Roll. No:

Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore – 641 043
Bachelor's Degree Examination – November 2017

V Semester

Class : III UG **Time : 3 Hours**
Major : Mathematics **Max. Marks : 100**

15BMAC13 Abstract Algebra - I

Part – A **10 x 1 = 10**
Choose the Correct Answer

1. Let G be a group and $a \in G$. Then (a) = { a^i / i = 0, $\pm$ 1, $\pm$ 2, ... } is called the ------------ generated by 'a'.
   a. normal subgroup b. cyclic subgroup
   c. quotient group d. power group

2. If H is a subgroup of a group G and if $a \in G$, then the right coset Ha is defined as
   a. { ah / $h \in H$} b. { $h \in H$ / ha = a h}
   c. { h a / $h \in H$} d. { $h \in H$ / ha = e}

3. G is a finite abelian group and P | 0 (G). Then there is an element $a \neq e$ in G such that
   a. $a^P = e$ b. $a^P-1 = e$
   c. $a^P = a$ d. $a^P+1 = e$.

4. A subgroup N of a group G is a normal subgroup of G if for every $g \in G$ and $n \in N$
   a. $g\, n \in G$ b. $g\, n \in N$
   c. $gng^{-1} \in G$ d. $gng^{-1} \in N$

5. If G is a cyclic group of order 7, then the mapping : $\phi\; a^i \to a^{2i}$ is an automorphism of G of order ----------------------
   a. 1 b. 3
   c. 5 d. 7

6. An automorphism of a group G is ------------
   a. an isomorphism of G ontoG b. a homorphism of G onto G
   c. an isomorphism of G into G d. a homorphism of G into G

7. Which of the following is a finite field?
   a. The set of complex numbers C b. The set of rational numbers R
   c. The set of real quaternions d. Jp where p is a prime

8. A communicative ring without zero divisors is called -----------------
   a. a field b. an integral domain
   c. a division ring d. an ideal

9. A ring R' is called an over – ring or an extension of a ring R if R. can be --------- in R'.
   a. imbedded b. Shrinked
   c. reduced d. added

10. If U is an ideal of R and $1 \in U$, then
    a. U is a maximal ideal of R b. U = R
    c. U = (e) d. U is a multiplicative subgroup of R

Part – B   5 x 6 = 30

Answer the following

Answer should not exceed 400 words or two pages

11.a. If G is a finite group and a$\in$ G, then prove that 0 (a) / 0 (G).

(Or)

11.b. If G is a finite group whose order is a prime number p, then prove that G is a cyclic group.

12.a. Prove that N is a normal subgroup of G if an only gNg^{-1} = N for every g$\in$ G.

(Or)

12.b. If ϕ is a homomorphism of G into G, then prove that

i. ϕ (e) = e, the unit element of G

ii. $\phi(x^{-1}) = \phi(x)^{-1}$ for all x$\in$ G.

13.a. Set G be a group and ϕ an automoprhism of G. If a$\in$G is of order 0(a) > 0, then Prove that 0 (ϕ(a)) = 0 (a).

(Or)

13.b. If G is a group, prove that the set of automomorphism of G is also a group.

14.a. Define a ring homomorphism. If : ϕ R $\rightarrow$ R' is a ring homomorphism, prove that ϕ (0)=0and ϕ (-a) = - ϕ (a) for every a$\in$ R.

(Or)

14.b. If R is a ring, then for all a, b$\in$ R, show that

(i) a0 = 0 a = 0   (ii) a (-b) = (-a) b = - (ab).

15.a. Define (i) an ideal and (ii) a maximal ideal.

(Or)

15.b. Let R be the ring of all the real – valued, continuous function on the interval [0,1]. Prove that M = { f(x) $\in$ R / f $\left(\frac{1}{2}\right)$ = 0} is a maximal ideal of R.

Part – C   5 x 12 = 60

Answer the following

Answer should not exceed 800 words or four pages

16.a. Let H, K be subgroup of a group G. Define HK and prove that HK is a subgroup of G if and only if HK = KH.

(Or)

16.b. If G is a finite group and H is a subgroup of G, then prove that 0(H) / 0(G).

17.a.i. Prove that the subgroup N of G is a normal subgroup of G if and only if every left co set of N is G is right coset of N in G.

ii. If N is a normal subgroup of G, prove that the product of two right cosets of N in G is again a right coset of N in G. r/.

(Or)

17.b. Define group homomorphism and the Kernel of a group homomorphism. If ϕ is a Homomorphism of G into G with kernel K, then prove that K is a normal subgroup of G.

18.a. State and prove Cayley's theorem.

(Or)

18.b. Prove that I (G) $\approx$ $G/_{Z}$ where I (G) is the group of inner automorphisms of G and Z in the centre of G.

19.a. If U is an ideal of the ring R, prove that R / U is a ring and is a homomorphic image of R.

(Or)

19.b. Prove that a finite integral domain is a field.

20.a. Let R be a commutative ring with unite element whose only ideals are (0) and R itself. Prove that R is a field.

(Or)

20.b. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if $R/_{M}$ is a field.

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