A STUDY ON LATTICE STRUCTURE OF NEUTROSOPHIC SOFT SETS

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Partial Fulfilment of the Requirements for the

Degree of Master of Science (M.Sc.)

By

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DECLARATION

DECLARATION

I declare that the thesis entitled **"A Study on Lattice Structure of Neutrosophic Soft Sets"** submitted by me for the degree of **Master of Science (M.Sc.)** is the record of work carried out by me during the period from December 2022 to May 2023 under the guidance of **Dr. C. Antony Crispin Sweety, M.Sc., B.Ed., M.Phil., Ph.D.,** Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this institute or any other University or other similar institution of Higher Learning.

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Signature of the Candidate

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Signature of the Supervisor

CERTIFICATE

CERTIFICATE

I certify that the thesis entitled **"A Study on Lattice Structure of Neutrosophic Soft Sets"** submitted for the degree of **Master of Science (M.Sc.)** by Ms. Arunaprabha K is the record of research work carried out by her during the period from December 2022 to May 2023 under my guidance and supervision, and that this work has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship or other Titles in this institute or any other University or institution of Higher Learning.

Signature of the Head of the Department

Signature of the Supervisor

Signature of the Director

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K. Arunaprabha

CONTENTS

SYNOPSIS

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In this study the algebraic structure of Neutrosophic soft set is studied using the operations.

The basic definitions of Fuzzy set and Intuitionistic fuzzy soft set are outlined in chapter 1.

Two aspects of the study are considered in chapter 1 and chapter 2. The first aspect is the study of basic definition of Fuzzy set, Soft set and the definition for the Neutrosophic soft set. The second aspect is the study of lattice structure on Neutrosophic soft sets using the operations union, intersection, restricted union and restricted intersection. And some properties like absorption, distributive and modular are also studies using the above operations. The ordering relation for two Neutrosophic soft set is studied.

Then the Neutrosophic soft equalities is studies. Then the image and inverse image of the Neutrosophic soft sets is studied.

CHAPTER 1

Introduction

It is unable to use traditional methodologies to successfully handle complicated issues in economics, technology, and the environment. Three ideas—the idea of probability, the idea of fuzzy sets, and the idea of interval mathematics—can be thought of as methods in mathematics for handling ambiguity. The concept of probabilities, the concept of fuzzy sets, the concept of vague sets, the concept of interval mathematics, and the concept of rough sets are among the research hypotheses that can be used to handle ambiguity instead of conventional mathematical techniques.

Each of these methods for dealing with uncertainties has benefits as well as inherent drawbacks. The concept of a soft set was created by Molodtsov [1] as a brand-new mathematical tool for addressing ambiguity that is free from the flaws that have plagued the conventional theoretical procedures in order to get past these challenges. Several applications for soft sets were put out by Molodtsov. This methodology has demonstrated success in a number of areas, including forecasting [9], data analysis [8, 7], and decision-making [2–6].

Research on soft sets has been very active, and many significant achievements have been made in the theoretical area. Maji et al. [10] performed a thorough theoretical study on soft sets and introduced various algebraic operations to the field of soft set theory. In addition, Ali et al. [11] proposed and examined some new algebraic methods for soft sets. Sezgin and Atagün [12] demonstrated that specific DeMorgan's laws apply to various operations on soft sets in the context of soft set theory and analysed the fundamental characteristics of such operations as intersection, extended intersection, restricted union, and restricted difference. The terms "mapping on classes of fuzzy soft sets and soft inverse images of fuzzy soft sets" were defined by Kharal and Ahmad [13,14], respectively, and numerous features of each were examined.

Review of Literature

D. Molodtsov, (1999), analyzed a general mathematical tool for handling ambiguous, fuzzily specified things is provided by the soft set theory. This paper's major goals are to establish the fundamental concepts of the theory of soft sets, to illustrate the theory's initial findings, and to analyse some potential future issues.

P.K. Maji, R. Biswas, A.R. Roy, (2001), The concept of a soft expert set to fuzzy soft expert set was defined, which will be more effective and useful. Also define its basic operations, namely complement, union, intersection, AND and OR. Then application of this concept in decision making problem was defined. Finally, Amapping on fuzzy soft expert classes and its properties was proposed.

P.K. Maji, R. Biswas and A.R. Roy, (2001), Introduce the concept of possibility fuzzy soft expert set and also define some basic operations and their properties. And given an applicstion of this theory in solving a decision making problem.

P.K. Maji et al., (2003), analyzed the Molodtsov-initiated theory of soft sets. With examples, the authors define the terms null soft set, absolute soft set, complement of a soft set, subset and superset of a soft set, and equality of two soft sets. Soft binary operations like AND, OR, as well as the union and intersection operations, are defined. In soft set theory, a number of results, including De Morgan's laws, are confirmed.

P.K. Maji, A.R. Roy, R. Biswas, (2004), The brief explanation of notion of soft set, fuzzy soft set and intuitionistic fuzzy soft set was introduced. Then extend the Jurio et al construction method of converting fuzzy set into intuitionistic fuzzy set to fuzzy soft set into intuitionistic fuzzy soft set. And consider a problem of decision making in fuzzy soft set and presented a method to generalize it into intuitionistic fuzzy soft set based decision making problem for modelling the problem in a better way. In the process the construction method and score function of intuitionistic fuzzy number was used.

D. Chen et al., (2005), described the uses of soft set parameterization reduction. And drawn attention to the fact that the soft set reduction results presented in are inaccurate.

A.R. Roy and P.K. Maji, (2007), presented a unique approach to object recognition using erroneous multiobserver data is given. The technique entails creating a Comparison Table for decision-making out of a fuzzy soft set in a parametric sense.

N. Çğman, (2007), The basic properties of soft sets were introduced, and compare soft sets to the related concepts of fuzzy sets and rough sets. Then given a definition of soft groups, and derive their basic properties using Molodtsov's definition of the soft sets.

Y. Zou and Z. Xiao, (2008), contributed to the conventional soft sets, the weight of each possible choice value is determined by the distribution of other objects, and the decision value of an object with incomplete information is derived by weighted-average of all possible choice values.

F. Feng, (2008), Molodtsov introduced the concept of soft sets, which can be seen as a new mathematical tool for dealing with uncertainty. In this paper, the study of soft semirings by using the soft set theory is initiated. The notions of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semiringhomomorphisms are introduced, and several related properties are investigated.

Y.B. Jun, (2008), In this paper the notion of soft sets by Molodtsov to the theory of BCK/BCI-algebras is applied. The notion of soft BCK/BCI-algebras and soft subalgebras are introduced, and their basic properties are derived.

Y.B. Jun and C.H. Park, (2008), The notion of soft p-ideals and p-idealistic soft BCI-algebras is introduced, and then investigate their basic properties. Using soft sets, the characterizations of (fuzzy) p-ideals in BCI-algebras is introduced. Then the relations between fuzzy p-ideals and p-idealistic soft BCI-algebras is given.

Z. Xiao et al., (2009), described the foundation of a government's policy-making and direction for a healthier international trade development is forecasting the export and import volume.

M. Irfan Ali, (2009), The theory of soft sets, which can be thought of as a novel mathematical approach to vagueness, was first developed by Molodtsov.

A. Aygünoğlu and H. Aygün, (2009), In this paper the concept of fuzzy soft group is introduced and in the meantime, some of their properties and structural characteristics are discussed and studied. Furthermore, definitions of fuzzy soft function and fuzzy soft homomorphism are defined and the theorems of homomorphic image and homomorphic pre-image are given. After that, the definition of normal fuzzy soft group is given and some of its basic properties are studied.

Y.B. Jun, K.J. Lee and C.H. Park, (2009), The notions of (transitive) soft d-algebras, soft edge d-algebras, soft d^*-algebras, soft d-ideals, soft $d^\#$-ideals, soft d^*-ideals, and -idealistic ($d^\#$-idealistic, or d^*-idealistic) soft d-algebras are introduced. Also, their related properties are surveyed.

F. Feng et al., (2010), studied the use of fuzzy soft sets in decision-making, Roy-Maji method's reliability and highlight its real drawbacks.

P.K. Maji et al., (2010), discussed a basic mathematics to address a decision-making problem by applying the notion of soft sets.

K. Qin and Z. Hong, (2010), The idea of soft sets, which can be thought of as a new mathematical tool for dealing with uncertainty, was first developed by Molodtsov. We discuss the algebraic structure of soft sets in this essay.

P. Majumdar and S.K. Samanta, (2010), The generalised fuzzy soft sets was defined and study some of their properties. Application of generalised fuzzy soft sets in decision making problem and medical diagnosis problem has been defined.

F. Koyuncu, B. Tanay, (2010), The main purpose of this paper is to introduce basic notions of soft rings, which are actually a parametrized family of subrings of a ring, over a ring **R**. Moreover, the concept of the soft ring homomorphism is introduced.

J. Zhan and Y.B. Jun, (2010), By means of ∈-soft sets and q-soft sets, some characterizations of (implicative, positive implicative and fantastic) filteristic soft BL-algebras are investigated. Finally, we prove that a soft set is an implicative filteristic soft BL-algebra if and only if it is both a positive implicative filteristic soft BL-algebra and a fantastic filteristic soft BL-algebra.

A. Sezgin, (2011), demonstrated the applicability of specific De Morgan's laws to various soft set operations.

Y. Jiang, (2011), The adjustable approach to fuzzy soft sets based decision making was generalize in this paper. Concretely, an adjustable approach to intuitionistic fuzzy soft sets based decision making by using level soft sets of intuitionistic fuzzy soft sets and give some illustrative examples. The properties of level soft sets are presented and discussed. Moreover, they also introduce the weighted intuitionistic fuzzy soft sets and investigate its application to decision making.

1. Preliminaries

Definition 1.1

The idea of the fuzzy sets, was first established by Zadeh in 1965 [29], and provides an appropriate framework for describing and interpreting fuzzy concepts by enabling partial memberships.

Assume that $\mathfrak{X}$ be a non-empty set. A fuzzy subset α of $\mathfrak{X}$ is defined as a mapping from $\mathfrak{X}$ into [0, 1], where [0, 1] is a usual interval of real numbers. The collection of all fuzzy sets of $\mathfrak{X}$ is defined by $\mathfrak{F}(\mathfrak{X})$. For α *and* $\gamma \in \mathfrak{F}(\mathfrak{X})$, by $\alpha \subseteq \gamma$. We mean $\alpha(x) \leq \gamma(x)$ for every $x \in \mathfrak{X}$. With the min-max system introduced by Zadeh [29], fuzzy union and intersection of α *and* γ, defined by $(\alpha \cup \gamma)$ and $(\alpha \cap \gamma)$, are defined in the fuzzy subsets of $\mathfrak{X}$ by

$$(\alpha \cup \gamma)(x) = \max \{ \alpha(x),\ \gamma(x)\} \qquad (\alpha \cap \gamma)(x) = \min \{ \alpha(x),\ \gamma(x)\}$$

For every $x \in \mathfrak{X}$, respectively

A fuzzy subset α of $\mathfrak{X}$ of the form

$$\alpha(y) = \begin{cases} \mathfrak{r}(\neq 0) & if\ y = x \\ 0 & otherwise \end{cases}$$

is said to be a fuzzy point with support x and value $\mathfrak{r}$ and is denoted by $x_{\mathfrak{r}}$, where $\mathfrak{r} \in (0, 1]$.

In what follows, let $\eta, \vartheta \in [0, 1]$ be such that $\eta < \vartheta$. For a fuzzy point $x_{\mathfrak{r}}$ and a fuzzy subset α of $\mathfrak{X}$, we say that

(1) $x_{\mathfrak{r}} \in_{\eta} \alpha$ if $\alpha(x) \geq \mathfrak{r} > \eta$.

(2) $x_{\mathfrak{r}}\, q_{\vartheta}\, \alpha$ if $\alpha(x) + \mathfrak{r} > 2\vartheta$.

(3) $x_{\mathfrak{r}} \in_{\eta} \vee q_{\vartheta}\, \alpha$ if $x_{\mathfrak{r}} \in_{\eta} \alpha$ or $x_{\mathfrak{r}} q_{\vartheta}\, \alpha$.

It is worth noting that the concepts of ‘‘$\in_{\eta}$’’ and ‘‘q_{ϑ}’’ are extensions of those of ‘‘∈’’ and ‘‘q’’ defined in Pu and Liu [30], respectively. Let us now introduce a new ordering relation on $\mathfrak{F}(\mathfrak{X})$, denoted by ‘‘$\subseteq \vee q_{(\eta,\vartheta)}$’’, as follows: $\forall \alpha, \gamma \in \mathfrak{F}(\mathfrak{X})$.

By $\alpha \subseteq \vee q_{(\eta,\vartheta)}\, \gamma$ we mean that $x_{\mathfrak{r}} \in_{\eta} \alpha$ implies $x_{\mathfrak{r}} \in_{\eta} \vee q_{\vartheta}\, \gamma$ for all $x \in \mathfrak{X}$ and $\mathfrak{r} \in (\eta, 1]$.

Definition 1.2 [31]

Assume that intuitionistic fuzzy set M of a non-empty set $\mathfrak{X}$ is of the form.

M = $\{\langle x, \alpha_M(x),\ \beta_M(x)\rangle / x \in \mathfrak{X}\}$

Where the functions $\alpha_M\colon \mathfrak{X} \to [0, 1]$ and $\beta_M\colon \mathfrak{X} \to [0, 1]$ denote the degree of membership $(namely\ \alpha_M(x))$ and degree of non membership $(namely\ \beta_M(x))$ of each element $x \in \mathfrak{X}$ to the set M, respectively and $0 \leq \alpha_M(x) + \beta_M(x) \leq 1$ for every $x \in \mathfrak{X}$.

Definition 1.3 [31]

Let M = $\{\langle x, \alpha_M(x),\ \beta_M(x)\rangle / x \in \mathfrak{X}\}$ and N = $\{\langle x, \alpha_N(x),\ \beta_N(x)\rangle / x \in \mathfrak{X}\}$ be intuitionistic fuzzy sets of $\mathfrak{X}$. Then:

(1) M $\subseteq N$ if and only if $\alpha_M(x) \leq \alpha_N(x)$ and $\beta_M(x) \geq \beta_N(x)$ for every $x\epsilon$ $\mathfrak{X}$.

(2) $M \cap N = \{\langle \mathfrak{X}, \min\{\alpha_M(x), \alpha_N(x)\},\ \max\{\beta_M(x), \beta_N(x)\}\rangle /\ x\epsilon\ \mathfrak{X}\}$,

(3) $M \cup N = \{\langle \mathfrak{X}, \max\{\alpha_M(x), \alpha_N(x)\},\ \min\{\beta_M(x), \beta_N(x)\}\rangle /\ x\epsilon\ \mathfrak{X}\}$.

For the purpose of the clarity, we use M = (α_M, β_M) to denote the intuitionistic fuzzy soft sets M = $\{\langle x, \alpha_M(x),\ \beta_M(x)\rangle / x \epsilon\ \mathfrak{X}\}$. The set of all intuitionistic fuzzy soft sets of $\mathfrak{X}$ is denoted by $\mathfrak{I}\ \mathfrak{F}(\mathfrak{X})$. Denoted by 1_x and 1^x the intuitionistic fuzzy sets of $\mathfrak{X}$ defined by $\alpha_{1_x}(x)$ = 0, $\beta_{1_x}(x)$= 1 and $\alpha_{1^x}(x)$= 1, $\beta_{1^x}(x)$= 0, respectively, for all $x \in \mathfrak{X}$.

Based on the ordering relation "$\subseteq \vee q_{(\eta,\vartheta)}$" on $\mathfrak{F}(\mathfrak{X})$, we define a new ordering relation "$\sqsubseteq_{(\eta,\vartheta)}$" on $\mathfrak{I}\ \mathfrak{F}(\mathfrak{X})$ as follows.

For any two intuitionistic fuzzy sets M and N, by M $\sqsubseteq_{(\eta,\vartheta)} N$ we mean that $\alpha_M \subseteq \vee q_{(\eta,\vartheta)} \alpha_N$ and $\beta_M \subseteq \vee q_{(1-\eta,1-\vartheta)} \beta_N$. Clearly M $\subseteq N$ implies M $\sqsubseteq_{(\eta,\vartheta)} N$ by lemma 2.1. M and N are said to be equal, denoted by M $\asymp_{(\eta,\vartheta)}$N, If M $\sqsubseteq_{(\eta,\vartheta)} N$ and N $\sqsubseteq_{(\eta,\vartheta)} M$. Lemma 2.1 and 2.2 gives that "$\asymp_{(\eta,\vartheta)}$" is an equivalence relation on $\mathfrak{I}\ \mathfrak{F}(\mathfrak{X})$. It is also worth noticing that M $\asymp_{(\eta,\vartheta)} N$ if and only if $\max\{\min\{\alpha_M(x), \vartheta\}, \eta\} = \max\{\min\{\alpha_N(x), \vartheta\}, \eta\}$ and $\max\{\min\{\beta_M(x), 1 - \vartheta\}, 1 - \eta\} = \max\{\min\{\beta_N(x), 1 - \vartheta\}, 1 - \eta\}$ by lemma 2.1 for all $x \in \mathfrak{X}$.

Definition 1.4 [1]

A pair (ψ, E) is called a soft set over $\mathfrak{U}$, where ψ is a mapping given by ψ : E → P($\mathfrak{U}$).

In other words, a soft set over $\mathfrak{U}$ is a parameterized collection of subsets of the universe $\mathfrak{U}$. For $\tau\ \epsilon$ E, $\psi(\tau)$ may be considered as the set of τ-approximate elements of the soft set (ψ, E).

Definition 1.5 [10]

For two soft sets $\langle\psi, M\rangle$and $\langle\phi, N\rangle$ over $\mathfrak{U}$, $\langle\psi, M\rangle$ is called a soft subset of $\langle\phi, N\rangle$ if,

(1) $M \subseteq N$ and

(2) for all $\tau \in M, \psi(\tau) \subseteq \phi(\tau)$.

Definition 1.6 [15]

Let (ψ, M) and (ϕ, N) be two soft sets over the universe $\mathfrak{U}$. $(\psi, M) \asymp_s (\phi, N)$ if for all $\tau \in M \cup N$, $\tau \in M \cap N$ implies $\psi(\tau) = \phi(\tau)$, $\tau \in M - N$ implies $\psi(\tau) = \emptyset$, and $\tau \in N - M$ implies $\phi(\tau) = \emptyset$.

Definition 1.7 [15]

Let (ψ, M) and (ϕ, N) be two soft sets over the universe $\mathfrak{U}$. $(\psi, M) \asymp^s (\phi, N)$ if for all $\tau \in M \cup N$, $\tau \in M \cap N$ implies $\psi(\tau) = \phi(\tau)$, $\tau \in M - N$ implies $\psi(\tau) = \mathfrak{U}$, and $\tau \in N - M$ implies $\phi(\tau) = \mathfrak{U}$.

By introducing the concept of intuitionistic fuzzy sets into the theory of soft sets, maji et al. [18] proposed the concept of the intuitionistic fuzzy soft sets as follows.

Definition 1.8 [18]

Let $\mathfrak{U}$ be an initial universe set, E a set of parameters and M $\subseteq E$. Then (ψ, M) is called an intuitionistic fuzzy soft set over $\mathfrak{U}$ where ψ is a mapping given by : M → $\mathfrak{I}\,\mathfrak{F}(\mathfrak{U})$.

In general, for every $\tau \in M, \psi(\tau)$ is an intuitionistic fuzzy set of $\mathfrak{U}$ and it is called intuitionistic fuzzy value set of parameter τ. Clearly, $\psi(\tau)$ can be written as an intuitionistic fuzzy set such that $\psi(\tau) = \{\langle x, \alpha_{\psi(\tau)}(x), \beta_{\phi(\tau)}(x)\rangle | x \in \mathfrak{U}\}$, where $\alpha_{\psi(\tau)}$ and $\beta_{\phi(\tau)}$ are the membership and non-membership functions, respectively. The set of all intuitionistic fuzzy soft sets over $\mathfrak{U}$ with parameters from E is called an intuitionistic fuzzy soft class, and it is denote by $\mathfrak{T}\,\mathfrak{F}\,\mathfrak{I}(\mathfrak{U}, E)$.

Definition 1.9 [18]

Let $\langle \psi, M \rangle$ and $\langle \phi, N \rangle$ be intuitionistic two fuzzy soft sets over $\mathfrak{U}$. We say that $\langle \psi, M \rangle$ is an intuitionistic fuzzy soft subset of $\langle \phi, N \rangle$ and write $\langle \psi, M \rangle \Subset \langle \phi, N \rangle$ if

(1) $M \subseteq N$;

(2) For any $\tau \in M, \psi(\tau) \subseteq \phi(\tau)$, that is, for all $x \in \mathfrak{U}$ and $\tau \in M$, $\alpha_{\psi(\tau)}(x) \leq \alpha_{\phi(\tau)}(x)$ and $\beta_{\psi(\tau)}(x) \geq \beta_{\phi(\tau)}(x)$. $\langle \psi, M \rangle$ and $\langle \phi, N \rangle$ are said to be intuitionistic fuzzy soft equal and write $\langle \psi, M \rangle = \langle \phi, N \rangle$ if $\langle \psi, M \rangle \Subset \langle \phi, N \rangle$ and $\langle \phi, N \rangle \Subset \langle \psi, M \rangle$.

Definition 1.10 [18]

The union of two intuitionistic fuzzy soft sets $\langle \psi, M \rangle$ *and* $\langle \phi, N \rangle$ over $\mathfrak{U}$ is an intuitionstic fuzzy soft set denoted by $\langle \omega, R \rangle$, where $R = M \cup N$ and

$$\omega(\tau) = \begin{cases} \psi(\tau) & \text{if } \tau \in M - N, \\ \varphi(\tau) & \text{if } \tau \in N - M, \\ \psi(\tau) \cup \varphi(\tau) & \text{if } \tau \in M \cap N, \end{cases}$$

For all $\tau \in R$. this is denoted by $\langle \omega, R \rangle = \langle \psi, M \rangle \breve{\cup} \langle \phi, N \rangle$.

Definition 1.11 [18]

Let $\langle \psi, M \rangle$ and $\langle \phi, N \rangle$ be two intuitionistic fuzzy soft sets over $\mathfrak{U}$ such that $M \cap N \neq \emptyset$. The restricted intersection of $\langle \psi, M \rangle$ and $\langle \phi, N \rangle$ is defined to be the intuitionistic fuzzy soft sets $\langle \omega, R \rangle$, where $R = M \cap N$ and $\omega(\tau) = \psi(\tau) \cap \phi(\tau)$ for all $\tau \in R$. This is denoted by $\langle \omega, R \rangle = \langle \psi, M \rangle \Cap \langle \phi, N \rangle$.

Definition 1.12 [18]

An intuitionistic fuzzy soft set $\langle \psi, M \rangle$ over $\mathfrak{U}$ is said to be a relative null intuitionistic fuzzy soft set (with respect to the parameter set M), denoted by $\emptyset_M$, if $\psi(\tau) = 1_{\mathfrak{U}}$ for all $\tau \in M$.

Definition 1.13 [18]

An intuitionistic fuzzy soft set $\langle \psi, M \rangle$ over $\mathfrak{U}$ is said to be a relative whole intuitionistic fuzzy soft set (with respect to the parameter set M), denoted by Σ_M, if $\psi(\tau) = 1^{\mathfrak{U}}$ for all $\tau \in M$.

The concepts of intersection and restricted union of two intuitionistic fuzzy soft sets can be studied as follows.

Definition 1.14 [18]

The intersection of two intuitionistic fuzzy soft sets $\langle\psi, M\rangle$ *and* $\langle\phi, N\rangle$ over $\mathfrak{U}$ is an intuitionstic fuzzy soft set denoted by $\langle\omega, R\rangle$, where R = M ⋃ N and

$$\omega(\tau) = \begin{cases} \psi(\tau) & \text{if } \tau \in M - N, \\ \varphi(\tau) & \text{if } \tau \in N - M, \\ \psi(\tau) \cap \varphi(\tau) & \text{if } \tau \in M \cap N, \end{cases}$$

For all $\tau \in R$. This is denoted by $\langle\omega, R\rangle = \langle\psi, M\rangle \breve{\Cap} \langle\phi, N\rangle$.

Definition 1.15 [18]

Let $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ be two intuitionistic fuzzy soft sets over $\mathfrak{U}$ such that M $\cap N \neq \emptyset$. The restricted union of $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ is defined to be the intuitionistic fuzzy soft sets $\langle\omega, R\rangle$, where R = M ⋂ N and $\omega(\tau) = \psi(\tau) \cup \phi(\tau)$ for all $\tau \in R$. This is denoted by $\langle\omega, R\rangle = \langle\psi, M\rangle \Cup \langle\phi, N\rangle$

As a generalization of the definition of the relative complement of a soft set introduced in Ali et al.[11].

Definition 1.16 [18]

The relative complement of an intuitionistic soft set $\langle\psi, M\rangle$ over $\mathfrak{U}$ is denoted by $\langle\psi, M\rangle^r$ and is defined by $\langle\psi^r, M\rangle$

Where for all $\tau \in M$, $\alpha_{\psi^r(\tau)} = \beta_{\psi(\tau)}$and $\beta_{\psi^r(\tau)} = \alpha_{\psi(\tau)}$, that is $\psi^r(\tau) = (\alpha_{\psi(\tau)}, \beta_{\psi(\tau)})$.

Clearly $(\langle\psi, M\rangle^r)^r = \langle\psi, M\rangle$

CHAPTER 2

2. A study on lattice structure of Neutrosophic soft sets

In this study the operation, properties and algebraic structure of neutrosophic soft sets is studied. The lattice structure of in neutrosophic soft sets is also studied. Then the expressions of (T, I, F)- neutrosophic soft equalities are studied and their basic properties are also studied. The connection between (T, I, F)- neutrosophic soft equalities and soft equalities is studied. The expression of a mapping on neutrosophic soft classes is studied and several properties of the image and the inverse image of neutrosophic soft sets are studied.

Introduction

It is unable to employ traditional approaches to effectively resolve complex problems in engineering, economics, and the environment due to a number of inherent uncertainties. The three theories of mathematical tools for handling uncertainties: the theory of probability, the theory of fuzzy sets, and the interval mathematics, but each of these hypotheses has its own drawbacks. Traditional mathematical methods cannot be used to manage uncertainties, but a variety of current theories, including probability theory, the theory of fuzzy sets, the theory of intuitionistic fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets are used.

However, each of these approaches of dealing with uncertainties has advantages as well as innate disadvantages. Molodtsov [1] developed the idea of soft set as a new mathematical tool for handling ambiguities that is free from the issues that have plagued the traditional theoretical techniques in order to get around these problems. Molodtsov suggested a number of uses for soft sets. This approach has proven effective in a variety of domains, including forecasting [9], data analysis [8, 7], and decision-making [2–6].

Research on soft sets has been very active, and many significant achievements have been made in the theoretical area. Maji et al. [10] performed a thorough theoretical study on soft sets and introduced various algebraic operations to the field of soft set theory. In addition, Ali et al. [11] proposed and examined some new algebraic methods for soft sets. Sezgin and Atagün [12] demonstrated that specific DeMorgan's laws apply to various operations on soft sets in the context of soft set theory and analysed the fundamental characteristics of such operations as intersection, extended intersection, restricted union, and restricted difference. The terms "mapping on classes of fuzzy soft sets and soft inverse images of fuzzy soft sets" were defined by Kharal and Ahmad [13,14], and numerous features of each were examined.

Further research into the soft set's algebraic structure was conducted by Qin and Hong [15], and also created the soft quotient algebra. The concepts of classical soft sets and fuzzy soft sets were extended by Maji et al. [16] and Majumdar and Samanta [17]. Maji et al.'s classical soft sets were expanded to intuitionistic fuzzy soft sets in Maji et al.[18] work, which was further examined in Maji et al.'s [19] and Jiang et al.'s [20] papers. Soft sets were compared to related ideas such as fuzzy sets and rough sets by Aktaş and Man [21]. Additionally, they developed other related features and created the idea of soft groups.

Aygünolu and Aygün [22] looked into normal fuzzy soft groups and explained how fuzzy soft sets can be used to group theory. Soft set theory was used by Feng et al. [23] to analyse soft semirings. Jun [24] first proposed the idea of soft BCK/BCI-algebras and researched it. The use of soft sets in the ideal theory of BCK/BCI-algebras and in d-algebras were explained by Jun and Park [25] and Jun et al. [26], respectively. Soft rings were first presented and researched by Koyuncu and Tanay [27]. Based on -soft sets and q-soft sets, Zhan and Jun [28] described the (implicative, positive implicative, and fantastic) filteristic soft BL-algebras. Smarandache introduced the concept of neutrosophic algebraic structure and neutrosophic N-algebraic structures. Bijan Davbaz introduced neutrosophic ideals of neutrosophic KU-algebra.

The main study of this paper is operating characteristics and algebraic structure of neutrosophic soft sets. The rest of the document is structured as follows. A few fundamental ideas are summarised in Part 2 and will be used throughout the study. The lattice structures of neutrosophic soft sets are studied in Part 3. The characteristics of (T, I, F)-neutrosophic soft equalities and (T, I, F) are studied in Part 4. The idea of a mapping on neutrosophic soft classes is studied in Part 5, and also studied the image and inverse image of neutrosophic soft sets.

The lattice structures of Neutrosophic soft sets

Definition 2.1

Let $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ be two neutrosophic soft sets over $\mathfrak{U}$ such that M $\cap N \neq \emptyset$. The restricted intersection of $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ is defined to be the neutrosophic soft sets $\langle\omega, R\rangle$, where R = M $\cap N$ and $\omega(\tau) = \psi(\tau) \cap \phi(\tau)$ for all $\tau \in R$. This is denoted by $\langle\omega, R\rangle = \langle\psi, M\rangle \Cap \langle\phi, N\rangle$.

Definition 2.2

An neutrosophic soft set $\langle\psi, M\rangle$ over $\mathfrak{U}$ is said to be a relative null neutrosophic soft set (with respect to the parameter set M), denoted by $\emptyset_M$, if $\psi(\tau) = 1_{\mathfrak{U}}$ for all $\tau \in M$.

Definition 2.3

An neutrosophic soft set $\langle\psi, M\rangle$ over $\mathfrak{U}$ is said to be a relative whole neutrosophic soft set (with respect to the parameter set M), denoted by Σ_M, if $\psi(\tau) = 1^{\mathfrak{U}}$ for all $\tau \in M$.

Definition 2.4

Let $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ be two neutrosophic soft sets over $\mathfrak{U}$ such that M $\cap N \neq \emptyset$. The restricted union of $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ is defined to be the neutrosophic soft sets $\langle\omega, R\rangle$, where R = M $\cap N$ and $\omega(\tau) = \psi(\tau) \cup \phi(\tau)$ for all $\tau \in R$. This is denoted by $\langle\omega, R\rangle = \langle\psi, M\rangle \Cup \langle\phi, N\rangle$

Definition 2.5

The relative complement of an neutrosophic soft set $\langle\psi, M\rangle$ over $\mathfrak{U}$ is denoted by $\langle\psi, M\rangle^r$ and is defined by $\langle\psi^r, M\rangle$

Where for all $\tau \in M$, $\alpha_{\psi^r(\tau)} = \beta_{\psi(\tau)}$ and $\beta_{\psi^r(\tau)} = \alpha_{\psi(\tau)}$,

Clearly $(\langle\psi, M\rangle^r)^r = (T_{\psi(\tau)}, 1 - I_{\psi(\tau)}, F_{\psi(\tau)})$

In this portion, the operations properties and lattice structure of Neutrosophic soft sets is studied.

Theorem 2.6

Let $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ be two neutrosophic soft sets over $\mathfrak{U}$.

(1)$\left(\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle\right)^{\mathfrak{r}}= \langle\psi, M\rangle^{\mathfrak{r}} \breve{\cap} \langle\phi, N\rangle^{\mathfrak{r}}$

and $\left(\langle\psi, M\rangle \breve{\cap} \langle\phi, N\rangle\right)^{\mathfrak{r}}= \langle\psi, M\rangle^{\mathfrak{r}} \breve{\cup} \langle\phi, N\rangle^{\mathfrak{r}}$

(2) If $M \cap N \neq \emptyset$, then $(\langle\psi, M\rangle \Cup \langle\phi, N\rangle)^{\mathfrak{r}} = \langle\psi, M\rangle^{\mathfrak{r}} \Cap \langle\phi, N\rangle^{\mathfrak{r}}$

and $(\langle\psi, M\rangle \Cap \langle\phi, N\rangle)^{\mathfrak{r}} = \langle\psi, M\rangle^{\mathfrak{r}} \Cup \langle\phi, N\rangle^{\mathfrak{r}}$.

Proof. Assume that $(\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle)^{\mathfrak{r}} = \langle\omega, M \cup N\rangle$ and $\langle\psi, M\rangle^{\mathfrak{r}} \breve{\cap} \langle\phi, N\rangle^{\mathfrak{r}} = \langle\varrho, M \cup N\rangle$ for any $\tau \in M \cup N$, we look about the below cases.

Case 1: $\tau \in M - N$ then $\omega(\tau) = \psi^{\mathfrak{r}}(\tau) = \varrho(\tau)$.

Case 2: $\tau \in N - M$ then $\omega(\tau) = \phi^{\mathfrak{r}}(\tau) = \varrho(\tau)$.

Case 3: $\tau \in M \cap N$

then $\omega(\tau) = (\psi^{\mathfrak{r}}(\tau) \cap \phi^{\mathfrak{r}}(\tau)) = \varrho(\tau)$.

Hence ω and ϱ are the same operators, and thus

$(\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle)^{\mathfrak{r}} = \langle\psi, M\rangle^{\mathfrak{r}} \breve{\cap} \langle\phi, N\rangle^{\mathfrak{r}}$

The below result is with respect to the operations $\breve{\cup}$ and $\Cap$.

Theorem 2.7

Let $\langle\psi, M\rangle, \langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \breve{\cup} \Sigma_E = \Sigma_E$ and $\langle\psi, M\rangle \Cap \Sigma_E = \langle\psi, M\rangle$.

(2) $\langle\psi, M\rangle \breve{\cup} \langle\psi, N\rangle = \langle\psi, M\rangle$ and $\langle\psi, M\rangle \Cap \langle\psi, N\rangle = \langle\psi, M\rangle$.

The below result can be easily deduced.

Theorem 2.8

Let $\langle\psi, M\rangle, \langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle = \langle\phi, N\rangle \breve{\cup} \langle\psi, M\rangle$

(2) $(\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle) \breve{\cup} \langle\omega, R\rangle = \langle\psi, M\rangle \breve{\cup} (\langle\phi, N\rangle \breve{\cup} \langle\omega, R\rangle)$

Theorem 2.9

Let $\langle\psi, M\rangle, \langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \breve{\cap} \emptyset_E = \emptyset_E$

(2) $\langle\psi, M\rangle \breve{\cap} \langle\psi, M\rangle = \langle\psi, M\rangle$

(3) $\langle\psi, M\rangle \breve{\cap} \langle\phi, N\rangle = \langle\phi, N\rangle \breve{\cap} \langle\psi, M\rangle$

(4) $(\langle\psi, M\rangle \breve{\cap} \langle\phi, N\rangle) \breve{\cap} \langle\omega, R\rangle = \langle\psi, M\rangle \breve{\cap} (\langle\phi, N\rangle \breve{\cap} \langle\omega, R\rangle)$

Theorem 2.10

Let $\langle\psi, M\rangle, \langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \Cup \emptyset_E = \emptyset_E$

(2) $\langle\psi, M\rangle \Cup \langle\psi, M\rangle = \langle\psi, M\rangle$

(3) $\langle\psi, M\rangle \Cup \langle\phi, N\rangle = \langle\phi, N\rangle \Cup \langle\psi, M\rangle$

(4) $(\langle\psi, M\rangle \Cup \langle\phi, N\rangle) \Cup \langle\omega, R\rangle = \langle\psi, M\rangle \Cup (\langle\phi, N\rangle \Cup \langle\omega, R\rangle)$

Theorem 2.11

Let $\langle\psi, M\rangle, \langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \Cap \langle\phi, N\rangle = \langle\phi, N\rangle \Cap \langle\psi, M\rangle$

(2) $(\langle\psi, M\rangle \Cap \langle\phi, N\rangle) \Cap \langle\omega, N\rangle = \langle\psi, M\rangle \Cap (\langle\phi, N\rangle \Cap \langle\omega, N\rangle)$

The below theorem shows that the absorption law with respect to the operations $\breve{\cup}$ and $\Cap$ holds.

Theorem 2.12

Let $\langle\psi, M\rangle$and $\langle\phi, N\rangle$ be two neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $(\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle) \Cap \langle\psi, M\rangle = \langle\psi, M\rangle$.

(2) $(\langle\psi, M\rangle \Cap \langle\phi, N\rangle)\ \breve{\cup}\langle\psi, M\rangle = \langle\psi, M\rangle$.

Proof. Assume that $(\langle\psi, M\rangle\breve{\cup}\langle\phi, N\rangle) \Cap \langle\psi, M\rangle = \langle\omega, (M \cup N) \cap M\rangle$ for any $\tau \in M$, we consider the following cases.

Case 1: $\tau \in M$. Then $\omega(\tau) = \big(\psi(\tau) \cup \phi(\tau)) \cap \psi(\tau)\big) = \psi(\tau)$.

Case 2: $\tau \notin M$. Then $\omega(\tau) = \psi(\tau) \cap \psi(\tau) = \psi(\tau)$.

Hence, ψ and ω are the same operators, and thus $(\langle\psi, M\rangle\breve{\cup}\langle\phi, N\rangle) \Cap \langle\psi, M\rangle = \langle\psi, M\rangle$.

The below theorem shows that the absorption law with respect to the operations $\breve{\cap}$ and $\Cup$ holds.

Theorem 2.13

Let $\langle\psi, M\rangle$and $\langle\phi, N\rangle$ be two neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $(\langle\psi, M\rangle\ \breve{\cap}\ \langle\phi, N\rangle) \Cup \langle\psi, M\rangle = \langle\psi, M\rangle$.

(2) $(\langle\psi, M\rangle \Cup \langle\phi, N\rangle)\ \breve{\cap}\ \langle\psi, M\rangle = \langle\psi, M\rangle$.

Proof. The proof is similar to that of Theorem 3.7

The absorption law with respect to operations $\breve{\cap}$ and $\breve{\cup}$, $\Cap$ and $\Cup$ may not hold in general as shown in the following example.

Example 2.14

Let $\mathfrak{U}$ be the universe, $\mathrm{E} = \{\tau_1, \tau_2, \tau_3\}$, $\mathrm{M} = \{\tau_1, \tau_2\}$ and $\mathrm{N} = \{\tau_1, \tau_3\}$. Let $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ be any neutrosophic soft sets over $\mathfrak{U}$.

Suppose that $(\langle\psi, M\rangle\ \breve{\cap}\ \langle\phi, N\rangle)\ \breve{\cup}\ \langle\psi, M\rangle = \langle\omega, M \cup N\rangle$

and $(\langle\psi, M\rangle \Cap \langle\phi, N\rangle) \Cup \langle\psi, M\rangle = \langle\varrho, M \cup N\rangle$

Since $\mathrm{M} \subset E = \mathrm{M} \cup N$ and $\mathrm{M} \cap N = \{\tau_1\} \subset \mathrm{M}$,

we have $(\langle\psi, M\rangle\ \breve{\cap}\ \langle\phi, N\rangle\ \breve{\cup}\ \langle\psi, M\rangle \neq \langle\psi, M\rangle$ and $(\langle\psi, M\rangle \Cap \langle\phi, N\rangle) \Cup \langle\psi, M\rangle \neq \langle\psi, M\rangle$.

The below theorem shows that the distributive law with respect to the operations $\Cap$ and $\breve{\cup}$ holds.

Theorem 2.15

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \Cap (\langle\phi, N\rangle \breve{\cup} \langle\omega, R\rangle) = (\langle\psi, M\rangle \Cap \langle\phi, N\rangle) \breve{\cup} (\langle\psi, M\rangle \Cap \langle\omega, R\rangle)$.

(2) $\langle\psi, M\rangle \breve{\cup} (\langle\phi, N\rangle \Cap \langle\omega, R\rangle) = (\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle) \Cap (\langle\psi, M\rangle \breve{\cup} \langle\omega, R\rangle)$.

Proof. Suppose that

$$\langle\psi, M\rangle \Cap (\langle\phi, N\rangle \breve{\cup} \langle\omega, R\rangle) = \langle\varrho, M \cap (N \cup R)\rangle,$$

$$(\langle\psi, M\rangle \Cap \langle\phi, N\rangle) \breve{\cup} (\langle\psi, M\rangle \Cap \langle\omega, R\rangle) = \langle\varsigma, (M \cap N) \cup (M \cap R)\rangle$$
$$= \langle\varsigma, M \cap (N \cup R)\rangle.$$

For any $\tau \in M \cap (N \cup R)$, it shows that $\tau \in M$ and $\tau \in N \cup R$. We Consider the below cases.

Case 1: $\tau \in M, \tau \notin N$ and $\tau \in R$. Then $\varrho(\tau) = \psi(\tau) \cap \omega(\tau) = \varsigma(\tau)$.

Case 2: $\tau \in M, \tau \in N$ and $\tau \notin R$. Then $\varrho(\tau) = \psi(\tau) \cap \omega(\tau) = \varsigma(\tau)$.

Case 3: $\in M$, $\epsilon \in N$ and $\epsilon \in R$. Then $\varrho(\tau) = \psi(\tau) \cap \big(\phi(\tau) \cup \omega(\tau)\big) = \big(\psi(\tau) \cap \phi(\tau)\big) \cup \big(\psi(\tau) \cap \omega(\tau)\big) = \varsigma(\tau)$.

Hence, ϱ and ς are the same operators, and thus$\langle\psi, M\rangle \Cap (\langle\phi, N\rangle \breve{\cup} \langle\omega, R\rangle) = (\langle\psi, M\rangle \Cap \langle\phi, N\rangle) \breve{\cup} (\langle\psi, M\rangle \Cap \langle\omega, R\rangle)$.

The below theorem shows that the distributive law with respect to the operations $\breve{\cap}$ and $\Cup$ holds.

Theorem 2.16

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \Cup (\langle\phi, N\rangle \breve{\cap} \langle\omega, R\rangle) = (\langle\psi, M\rangle \Cup \langle\phi, N\rangle) \breve{\cap} (\langle\psi, M\rangle \Cup \langle\omega, R\rangle)$.

(2) $\langle\psi, M\rangle \breve{\cap} (\langle\phi, N\rangle \Cup \langle\omega, R\rangle) = (\langle\psi, M\rangle \breve{\cap} \langle\phi, N\rangle) \Cup (\langle\psi, M\rangle \breve{\cap} \langle\omega, R\rangle)$.

Proof. The proof is similar to that of Theorem 3.10

The below theorem shows that the distributive law with respect to the operations ⋒ and ⋓ holds.

Theorem 2.17

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \Cup (\langle\phi, N\rangle \Cap \langle\omega, R\rangle) = (\langle\psi, M\rangle \Cup \langle\phi, N\rangle) \Cap (\langle\psi, M\rangle \Cup \langle\omega, R\rangle)$.

(2) $\langle\psi, M\rangle \Cap (\langle\phi, N\rangle \Cup \langle\omega, R\rangle) = (\langle\psi, M\rangle \Cap \langle\phi, N\rangle) \Cup (\langle\psi, M\rangle \Cap \langle\omega, R\rangle)$.

Proof. The proof is similar to that of Theorem 3.10.

The below theorem shows that the modular law with respect to the operations $\breve{\cap}$ and $\breve{\cup}$holds.

Theorem 2.18

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Such that$\langle\omega, R\rangle \subseteq \langle\psi, M\rangle$.

Then $\langle\psi, M\rangle\breve{\cap} (\langle\phi, N\rangle\breve{\cup}\langle\omega, R\rangle) \Subset (\langle\psi, M\rangle\breve{\cap}\langle\phi, N\rangle) \breve{\cup}\langle\omega, R\rangle$

and $\langle\psi, M\rangle\breve{\cap} (\langle\phi, N\rangle\breve{\cup}\langle\omega, R\rangle) = (\langle\psi, M\rangle\breve{\cap}\langle\phi, N\rangle) \breve{\cup}\langle\omega, R\rangle$ if $M \subseteq N$.

Proof. Assume that

$\langle\psi, M\rangle\breve{\cap} (\langle\phi, N\rangle\breve{\cup}\langle\omega, R\rangle) = \langle\varrho, M \cup (N \cup R)\rangle$,

$(\langle\psi, M\rangle\breve{\cap}\langle\phi, N\rangle) \breve{\cup}\langle\omega, R\rangle = \langle\varsigma, (M \cup N) \cup R\rangle$.

For any $\tau \in M \cup (N \cup R)$, we look about the below cases.

Case 1: $\tau \in M, \tau \in N$ and $\tau \notin R$. Then $\varrho(\tau) = \psi(\tau) \cap \omega(\tau) = \varsigma(\tau)$.

Case 2: $\tau \in M, \tau \notin N$ and $\tau \in R$.

Then $\varrho(\tau) = \psi(\tau) \cap \omega(\tau) = \omega(\tau) \subseteq \psi(\tau) = \psi(\tau) \cup \omega(\tau) = \varsigma(\tau)$.

Case 3: $\tau \in M, \tau \notin N$ and $\tau \notin R$. Then $\varrho(\tau) = \psi(\tau) = \varsigma(\tau)$.

Case 4: $\tau \in M$,$\tau \in N$ and$\tau \in R$. Then $\varrho(\tau) = \psi(\tau) \cap (\phi(\tau) \cup \omega(\tau)) = (\psi(\tau) \cap \phi(\tau)) \cup (\psi(\tau) \cap \omega(\tau)) = (\psi(\tau) \cap \phi(\tau)) \cup \omega(\tau) = \varsigma(\tau)$.

Case 5: $\notin M$, $\tau \in N$ and $\tau \notin R$. Then $\varrho(\tau) = \phi(\tau) = \varsigma(\tau)$.

Hence, Then$\langle\psi, M\rangle \breve{\cap} (\langle\phi, N\rangle \breve{\cup} \langle\omega, R\rangle) \Subset (\langle\psi, M\rangle \breve{\cap} \langle\phi, N\rangle) \breve{\cup} \langle\omega, R\rangle$. If $M \subseteq N$, then cases 2 and 3 do not hold.

Therefore$\langle\psi, M\rangle \breve{\cap} (\langle\phi, N\rangle \breve{\cup} \langle\omega, R\rangle) = (\langle\psi, M\rangle \breve{\cap} \langle\phi, N\rangle) \breve{\cup} \langle\omega, R\rangle$.

Theorem 2.19

(1) ($\mathfrak{N}\mathfrak{I}$ $(\mathfrak{A}, E)$,$\breve{\cup}$,$\Cap$) is a complete distributive lattice under the ordering relation "$\Subset$".

(2) ($\mathfrak{N}\mathfrak{I}$ $(\mathfrak{A}, E)$, $\Cup$, $\breve{\cap}$) is a complete distributive lattice under the ordering relation "$\Subset_1$". Where for any $\langle\psi, M\rangle, \langle\phi, N\rangle \in \mathfrak{N}\mathfrak{I}$ $(\mathfrak{A}, E)$,$\langle\psi, M\rangle \Subset_1 \langle\phi, N\rangle$ iff $N \subseteq M$ and $\psi(\tau) \subseteq \phi(\tau)$ for any $\tau \in N$.

Proof. By Theorems 3.2, 3.3, 3.6, 3.7 and 3.10, ($\mathfrak{N}\mathfrak{I}$ $(\mathfrak{A}, E)$, $\breve{\cup}$, $\Cap$) is a distributive lattice. For any $\langle\psi, M\rangle, \langle\phi, N\rangle \in \mathfrak{N}\mathfrak{I}$ $(\mathfrak{A}, E)$, it is easy to see that $\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle$ and $\langle\psi, M\rangle \Cap \langle\phi, N\rangle$ are the least upper bound and the greatest lower bound of $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$, respectively. There is no difficulty in replacing the $\{\langle\psi, M\rangle, \langle\phi, N\rangle\}$ with an arbitrary collection of $\mathfrak{N}\mathfrak{I}$ $(\mathfrak{A}, E)$ and so ($\mathfrak{N}\mathfrak{I}$ $(\mathfrak{A}, E)$, $\breve{\cup}$, $\Cap$) is complete. Hence ($\mathfrak{N}\mathfrak{I}$ $(\mathfrak{A}, E)$, $\breve{\cup}$, $\Cap$) is a complete distributive lattice.

Assume the neutrosophic soft sets over a definite parameter set. $M \subseteq E$and

$\mathfrak{N}\mathfrak{I}_M(\mathfrak{A}) = \{\langle\psi, M\rangle | \psi : M \rightarrow \mathfrak{N}\mathfrak{F}(\mathfrak{U})\}$

Be the set of neutrosophic soft set over the universe $\mathfrak{U}$ and the parameter set M.It is trivial to verify that $\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle, \langle\psi, M\rangle \breve{\cap} \langle\phi, N\rangle, \langle\psi, M\rangle \Cup \langle\phi, N\rangle, \langle\psi, M\rangle \Cap \langle\phi, N\rangle \in \mathfrak{T}\mathfrak{F}\mathfrak{I}_M(\mathfrak{A})$ for all $\langle\psi, M\rangle, \langle\phi, N\rangle \in \mathfrak{T}\mathfrak{F}\mathfrak{I}_M(\mathfrak{A})$.

Corollary 2.20

$(\mathfrak{T}\mathfrak{F}\mathfrak{J}_M(\mathfrak{A}),\ \breve{\cup},\ \Cap)$ and $(\mathfrak{T}\mathfrak{F}\mathfrak{J}_M(\mathfrak{A}),\ \Cup,\ \breve{\cap})$are sub lattices of $(\mathfrak{T}\mathfrak{F}\mathfrak{J}(\mathfrak{A}, E),\ \breve{\cup},\ \Cap)$ and $(\mathfrak{T}\mathfrak{F}\mathfrak{J}(\mathfrak{A}, E),\ \Cup, \breve{\cap})$ with minimal element $\emptyset_M$and maximal element Σ_M, respectively.

Definition 2.21

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Define a neutrosophic soft set $\langle\psi, M\rangle : \langle\phi, N\rangle$over $\mathfrak{U}$ by

$\langle\psi, M\rangle : \langle\phi, N\rangle = \breve{\cup}\{\langle\omega, R\rangle \in \mathfrak{T}\mathfrak{F}\mathfrak{J}(\mathfrak{A}, E) \mid \langle\psi, M\rangle \Cap \langle\omega, R\rangle \Subset \langle\phi, N\rangle\}$.

Lemma 2.22

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1)$\langle\psi, M\rangle \Subset \langle\phi, N\rangle$implies$\langle\psi, M\rangle \breve{\cup} \langle\omega, R\rangle \Subset \langle\phi, N\rangle \breve{\cup} \langle\omega, R\rangle$

and $\langle\psi, M\rangle \Cap \langle\omega, R\rangle \Subset \langle\phi, N\rangle \Cap \langle\omega, R\rangle$.

(2) $\langle\psi, M\rangle \Cap \langle\psi, M\rangle : \langle\omega, R\rangle \Subset \langle\omega, R\rangle$.

(3) $\langle\psi, M\rangle \Subset \langle\phi, N\rangle$implies$\langle\phi, N\rangle : \langle\omega, R\rangle \Subset \langle\psi, M\rangle : \langle\omega, R\rangle$

and $\langle\omega, R\rangle : \langle\psi, M\rangle \Subset \langle\omega, R\rangle : \langle\phi, N\rangle$.

(4) $\langle\psi, M\rangle \Cap \langle\phi, N\rangle \Subset \langle\omega, R\rangle$Iff $\langle\phi, N\rangle \Subset \langle\psi, M\rangle : \langle\omega, R\rangle$.

Proof. (1) Suppose that $\langle\psi, M\rangle \breve{\cup} \langle\omega, R\rangle = \langle\varrho, M \cup R\rangle$ and $\langle\phi, N\rangle \breve{\cup} \langle\omega, R\rangle = \langle\varsigma, N \cup R\rangle$.

From $\langle\psi, M\rangle \Subset \langle\phi, N\rangle$, we have $M \subseteq N$ and $\psi(\tau) \subseteq \phi(\tau)$ for any $\tau \in M$.

Now for any $\tau \in M \cup R$, we consider the following cases.

Case 1: $\tau \in M - R$. Then $\tau \in N - R$. Hence $\varrho(\tau) = \psi(\tau) \subseteq \phi(\tau) = \varsigma(\tau)$.

Case 2: $\tau \in (N \cap R) - M$. Then $\varrho(\tau) = \omega(\tau) \subseteq \phi(\tau) \cup \omega(\tau) = \varsigma(\tau)$.

Case 3: $\tau \in R - N$. Then $\tau \in R - M$. Hence $\varrho(\tau) = \omega(\tau) = \varsigma(\tau)$.

Case 4: $\tau \in M \cap R$. Then $\tau \in N \cap R$. Hence $\varrho(\tau) = \psi(\tau) \cup \omega(\tau) \subseteq \phi(\tau) \cup \omega(\tau) = \varsigma(\tau)$.

(2) Let $\langle\psi, M\rangle : \langle\omega, R\rangle = \breve{\bigcup}_{i\in\Lambda}\{\langle\phi_i, N_i\rangle \in \mathfrak{N}\,\mathfrak{J}\,(\mathfrak{U}, E) | \langle\psi, M\rangle \Cap \langle\phi_i, N_i\rangle \Subset \langle\omega, R\rangle\} = \langle I, \cup_{i\in\Lambda} N_i\rangle$, where Λ is an indexset. For any $i \in \Lambda$, by $\langle\psi, M\rangle \Cap \langle\phi_i, N_i\rangle \Subset \langle\omega, R\rangle$, we have $M \cap N_i \subseteq R$ and $\psi(\tau) \cap \phi_i(\tau) \subseteq \omega(\tau)$ for any $\tau \in M \cap N_i$.

Hence $M \cap \cup_{i\in\Lambda} N_i = \bigcup_{i\in\Lambda}(M \cap N_i) \subseteq R$ and it is trivial to verify that $\psi(\tau) \cap \varrho(\tau) \subseteq \omega(\tau)$ for any $\tau \in M \cap \bigcup_{i\in\Lambda} N_i$.

Hence $\langle\psi, M\rangle \Cap (\langle\psi, M\rangle : \langle\omega, R\rangle) \Subset \langle\omega, R\rangle$.

Extend the operations $(\mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E), \breve{\bigcup}, \Cap)$ to $\mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E) \cup \emptyset$ by defining

$\langle\psi, M\rangle \breve{\cup} \emptyset = \langle\psi, M\rangle$ and $\langle\psi, M\rangle \Cap \emptyset = \emptyset$

for all $\langle\psi, M\rangle \in \mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E)$, where $\emptyset \in \mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E)$.

Then $(\mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E) \cup \emptyset, \breve{\cup}, \Cap, \emptyset, \Sigma_E)$ is a bounded lattice with minimal Element $\emptyset$ and maximal element Σ_E by Theorems 3.2, 3.3, 3.6and 3.15. As a consequence of Lemma 3.18, we can obtain the following result.

Theorem 2.23

$(\mathfrak{N}\,\mathfrak{J}(\mathfrak{A}, E) \cup \emptyset, \breve{\cup}, \Cap, :, \emptyset, \Sigma_E))$ is a complete distributive residuated lattice under the ordering relation "$\Subset$".

Definition 2.24

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Define $\langle\psi, M\rangle :_1 \langle\phi, N\rangle$ over $\mathfrak{U}$ by

$\langle\psi, M\rangle :_1 \langle\phi, N\rangle = \Cup \{\langle\omega, R\rangle \in \mathfrak{N}\,\mathfrak{J}\,(\mathfrak{A}, E) \mid \langle\psi, M\rangle \breve{\cap} \langle\omega, R\rangle \Subset_1 \langle\phi, N\rangle$.

Lemma 2.25

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$ and $\langle\omega, R\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. Then:

(1) $\langle\psi, M\rangle \Subset_1 \langle\phi, N\rangle$ implies $\langle\psi, M\rangle \breve{\cap} \langle\omega, R\rangle \Subset_1 \langle\phi, N\rangle \breve{\cap} \langle\omega, R\rangle$

and $\langle\psi, M\rangle \Cup \langle\omega, R\rangle \Subset_1 \langle\phi, N\rangle \Cup \langle\omega, R\rangle$.

(2) $\langle\psi, M\rangle \breve{\cap} \langle\psi, M\rangle :_1 \langle\omega, R\rangle \Subset_1 \langle\omega, R\rangle$.

(3) $\langle\psi, M\rangle \Subset_1 \langle\phi, N\rangle$ implies $\langle\phi, N\rangle :_1 \langle\omega, R\rangle \Subset_1 \langle\psi, M\rangle :_1 \langle\omega, R\rangle$

and $\langle\omega, R\rangle :_1 \langle\psi, M\rangle \Subset_1 \langle\omega, R\rangle :_1 \langle\phi, N\rangle$.

(4) $\langle\psi, M\rangle \breve{\sqcap} \langle\phi, N\rangle \Subset_1 \langle\omega, R\rangle$ Iff $\langle\phi, N\rangle \Subset_1 \langle\psi, M\rangle :_1 \langle\omega, R\rangle$.

Proof. The proof is similar to that of Lemma 3.18.

Extend the operations ($\mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E)$, $\Cup$, $\breve{\sqcap}$) to $\mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E) \cup \Sigma$ by defining

$\langle\psi, M\rangle \Cup \Sigma = \Sigma$ and $\langle\psi, M\rangle \breve{\sqcap} \Sigma = \langle\psi, M\rangle$

for all $\langle\psi, M\rangle \in \mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E)$, where $\Sigma \in \mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E)$. Then ($\mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E) \cup \Sigma$, $\Cup$, $\breve{\sqcap}$, $\emptyset_E$, Σ) is a bounded lattice with minimal element $\emptyset_E$ and maximal element Σ by Theorems 3.4, 3.5and 3.15. As a consequence of Lemma 3.21, we can obtain the following result.

Theorem 2.26

($\mathfrak{N}\,\mathfrak{J}(\mathfrak{A}, E) \cup \Sigma$, $\Cup$, $\breve{\sqcap}$, $:_1$, $\emptyset_E$, Σ) is a complete distributive residuated lattice under the ordering relation "$\Subset_1$".

(T, I, F) neutrosophic soft equalities $\doteq_{(T,I,F)}$ and $\triangleq_{(T,I,F)}$

Qin and Hong proposed the properties of soft equalities $\approx_S$ and $\approx^S$ on soft sets. In general we learn about the properties of (T, I, F)- neutrosophic soft equalities $\doteq_{(T,I,F)}$and $\triangleq_{(T,I,F)}$ on neutrosophic soft sets.

Definition 2.27

Let $\langle\psi$, M$\rangle$ and $\langle\phi$, N$\rangle$ be two neutrosophic soft sets over $\mathfrak{U}$. We say that $\langle\psi$, M$\rangle$ is a (T, I, F) –neutrosophic soft subset of $\langle\phi$, N$\rangle$ and write $\langle\psi, M\rangle \Subset_{(T,I,F)} \langle\phi, N\rangle$ if,

(1) $M \subseteq N$;

(2) for any $\tau \in M, \psi(\tau) \sqsubseteq_{(\eta,\vartheta)} \phi(\tau)$.

$\langle\psi$, M$\rangle$ and $\langle\phi$, N$\rangle$ are said to be $(T, I, F) - neutrosophic$ soft equal and write

$\langle\psi, M\rangle =_{(T,I,F)} \langle\phi, N\rangle$ if $\langle\psi, M\rangle \Subset_{(T,I,F)} \langle\phi, N\rangle$ and $\langle\phi, N\rangle \Subset_{(T,I,F)} \langle\psi, M\rangle$.

It is worth noticing that if $\langle\psi, M\rangle \Subset \langle\phi, N\rangle$ then $\langle\psi, M\rangle \Subset_{(T,I,F)} \langle\phi, N\rangle$

For any r $\in$ [0, 1],$\langle\psi, M\rangle \in \mathfrak{N}\,\mathfrak{J}(\mathfrak{U}, E)$ and $\tau \in M$, denote

$\psi_{\mathfrak{r}}^{(T,I,F)}(\tau) = \{x \in \mathfrak{U} | x_{\mathfrak{r}} \in_{\eta} \alpha_{\psi(\tau)}\}$,

$\langle\psi\rangle_{\mathfrak{r}}^{(T,I,F)}(\tau) = \{x \in \mathfrak{U} | x_{\mathfrak{r}} q_{\vartheta}\, \alpha_{\psi(\tau)}\}$,

$[\psi]_{\mathfrak{r}}^{(T,I,F)}(\tau) = \{x \in \mathfrak{U} | x_{\mathfrak{r}} \in_{\eta} \vee q_{\vartheta}\, \alpha_{\psi(\tau)}\}$,

$\hat{\psi}_{\mathfrak{r}}^{(T,I,F)}(\tau) = \{x \in \mathfrak{U} | \beta_{\psi(\tau)}(x) \leq \mathfrak{r}\}$,

$\widehat{\langle\psi\rangle}_{\mathfrak{r}}^{(T,I,F)}(\tau) = \{x \in \mathfrak{U} | \beta_{\psi(\tau)}(x) + \mathfrak{r} < 2(1-\vartheta)\}$

And

$\widehat{[\psi]}_{\mathfrak{r}}^{(T,I,F)}(\tau) = \left\{x \in \mathfrak{U} \middle| \beta_{\psi(\tau)}(x) \leq \mathfrak{r} \ or\ \beta_{\psi(\tau)}(x) + \mathfrak{r} < 2(1-\vartheta)\right\}$.

Then $\left(\psi_{\mathfrak{r}}^{(T,I,F)}, M\right)$, $\left(\langle\psi\rangle_{\mathfrak{r}}^{(T,I,F)}, M\right)$, $\left([\psi]_{\mathfrak{r}}^{(T,I,F)}, M\right)$, $\left(\hat{\psi}_{\mathfrak{r}}^{(T,I,F)}, M\right)$, $\left(\widehat{\langle\psi\rangle}_{\mathfrak{r}}^{(T,I,F)}, M\right)$ and $\left(\widehat{[\psi]}_{\mathfrak{r}}^{(T,I,F)}, M\right)$ are soft sets over $\mathfrak{U}$.

Theorem. 2.28

Let $\langle\psi$, M$\rangle$ and $\langle\phi$, N$\rangle$ are two neutrosophic soft sets over $\mathfrak{A}$. If $\langle\psi$, M$\rangle$ is a (T, I, F)-neutrosophic soft subset of $\langle\phi$, N$\rangle$, then

(1) Soft set $(\psi_{\mathfrak{r}}^{(\eta,\vartheta)}, M)$ is a soft subset of $(\phi_{\mathfrak{r}}^{(\eta,\vartheta)}, N)$ for all $\mathfrak{r} \in (\eta, \vartheta]$;

(2) Soft set $(\langle\psi\rangle_{\mathfrak{r}}^{(T,I,F)}, M)$ is a soft subset of $(\langle\phi\rangle_{\mathfrak{r}}^{(T,I,F)}, N)$ for every $\mathfrak{r} \in (\vartheta$, $\min\{2\vartheta - \eta, 1\}]$;

(3) Soft set $([\psi]_{\mathfrak{r}}^{(T,I,F)}, M)$ is a soft subset of $([\phi]_{\mathfrak{r}}^{(T,I,F)}, N)$ for every $\mathfrak{r} \in (\eta$, $\min\{2\vartheta - \eta, 1\}]$;

(4) Soft set$\left(\hat{\psi}_{\mathfrak{r}}^{(T,I,F)}, M\right)$ is a soft subset of $\left(\hat{\phi}_{\mathfrak{r}}^{(T,I,F)}, N\right)$ for every $\mathfrak{r} \in [1-\vartheta, 1-\eta)$;

(5) soft set $\left(\langle\hat{\psi}\rangle_{\mathfrak{r}}^{(T,I,F)}, M\right)$is a soft subset of $\left(\langle\hat{\phi}\rangle_{\mathfrak{r}}^{(T,I,F)}, N\right)$ for every $\mathfrak{r} \in [\max\{1+\eta - 2\vartheta, 0\}, 1-\vartheta)$.

(6) Soft set $(\widehat{[\psi]}_{\mathfrak{r}}^{(T,I,F)}, M)$is a soft subset of $(\widehat{[\phi]}_{\mathfrak{r}}^{(T,I,F)}, N)$ for every $\mathfrak{r} \in [\max\{1 + \eta - 2\vartheta, 0\}, 1-\eta)$.

Proof:

(1) Let τ ∈ M, ŗ ∈ $(\eta,\vartheta]$ and $x \in \psi_{ŗ}^{(T,I,F)}$(τ). Then $x_{ŗ} \in_{\eta} \alpha_{\psi(\tau)}$, that is, $\alpha_{\psi(\tau)}(x) \geq$ ŗ$>\eta$. Since⟨ψ, M⟩ is a (T,I,F)- neutrosophic soft subset of ⟨ϕ, N⟩, there are $\max\{\alpha_{\psi(\tau)}(x), \eta\} \geq \min\{\alpha_{\psi(\tau)}(x), \vartheta\} \geq \{ŗ, \vartheta\}$ = ŗ, and so $\alpha_{\psi(\tau)}(x) \geq$ ŗ $> \eta$, Since ŗ $> \eta$, that is, $x \in \phi_{ŗ}^{(T,I,F)}$(τ). Therefore, (1) holds.

(2) Let τ ∈ M, ŗ ∈ $(\vartheta, \min\{2\vartheta - \eta, 1\}]$ and $x \in \langle\psi\rangle_{ŗ}^{(T,I,F)}$(τ). Then $x_{ŗ}\, q_{\vartheta}\, \alpha_{\psi(\tau)}$, that is, $\alpha_{\psi(\tau)}(x)$+ ŗ$> 2\vartheta$. Since ⟨ψ, M⟩ is a (T, I, F)-neutrosophic soft subset of ⟨ϕ, N⟩, we have $\max\{\alpha_{\psi(\tau)}(x), \eta\} \geq \min\{\alpha_{\psi(\tau)}(x), \vartheta\}$. Hence, by ŗ $>$ ϑ,

$$\max\{\alpha_{\psi(\tau)}(x) + ŗ, \eta + ŗ\} = \max\{\alpha_{\psi(\tau)}(x), \eta\} + ŗ$$

$$\geq \min\{\alpha_{\psi(\tau)}(x), \vartheta\} + ŗ$$

$$= \min\{\alpha_{\psi(\tau)}(x) + ŗ, \vartheta + ŗ\} > 2\xi.$$

From ŗ $\leq 2\vartheta - \eta$, that is, ŗ $+ \eta \leq 2\vartheta$, we have $\alpha_{\psi(\tau)}(x)$ + ŗ$> 2\vartheta$ and so $x \in \langle\phi\rangle_{ŗ}^{(T,I,F)}$(τ). Hence, (2) holds.

(3) Assume that τ ∈ M, ŗ ∈ $(\eta, \min\{2\vartheta - \eta, 1\}]$ and $x \in [\psi]_{ŗ}^{(T,I,F)}$(τ). Then $x_{ŗ} \in_{\eta} \vee q_{\vartheta}\, \alpha_{\psi(\tau)}$, that is, $\alpha_{\psi(\tau)}(x) \geq$ ŗ $> \eta$ or $\alpha_{\psi(\tau)}(x) > 2\vartheta -$ ŗ $\geq 2\vartheta - (2\vartheta - \eta) = \eta$. Because ⟨$\psi$, M⟩ is a (T, I, F)-neutrosophic soft subset of b ⟨ϕ, N⟩, there are $\max\{\alpha_{\psi(\tau)}(x), \eta\} \geq \min\{\alpha_{\psi(\tau)}(x), \vartheta\}$ and so $\alpha_{\psi(\tau)}(x) \geq \min\{\alpha_{\psi(\tau)}(x), \vartheta\}$ since $\eta < \min\{\alpha_{\psi(\tau)}(x), \vartheta\}$. We look about the following situations.

Case 1: ŗ ∈ $(\eta, \vartheta]$. Since ŗ ∈ $(\eta, \vartheta]$, we have $2\vartheta -$ ŗ $\geq \vartheta \geq$ ŗ. Then from $\alpha_{\psi(\tau)}(x) \geq$ ŗ or $\alpha_{\psi(\tau)}(x) > 2\vartheta -$ ŗ, we have $\alpha_{\psi(\tau)}(x) \geq \min\{\alpha_{\psi(\tau)}(x), \vartheta\} \geq$ ŗ. Hence $x_{ŗ} \in_{\eta} \alpha_{\psi(\tau)}$.

Case 2: ŗ ∈ $(\vartheta, 1]$. Because ŗ ∈ $(\vartheta, 1]$, we have $2\vartheta -$ ŗ $< \vartheta <$ ŗ. Then from $\alpha_{\psi(\tau)}(x) \geq$ ŗ or $\alpha_{\psi(\tau)}(x) > 2\vartheta -$ ŗ, there are $\alpha_{\psi(\tau)}(x) \geq \min\{\alpha_{\psi(\tau)}(x), \vartheta\} > 2\vartheta -$ ŗ. Hence $x_{ŗ} q_{\eta} \alpha_{\psi(\tau)}$.

Thus, in both cases, we have $x \in\phi_{\text{ɽ}}^{(T,I,F)}(\tau)$. Therefore, (3) holds.

(4) Let τ ∈ M, ɽ ∈ [1−ϑ, 1−η) and $x \in\hat{\psi}_{\text{ɽ}}^{(T,I,F)}(\tau)$. Then $\beta_{\psi(\tau)}(x) \leq$ ɽ. Since ⟨ψ, M⟩ is a (T, I, F)-neutrosophic soft subset of ⟨ϕ, N⟩, we have ɽ = max{ɽ, 1 − ϑ} ≥ max{$\beta_{\psi(\tau)}(x)$, 1−ϑ} ≥ min{$\beta_{\psi(\tau)}(x)$, 1 − η}, and so $\beta_{\psi(\tau)}(x)$ ≤ ɽ since ɽ< 1 − η , that is, $x \in\hat{\phi}_{\text{ɽ}}^{(T,I,F)}(\tau)$. Hence, (4) holds.

(5) Assume that τ ∈ M, ɽ ∈ [max {1 + η − 2ϑ, 0}, 1 − ϑ) and $x \in\langle\hat{\psi}\rangle_{\text{ɽ}}^{(T,I,F)}(\tau)$. Then $\beta_{\psi(\tau)}(x)$ + ɽ < 2(1 − ϑ), that is, $\beta_{\psi(\tau)}(x)$< 2(1 − ϑ) − ɽ ≤ 1 − η since 1 + η − 2ϑ ≤ ɽ. Because ⟨ψ, M⟩ is a (T, I, F)-neutrosophic soft subset of ⟨ϕ, N⟩, there are max{$\beta_{\psi(\tau)}(x)$, 1 − ϑ} ≥ min{$\beta_{\psi(\tau)}(x)$, 1 − η} and so max{$\beta_{\psi(\tau)}(x)$, 1 − ϑ} ≥ $\beta_{\psi(\tau)}(x)$ since 1 − η > max{$\beta_{\psi(\tau)}(x)$, 1 − ϑ}. Hence, by$x$ < 1 − ϑ,

$$2(1-\vartheta) > \max\{\beta_{\psi(\tau)}(x) + \text{ɽ}, 1-\vartheta+\text{ɽ}\} = \max\{\beta_{\psi(\tau)}(x), 1-\vartheta\} + \text{ɽ} \geq \beta_{\psi(\tau)}(x) + \text{ɽ}.$$

Hence $x \in\langle\hat{\phi}\rangle_{\text{ɽ}}^{(\eta,\vartheta)}(\tau)$. Therefore, (5) holds.

(6) Assume that τ ∈M, ɽ ∈ [max{1+η −2ϑ, 0}, 1− η) and $x \in[\widehat{\psi}]_{\text{ɽ}}^{(T,I,F)}(\tau)$. Then $\beta_{\psi(\tau)}(x)$ ≤ ɽ < 1 − η or $\beta_{\psi(\tau)}(x)$ < 2(1 − ϑ)− ɽ ≤ 1 − ϑ . because ⟨ψ, M⟩ is a (T, I, F)-neutrosophic soft subset of ⟨ϕ, N⟩, there are max{$\beta_{\psi(\tau)}(x)$, 1 − η} ≥ min{$\beta_{\psi(\tau)}(x)$, 1 − ϑ} and so max{$\beta_{\psi(\tau)}(x)$, 1 − ϑ} ≥ $\beta_{\psi(\tau)}(x)$ since 1 − η > max{$\beta_{\psi(\tau)}(x)$, 1 − ϑ}. We look about the following cases.

Case 1: ɽ ∈ [max {1 + η − 2ϑ, 0}, 1 − ϑ). Because ɽ ∈ [max {1 + η − 2ϑ, 0}, 1 − ϑ), there are 1 − ϑ < 2(1 − ϑ) − ɽ and ɽ< 2(1−ϑ)− ɽ. Now from $\beta_{\psi(\tau)}(x)$ ≤ ɽ or $\beta_{\psi(\tau)}(x)$< 2(1−ϑ)−ɽ, we have $\beta_{\psi(\tau)}(x)$ ≤ max{$\beta_{\psi(\tau)}(x)$, 1−ϑ} < 2(1−ϑ)− ɽ. Hence $x \in[\widehat{\phi}]_{r}^{(T,I,F)}(\tau)$.

Case 2: r ∈ [1−ϑ, 1−η). Because r ∈ [1−ϑ, 1−η), we have 2(1−ϑ) − ꞅ ≤ ꞅ. Then from $\beta_{\psi(\tau)}(x)$ ≤ ꞅ or $\beta_{\psi(\tau)}(x)$< 2(1−ϑ)− ꞅ, there are $\beta_{\psi(\tau)}(x)$ ≤ max{$\beta_{\psi(\tau)}(x)$, $1-\vartheta$} ≤ ꞅ. Hence $x \in \widehat{[\phi]}_{ꞅ}^{(T,I,F)}$(τ).

Thus, in both cases, $x \in \widehat{[\phi]}_{ꞅ}^{(\eta,\vartheta)}$(τ).Hence, (6) holds.

Now, for any ꞅ, s∈ [0, 1], ⟨ψ, M⟩∈𝔑 𝔍 (𝔄, E) and τ ∈ M, indicate

$$\psi_{(ꞅ,\hat{s})}^{(T\ I,F)}(\tau) = \psi_{ꞅ}^{(T,I,F)}(\tau) \cap \hat{\psi}_{ꞅ}^{(T,I\ \ F)}(\tau),$$

$$\psi_{(ꞅ,\widehat{(s)})}^{(T,I,F)}(\tau) = \psi_{ꞅ}^{(T,I,F)}(\tau) \cap \hat{\psi}_{ꞅ}^{(T,I,F)}(\tau),$$

$$\psi_{(ꞅ,\widehat{[s]})}^{(T,I\ F)}(\tau) = \psi_{ꞅ}^{(T,I,F)}(\tau) \cap \widehat{[\psi]}_{ꞅ}^{(T,I,F)}(\tau),$$

$$\psi_{(\langle ꞅ\rangle,\hat{s})}^{(T,I\ \ F)}(\tau) = \langle\psi\rangle_{ꞅ}^{(T,I\ \ F)}(\tau) \cap \hat{\psi}_{ꞅ}^{(T,I,F)}(\tau),$$

$$\psi_{(\langle ꞅ\rangle,\langle\hat{s}\rangle)}^{(T,I,F)}(\tau) = \langle\psi\rangle_{ꞅ}^{(T,I,F)}(\tau) \cap \widehat{\langle\psi\rangle}_{ꞅ}^{(T,I,F)}(\tau),$$

$$\psi_{(\langle ꞅ\rangle,\widehat{[s]})}^{(T,I,F)}(\tau) = \psi_{ꞅ}^{(T,I,F)}(\tau) \cap \widehat{[\psi]}_{ꞅ}^{(T,I,F)}(\tau),$$

$$\psi_{([ꞅ],\hat{s})}^{(T,I,F)}(\tau) = [\psi]_{ꞅ}^{(\eta,\vartheta)}(\tau) \cap \hat{\psi}_{ꞅ}^{(T,I,F)}(\tau),$$

$$\psi_{([ꞅ],\widehat{(s)})}^{(T,I,F)}(\tau) = [\psi]_{ꞅ}^{(T,I,F)}(\tau) \cap \widehat{\langle\psi\rangle}_{ꞅ}^{(T,I,F)}(\tau) \quad \text{And}$$

$$\psi_{([ꞅ],\widehat{[s]})}^{(T,I,F)}(\tau) = [\psi]_{ꞅ}^{(T,I,F)}(\tau) \cap \widehat{[\psi]}_{ꞅ}^{(T,I,F)}(\tau).$$

Then

$\left(\psi_{(ꞅ,\hat{s})}^{(T,I,F)}, M\right), \left(\psi_{(ꞅ,\widehat{(s)})}^{(T,I,F)}, M\right), \left(\psi_{(ꞅ,\widehat{[s]})}^{(T,I,F)}, M\right), \left(\psi_{(\langle ꞅ\rangle,\hat{s})}^{(T,I,F)}, M\right), \left(\psi_{(\langle ꞅ\rangle,\widehat{\langle s\rangle})}^{(T,I,F)}, M\right), \left(\psi_{(\langle ꞅ\rangle,\widehat{[s]})}^{(T,I,F)}, M\right), \left(\psi_{([ꞅ],\hat{s})}^{(T,I,F)}, M\right),$

$\left(\psi_{([ꞅ],\widehat{(s)})}^{(T,I\ \ F)}, M\right)$ *and* $\left(\psi_{([ꞅ],\widehat{[s]})}^{(T,I,F)}, M\right)$are soft sets over 𝔄.

The similarities between are described in the following theorem (T, I, F)- neutrosophic soft subsets and soft subsets.

Definition 2.29

Consider ⟨ψ, M⟩ and ⟨ϕ, N⟩ are two neutrosophic soft sets over $\mathfrak{A}$. It state that ⟨ψ, M⟩ $\triangleq_{(T,I,F)}$⟨ϕ, N⟩ if for every τ ∈M ∪N, τ ∈M ∩N suggests ψ(τ) $\asymp_{(T,I,F)}$ ϕ(τ), τ ∈ M − N suggests ψ(τ) $\asymp_{(T,I,F)}$ $1_{\mathfrak{A}}$ and τ ∈ N − M suggests ϕ(τ) $\asymp_{(T,I,F)}$ $1_{\mathfrak{A}}$.

Definition 2.30

Assume that ⟨ψ, M⟩ and ⟨ϕ, N⟩ are two neutrosophic soft sets over $\mathfrak{A}$. It state that ⟨ψ, M⟩$\triangleq_{(T,I,F)}$⟨ϕ, N⟩ if for every τ ∈ M ∪N, τ ∈M ∩N suggests ψ(τ) $\asymp_{(T,I,F)}$ ϕ(τ), τ ∈M − N suggests $\psi(\vartheta)$ $\asymp_{(T,I\,F)}$ $1^{\mathfrak{A}}$ and τ ∈N − M suggests ϕ(τ) $\asymp_{(T,I,F)}$ $1^{\mathfrak{A}}$.

Theorem 2.31

Let $\langle\psi, M\rangle$ and $\langle\phi, M\rangle$ be two neutrosophic soft sets over $\mathfrak{U}$. Then

(1) $\langle\psi, M\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle$ If and only if $\langle\psi, M\rangle\breve{\cup}\langle\phi, N\rangle \triangleq_{(T,I,F)} \langle\psi, M\rangle \Cap \langle\phi, N\rangle$

(2) $\langle\psi, M\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle$ If and only if $\langle\psi, M\rangle\breve{\cap}\langle\phi, N\rangle \triangleq_{(T,I,F)} \langle\psi, M\rangle \Cup \langle\phi, N\rangle$

(3) $\langle\psi, M\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle$ If and only if $\langle\psi, M\rangle\breve{\cup}\langle\phi, N\rangle \triangleq_{(T,I,F)} \langle\psi, M\rangle \Cap \langle\phi, N\rangle$

(4) $\langle\psi, M\rangle \triangleq_{(T,I\,F)} \langle\phi, N\rangle$ If and only if $\langle\psi, M\rangle\breve{\cap}\langle\phi, N\rangle \triangleq_{(T,I,F)} \langle\psi, M\rangle \Cup \langle\phi, N\rangle$

Theorem 2.32

Let $\langle\psi, M\rangle$ and $\langle\phi, N\rangle$ be two neutrosophic soft sets over $\mathfrak{U}$. $\langle\psi, M\rangle \triangleq_{(T,I\,F)} \langle\phi, N\rangle$or $\langle\psi, M\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle$, then

(1) $\langle\psi, M\rangle\breve{\cup}\langle\phi, N\rangle \asymp_{(T,I\;F)} \langle\psi, M\rangle\breve{\cap}\langle\phi, N\rangle$.

(2) $\langle\psi, M\rangle \Cup \langle\phi, N\rangle \asymp_{(T,I\;F)} \langle\psi, M\rangle \Cap \langle\phi, N\rangle$.

Theorem 2.33

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$, $\langle\omega, R\rangle$ and $\langle\varrho, S\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. If $\langle\psi, M\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle$and $\langle\omega, R\rangle \triangleq_{(T,I,F)} \langle\varrho, S\rangle$, then

(1) $\langle\psi, M\rangle \breve{\cup} \langle\omega, R\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle \breve{\cup} \langle\varrho, S\rangle$.

(2)$\langle\psi, M\rangle \Cap \langle\omega, R\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle \Cap \langle\varrho, S\rangle$.

Proof. Let $\langle\psi, M\rangle \breve{\cup} \langle\omega, R\rangle = \langle\sigma, M \cup R\rangle$ and $\langle\phi, N\rangle \Cap \langle\varrho, S\rangle = \langle\delta, N \cup S\rangle$. For any $\tau \in (M \cup R) \cap (N \cup S)$, then $\tau \in (M \cup R)$ and $\tau \in (N \cup S)$. Assume that $\tau \in M$ and $\tau \in S$. We consider the following cases.

Case 1: $\tau \in N$ and $\tau \in R$. Then $\tau \in M \cap N$ and $\tau \in R \cap S$. Hence $\psi(\tau) \asymp_{(T,I,F)} \phi(\tau)$ and $\omega(\tau) \asymp_{(T,I,F)} \varrho(\tau)$. If follows that $\sigma(\tau) = \psi(\tau) \cup \omega(\tau) \asymp_{(T,I,F)} \phi(\tau) \cup \varrho(\tau) = \delta(\tau)$.

Case 2: :$\tau \in N$ and $\tau \notin R$. Then $\tau \in M \cap N$ and $\tau \in S - R$.

Hence $\psi(\tau) \asymp_{(T,I,F)} \phi(\tau)$ and $\varrho(\tau) \asymp_{(T,I,F)} 1_{\mathfrak{U}}$. It follows that $\sigma(\tau) = \psi(\tau) \asymp_{(T,I,F)} \phi(\tau) \asymp_{(T,I,F)} \phi(\tau) \cup \varrho(\tau) = \delta(\tau)$.

Case 3: $\tau \notin N$and $\tau \in R$. Analogous to case 2, $\omega(\tau) \asymp_{(T,I,F)} \delta(\tau)$.

Case 4: $\tau \notin N$and $\tau \notin R$. Then $\tau \in M - N$and $\tau \in S - R$. Hence $\psi(\tau) \asymp_{(T,I\,F)} 1_{\mathfrak{U}}$ and $\varrho(\tau) \asymp_{(T,I,F)} 1_{\mathfrak{U}}$. Thus $\omega(\tau) = \psi(\tau) \asymp_{(T,I,F)} 1_{\mathfrak{U}} \asymp_{(T,I,F)} = \delta(\tau)$.

For any $\tau \in (M \cup R) - (N \cup S)$, we have $\tau \in (M \cup R)$, $\tau \notin N$ and $\tau \notin S$. We consider the following cases.

Case 1: $\tau \in M$and $\tau \in R$. Then $\tau \in M - N$and $\tau \in R - S$. Hence $\psi(\tau) \asymp_{(T,I,F)} 1_{\mathfrak{U}}$. Thus $\omega = \psi(\tau) \cup \omega(\tau) \asymp_{(T,I,F)} 1_{\mathfrak{U}} \cup 1_{\mathfrak{U}} \asymp_{(T,I,F)} 1_{\mathfrak{U}}$.

Case 2: $\tau \in M$ and $\tau \notin R$.Then $\tau \in M - N$. Hence $\psi(\tau) \asymp_{(T,I,F)} 1_{\mathfrak{U}}$ and so $\sigma(\tau) = \psi(\tau) \asymp_{(T,I,F)} 1_{\mathfrak{U}}$.

Case 3: $\tau \notin M$ and $\tau \in R$. Analogous to case 2, $\sigma(\tau) \asymp_{(T,I,F)} 1_{\mathfrak{U}}$.

Hence $\langle\psi, M\rangle \breve{\cup} \langle\omega, R\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle \breve{\cup} \langle\varrho, S\rangle$.

Theorem 2.34

$\triangleq_{(T,I,F)}$is a congruence relation with respect to $\breve{\cup}$ and $\Cap$ on $\mathfrak{N}\,\mathfrak{I}\,(\mathfrak{U})$.

Proof. It is straightforward by theorem 4.4

For any $\langle\psi, M\rangle \in \mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)$.

let$\langle\psi, M\rangle_{\doteq_{(T,I,F)}} = \{\langle\phi, N\rangle \in \mathfrak{N}\mathfrak{J}(\mathfrak{U}, E) | \langle\psi, M\rangle \doteq_{(T,I,F)} \langle\phi, N\rangle\}$ be the congruence class including $\langle\psi, M\rangle$ and

$\mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)| \doteq_{(T,I,F)} = \{\langle\psi, M\rangle_{\doteq_{(T,I,F)}} | \langle\psi, M\rangle \in \mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)\}$.

We define operations $\breve{\cup}_{\doteq_{(T,I,F)}}$ and $\Cap_{\doteq_{(T,I,F)}}$ on $\mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)| \doteq_{(T,I,F)}$ by

$\langle\psi, M\rangle_{\doteq_{(T,I,F)}} \breve{\cup}_{\doteq_{(T,I,F)}} \langle\phi, N\rangle_{\doteq_{(T,I,F)}} = (\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle)_{\doteq_{(T,I,F)}}$,

$\langle\psi, M\rangle_{\doteq_{(T,I,F)}} \Cap_{\doteq_{(T,I,F)}} \langle\phi, N\rangle_{\doteq_{(T,I,F)}} \neq (\langle\psi, M\rangle \Cap \langle\phi, N\rangle)_{\doteq_{(T,I,F)}}$.

Then $\breve{\cup}_{\doteq_{(T,I,F)}}, \Cap_{\doteq_{(T,I,F)}}$ are well defined.

$(\mathfrak{Y}\mathfrak{J}(\mathfrak{U}, E)/\doteq_{(T,I,F)}, \breve{\cup}_{\doteq_{(\eta,\vartheta)}}, \Cap_{\doteq_{(T,I,F)}})$

The neutrosophic soft quotient algebra (with respect to $\doteq_{(T,I,F)}$) over the universe $\mathfrak{U}$ and the parameter set E.

Theorem 2.35

The neutrosophic soft quotient algebra $(\mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)/\doteq_{(T,I,F)}, \breve{\cup}_{\doteq_{(T,I,F)}}, \Cap_{\doteq_{(T,I,F)}})$ is a distributive lattice.

Theorem 2.36

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle$, $\langle\omega, R\rangle$ and $\langle\varrho, S\rangle$ be neutrosophic soft sets over $\mathfrak{U}$. If $\langle\psi, M\rangle \doteq_{(T,I,F)} \langle\phi, N\rangle$and $\langle\omega, M\rangle \doteq_{(T,I,F)} \langle\varrho, N\rangle$, then

(1) $\langle\psi, M\rangle \breve{\cap} \langle\omega, R\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle \breve{\cap} \langle\varrho, S\rangle$.

(2)$\langle\psi, M\rangle \Cup \langle\omega, R\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle \Cup \langle\varrho, S\rangle$.

Theorem 2.37

$\triangleq_{(T,I,F)}$is a congruence relation with respect to $\breve{\cap}$ and $\Cup$ on $\mathfrak{N}\mathfrak{J}(\mathfrak{U})$.

Proof. It is straightforward by theorem 4.7

For any $\langle\psi, M\rangle \in \mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)$,

let $\langle\psi, M\rangle_{\triangleq(T,I,F)} = \{\langle\phi, N\rangle \in \mathfrak{N}\mathfrak{J}(\mathfrak{U}, E) | \langle\psi, M\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle\}$ be the congruence class including $\langle\psi, M\rangle$ and

$\mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)| \triangleq_{(T,I,F)} = \{\langle\psi, M\rangle_{\triangleq(T,I,F)} | \langle\psi, M\rangle \in \mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)\}$.

We define operations $\breve{\cap}_{\triangleq(T,I,F)}$ and $\Cup_{\triangleq(T,I,F)}$ on $\mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)| \triangleq_{(T,I,F)}$ by

$\langle\psi, M\rangle_{\triangleq(T,I,F)} \breve{\cap}_{\triangleq(T,I,F)} \langle\phi, N\rangle_{\triangleq(T,I,F)} = (\langle\psi, M\rangle \breve{\cap} \langle\phi, N\rangle)_{\triangleq(T,I,F)}$,

$\langle\psi, M\rangle_{\triangleq(T,I,F)} \Cup_{\triangleq(T,I,F)} \langle\phi, N\rangle_{\triangleq(T,I,F)} = (\langle\psi, M\rangle \Cup \langle\phi, N\rangle)_{\triangleq(T,I,F)}$.

Then $\breve{\cap}_{\triangleq(T,I\,F)}, \Cup_{\triangleq(T,I,F)}$ are well defined.

$(\mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)| \triangleq_{(T,I,F)}, \breve{\cap}_{\triangleq(T,I,F)}, \Cup_{\triangleq(T,I,F)})$

The neutrosophic soft quotient algebra (with respect to $\triangleq_{(T,I,F)}$) over the universe $\mathfrak{U}$ and the parameter set E.

Theorem 2.38

The neutrosophic soft quotient algebra $\mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)/\triangleq_{(T,I,F)}, \breve{\cap}_{\triangleq(T,I,F)}, \Cup_{\triangleq(T,I,F)})$ is a distributive lattice.

The image and the inverse image of neutrosophic soft sets

Definition 2.39

Consider $\mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)$ and $\mathfrak{N}\mathfrak{J}(\mathfrak{U}', E')$ be two neutrosophic soft classes, and let $: \mathfrak{U} \to \mathfrak{U}'$ and $\zeta : E \to E'$ be mappings.

Then a mapping $(\xi, \zeta) : \mathfrak{N}\mathfrak{J}(\mathfrak{U}, E) \to \mathfrak{N}\mathfrak{J}(\mathfrak{U}', E')$ is defined as, for $\langle\psi, M\rangle\, \mathfrak{N}\mathfrak{J}(\mathfrak{U}, E)$, the image of $\langle\psi, M\rangle$ under (ξ, ζ), denoted by $(\xi, \zeta)\langle\psi, M\rangle = \langle\xi(\psi), \zeta(M)\rangle$, is an neutrosophic soft set in $\mathfrak{N}\mathfrak{J}(\mathfrak{U}', E')$ given by

$$T_{\zeta(\psi)(\tau')}(x') = \begin{cases} \sup\limits_{\tau \in \xi^{-1}(\tau') \cap M, x \in \zeta^{-1}(\tau')} T_{\psi(\tau)}(x) & if\ \zeta^{-1}(x') \neq \phi, \\ 0 & otherwise, \end{cases}$$

$$I_{\zeta(\psi)(\tau')}(x') = \begin{cases} \sup_{\tau \in \xi^{-1}(\tau') \cap M, x \in \zeta^{-1}(\tau')} I_{\psi(\tau)}(x) & if\ \zeta^{-1}(x') \neq \phi, \\ 0 & otherwise, \end{cases}$$

$$F_{\zeta(\psi)(\tau')}(x') = \begin{cases} \inf_{\tau \in \xi^{-1}(\tau') \cap M, x \in \zeta^{-1}(\tau')} F_{\psi(\tau)}(x) & if\ \zeta^{-1}(x') \neq \phi, \\ 0 & otherwise, \end{cases}$$ and

for all $\tau' \in \xi$(M) and $x' \in \mathfrak{U}'$. For $\langle\psi'$, M$'\rangle \in \mathfrak{N}\,\mathfrak{J}\,(\mathfrak{U}'$, E$'$), the inverse image of $\langle\psi'$, M$'\rangle$ under (ξ, ς), denoted by $(\xi, \zeta)^{-1}\langle\psi'$, M$'\rangle$ = ($\xi^{-1}(\psi')$, ζ^{-1}(M$'$)) is an neutrosophic soft set in $\mathfrak{N}\,\mathfrak{J}\,(\mathfrak{U}$, E) given by

$$\alpha_{\zeta^{-1}(\psi')(\tau)}(x) = \alpha_{\psi'(\xi(\tau))}(\zeta(\tau)) \text{ and}$$

$$\beta_{\zeta^{-1}(\psi')(\tau)}(x) = \beta_{\psi'(\xi(\tau))}(\zeta(\tau)).$$

For every $\tau \in \xi^{-1}(N)$ and $x \in \mathfrak{U}$.

Theorem 2.40

Let $\langle\psi, M\rangle$, $\langle\phi, N\rangle \in \mathfrak{N}\,\mathfrak{J}\,(\mathfrak{U}$, E) and $\zeta : \mathfrak{U} \rightarrow \mathfrak{U}'$and $\xi : E \rightarrow E'$ be two mapping. Then

(1) $\langle\psi, M\rangle \Subset (\xi, \zeta)^{-1}((\xi, \zeta)\ \langle\psi, M\rangle)$.

(2) $((\xi, \zeta)\langle\psi, M\rangle)^r \Subset (\xi, \zeta)(\langle\psi, M\rangle^r)if\xi$ is surjective.

(3) $(\xi, \zeta)\big(\langle\psi, M\rangle \breve{\cup} \langle\phi, N\rangle\big) = (\xi, \zeta)\langle\psi, M\rangle \breve{\cup} (\xi, \zeta)\langle\phi, N\rangle$.

(4) $(\xi, \zeta)(\langle\psi, M\rangle \Cup \langle\phi, N\rangle) = (\xi, \zeta)\langle\psi, M\rangle \Cup (\xi, \zeta)\langle\phi, N\rangle$.

(5) $(\xi, \zeta)(\langle\psi, M\rangle \Cap \langle\phi, N\rangle) = (\xi, \zeta)\langle\psi, M\rangle \Cap (\xi, \zeta)\langle\phi, N\rangle$.

(6) if$\langle\psi, M\rangle \Subset_{(T,I,F)} \langle\phi, N\rangle$ then $(\xi, \zeta)\langle\psi, M\rangle \Subset_{(T,I,F)} (\xi, \zeta)\langle\phi, N\rangle$.

Theorem 2.41

Let $\langle\psi, M\rangle$, $\langle\phi, M\rangle \in \mathfrak{N}\,\mathfrak{J}\,(\mathfrak{U}, E)$ and ξ: $\mathfrak{U} \rightarrow \mathfrak{U}'$ and ζ: E $\rightarrow$ E$'$ be a mapping and an injective mapping, respectively.

(1) $\langle\psi, M\rangle = (\xi, \zeta)^{-1}((\xi, \zeta)\ \langle\psi, M\rangle)$ if ζ is also injective.

(2) (ξ, ζ)($\langle\psi$, M$\rangle$ $\breve{\sqcap}$ $\langle\phi$, N$\rangle$) =(ξ, ζ)$\langle\psi$, M$\rangle$ $\breve{\sqcap}$ (ξ, ζ)$\langle\phi$, N$\rangle$.

(3) (ξ, ζ)($\langle\psi$, M$\rangle$ $\Cup$ $\langle\phi$, N$\rangle$) =(ξ, ζ)$\langle\psi$, M$\rangle$ $\Cup$ (ξ, ζ)$\langle\phi$, N$\rangle$.

(4) If $\langle\psi, M\rangle \doteqdot_{(T,I,F)} \langle\phi, N\rangle$, then ($\xi$, ζ)$\langle\psi$, M$\rangle \doteqdot_{(T,I,F)}$ (ξ, ζ)$\langle\phi$, N$\rangle$.

(5) If $\langle\psi, M\rangle \triangleq_{(T,I,F)} \langle\phi, N\rangle$, then ($\xi$, ζ)$\langle\psi$, M$\rangle \triangleq_{(T,I,F)}$ (ξ, ζ)$\langle\phi$, N$\rangle$.

Theorem 2.42

Let $\langle\psi', M'\rangle$, $\langle\phi', M'\rangle \in \mathfrak{N}\,\mathfrak{J}\,(\mathfrak{U}', E')$ and ξ: $\mathfrak{U} \rightarrow \mathfrak{U}'$ and ζ: E → E′ be a mapping and an injective mapping, respectively.

(1) $(\xi,\zeta)\,((\xi,\zeta)^{-1}\langle\psi',\phi'\rangle) \Subset \langle\psi',\phi'\rangle$ and $(\xi,\zeta)\,((\xi,\zeta)^{-1}\langle\psi',\phi'\rangle) = \langle\psi',\phi'\rangle$ if both ξ and ζ are surjective.

(2) $((\xi,\zeta)^{-1}\langle\psi',M'\rangle)^r = (\xi,\zeta)^{-1}(\langle\psi',M'\rangle^r)$.

(3) $(\xi, \zeta)^{-1}(\langle\psi', M'\rangle \,\breve{\cup}\, \langle\phi', N'\rangle) = (\xi, \zeta)^{-1}\langle\psi', M'\rangle \,\breve{\cup}\, (\xi, \zeta)^{-1}\langle\phi', N'\rangle$.

(4) $(\xi, \zeta)^{-1}(\langle\psi', M'\rangle \,\breve{\cap}\, \langle\phi', N'\rangle) = (\xi, \zeta)^{-1}\langle\psi', M'\rangle \,\breve{\cap}\, (\xi, \zeta)^{-1}\langle\phi', N'\rangle$.

(5) $(\xi, \zeta)^{-1}(\langle\psi', M'\rangle \Cup \langle\phi', N'\rangle) = (\xi, \zeta)^{-1}\langle\psi', M'\rangle \Cup (\xi, \zeta)^{-1}\langle\phi', N'\rangle$.

(6) $(\xi, \zeta)^{-1}(\langle\psi', M'\rangle \Cap \langle\phi', N'\rangle) = (\xi, \zeta)^{-1}\langle\psi', M'\rangle \Cap (\xi, \zeta)^{-1}\langle\phi', N'\rangle$.

(7) If $\langle\psi', M'\rangle \circeq_{(T,I\,F)} \langle\phi', N'\rangle$, then $(\xi, \zeta)^{-1}\langle\psi', M'\rangle \circeq_{(T,I,F)} (\xi, \zeta)^{-1}\langle\phi', N'\rangle$.

(8) If $\langle\psi', M'\rangle \triangleq_{(T,I,F)} \langle\phi', N'\rangle$, then $(\xi, \zeta)^{-1}\langle\psi', N'\rangle \triangleq_{(T,I,F)} (\xi, \zeta)^{-1}\langle\phi', N'\rangle$.

CONCLUSION

Conclusion

In this study the Neutrosophic soft sets on algebraic structure using operation union, intersection, restricted union and restricted intersection is studied, and also studied some properties using the operations. Then the soft sets on the Neutrosophic soft sets are studied.

Ordering relation between the two Neutrosophic soft sets is also studied. In chapter 1 the definitions that are used throughout this paper is studied. In chapter 2, lattice structure on the Neutrosophic soft sets and the properties of the algebraic structure are studied. Then the Neutrosophic soft equalities is studied. Then the image and the inverse image of Neutrosophic soft set is also studied.

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