



INTRODUCTION

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*“Mathematics is the only science where one never knows
what one is talking about nor whether what is said is true”
Bertrand Russell*

The importance of general topological spaces rapidly increases in many fields of applications such as data mining. Information systems are basic tools for producing knowledge from data in any real-life field. Topological structures on the collection of data are suitable mathematical models for mathematizing not only quantitative data but also qualitative ones.

Closedness are basic concept for the study and investigation in general topological spaces. This concept has been generalized and studied by many authors. Levine [30] initiated the study of so-called generalized closed sets in a topological space in 1970 in order to extend many of the important properties of closed sets to a larger family. In 1987, Bhattacharyya and Lahiri [7] used semi open sets to define and investigate the notion of semi generalized closed sets in a topological space. In 1990 Arya. S.P and Nour. T [3] introduced the concepts of generalized semi closed sets using the semi closure of a subset with its open supersets to characterize the s-normality axiom. Chandrasekhara Rao and Joseph [11] introduced the concepts of semi star generalized open sets and semi star generalized closed sets in a topological space.

In 1963, the concept of bitopological space was introduced by Kelly [27]. Considerable effort had been made in obtaining appropriate generalizations of standard topological properties to bitopological category. Kelly initiated the study of separation axioms for bitopological spaces.

Maheshwari and Prasad [33] introduced semi open sets in bitopological spaces in 1977 and further properties of this notion were studied by Bose [8] in 1981. Fukutake [24] defined one kind of semi open sets in bitopological spaces and studied their properties in 1989.

Fukutake [23] also introduced generalized closed sets and pairwise generalized closure operator in bitopological spaces in 1986. Semi generalized closed sets and

generalized semi closed sets are extended to bitopological space by Khedr and Al-saadi [28]. In 2002, Fukutake, Sundaram and Sheik John [25] introduced the concept of w -closed sets, w -open sets and w -continuous maps in bitopological spaces. Maki et al [35] introduced generalized homeomorphism and gc -homeomorphism which are generalizations of homeomorphism in topological properties. Devi et al [21] defined and studied generalized semi- homeomorphism and gsc - homeomorphism in topological properties.

The g^* -closed sets in bitopological spaces were introduced by Sheik and Sundaram [44] in 2004. Lellis Thivagar and O. Ravi [29] introduced the concept of $(1, 2)^*$ -semi generalized closed sets and a new class of generalized functions called $(1, 2)^*$ -semi-generalized continuous maps in 2006. On the other hand Chandrasekhara Rao and Kannan [12] introduced the concepts of semi star generalized open sets and semi star generalized closed sets in a bitopological space.

The aim of this dissertation is to study some of the generalizations of closed sets in bitopological spaces. The following generalizations are analyzed,

1. Semi star generalized closed sets in bitopological spaces.
2. $(1,2)^*$ -semi generalized separation axioms bitopological spaces.
3. $(1,2)^*$ - generalized homeomorphisms in bitopological spaces.

To study these generalizations the following articles are chosen for our study

1. “Semi star generalized closed sets in bitopological spaces” due to Chandrasekhara Rao ,Kannan, and Narasimhan [17] and Kannan, Narasimhan ,Chandrasekhara Rao and M.Sundararaman[18]
2. “ $(1,2)^*$ -semi generalized separation axioms bitopological spaces” due to Lellis Thivagar,Nirmala Mariappan [32] and Nirmala Mariappan, Hatir ,Lellis Thivagar [39].
3. “ $(1,2)^*$ - generalized homeomorphisms in bitopological spaces” due to Ravi, Pious Missier and Salai Parkunan[41].

Chapter I is devoted to study of semi star generalized closed sets in bitopological spaces .

In the first section , preliminary results are discussed.

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In the second section, $\tau_1\tau_2$ - s^*g closed sets, Pairwise s^*g continuous functions and its properties are discussed.

In the third section, Pairwise S^*GO -connected spaces, S^*GO -compact space and their properties and characterizations are analyzed.

In the fourth section, $(\tau_1, \tau_2)^*$ -semi star generalized closed(open) sets and its properties are discussed.

In the fifth section. $(1,2)^*$ -semi star generalized continuous functions, pairwise semi star generalized T_S -spaces and its properties are analyzed.

Some of the interesting results discussed are as follows:

1. every pairwise s^*g continuous function is pairwise g -continuous.
2. every $\tau_1\tau_2$ -closed set is $(\tau_1, \tau_2)^*$ - s^*g closed.
3. every $(1,2)^*$ -continuous function is $(1,2)^*$ - s^*g continuous.

Chapter II is devoted to study of $(1,2)^*$ -semi generalized separation axioms in bitopological spaces .

In the first section, preliminary results are discussed.

In the second section, we study the concepts of $(1,2)^*$ -semi-generalized separation axioms and its properties are analyzed.

In the third section, $(1,2)^*$ - ψ separation axioms namely $(1,2)^*$ - $sg-T_i$ and $(1,2)^*$ - $\psi-T_i$ and its properties are discussed.

In the fourth and fifth section, $(1,2)^*$ -semi- g -regular spaces and $(1,2)^*$ -semi- g -normal spaces

It is interesting to note that

1. every singleton set $\{x\}$ of a bitopological space (X, τ_1, τ_2) is either $(1,2)^*$ nowhere dense or $(1,2)^*$ -preopen.
2. every $(1,2)^*$ -semi-normal, $(1,2)^*$ -semi-symmetric bitopological space (X, τ_1, τ_2) is $(1,2)^*$ -semi-regular.
3. every $(1,2)^*$ -semi- g -normal, $(1,2)^*$ -semi-symmetric bitopological space (X, τ_1, τ_2) is $(1,2)^*$ -semi- g -regular.

Chapter III is devoted to study of $(1,2)^*$ -generalized homeomorphisms in bitopological spaces .

The first section deals with preliminary results.

In the second section, characterizations and properties are analyzed.

The third and fourth section study the concepts of comparisons, $(1,2)^*$ -SGO compact spaces, $(1,2)^*$ -GSO-compact spaces and its properties.

Some of the interesting results analysed are as follows:

1. Every $(1,2)^*$ -homeomorphism is $(1,2)^*$ -gc-homeomorphism.
2. Every $(1,2)^*$ -g-open is $(1,2)^*$ -gs-open.
3. If f is $(1,2)^*$ -open and $(1,2)^*$ -gc irresolute, then f is $(1,2)^*$ -gs-irresolute.