \cup G_2$ be a neutrosophic bigraph which is edge glued given by the following figure



Thus both G_1 and G_2 are neutrosophic graphs with the edge joining the vertices v_5 and v_6 in G_1 and v'_1 and v'_6 in G_2.

Thus $G = G_1 \cup G_2$ is a edge glued neutrosophic bigraph.

$V(G_1) = \{ v_1, v_2, \ldots, v_{10}\}$ and

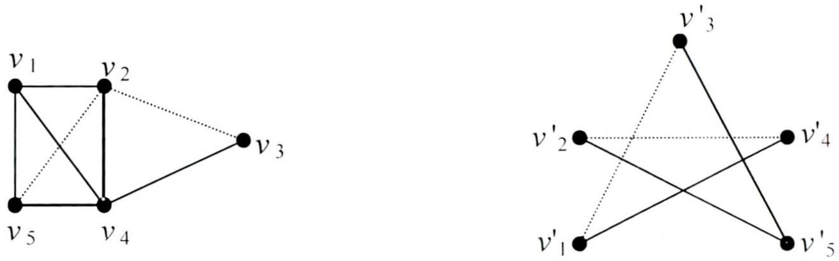
$V(G_1) = \{ v'_1, v'_2, \ldots, v'_6\}$

with every neutrosophic bigraph, there is a neutrosophic bimatrix associated with it.

Further with every weak neutrosophic bigraph we have a weak neutrosophic bimatrix associated with it.

Example 2.2.21

Let $G = G_1 \cup G_2$ be a neutrosophic bigraph given by the following figure .



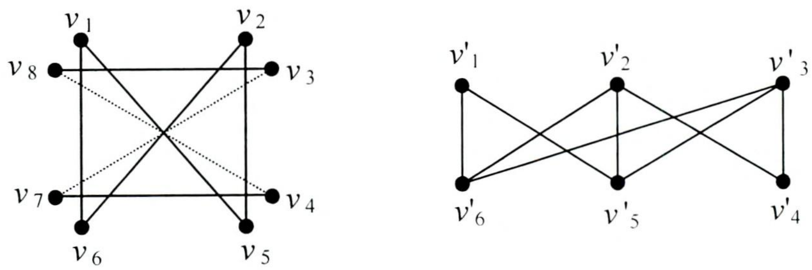
The associated neutrosophic bimatrix is given by M= $M_1 \cup M_2$.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & I & 1 & I \\ 0 & I & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & I & 0 & 1 & 0 \end{bmatrix} \end{array} \cup \begin{array}{c} \\ v'_1 \\ v'_2 \\ v'_3 \\ v'_4 \\ v'_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v'_1 & v'_2 & v'_3 & v'_4 & v'_5 \end{array} \\ \begin{bmatrix} 0 & 0 & I & 1 & 0 \\ 0 & 0 & 0 & I & 1 \\ I & 0 & 0 & 0 & 1 \\ 1 & I & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

We see both M_1 and M_2 are neutrosophic matrices so M is a neutrosophic bimatrix.

Example 2.2.22

Let G = $G_1 \cup G_2$ be a weak neutrosophic bigraph given by the following figure .



The related bimatrix M = $M_1 \cup M_2$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array}\begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & I \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & I & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \cup \begin{array}{c} \\ v'_1 \\ v'_2 \\ v'_3 \\ v'_4 \\ v'_5 \\ v'_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v'_1 & v'_2 & v'_3 & v'_4 & v'_5 & v'_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Clearly the bigraph has a associated bimatrix which is a weak neutrosophic bimatrix.