Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore -641043

Master's Degree Examination – December 2017
Semester III

Class: II P.G **Max. Marks: 60**
Major: Mathematics **Time : 3 Hours**

12MMAC13 Topology - I

Part - A **10 x ½ = 5**

Choose the correct Answer:

1. If X is a non empty set, then the set of all subsets of X will form a_____ topology on X.

(a) indiscrete (b) discrete (c) uniform (d) standard

2. The closure of the set $A=(0,1/2)$ in the subspace $Y=(0,1]$ is ____

(a) $[0,1]$ (b) $[0,1/2]$ (c) $(0,1)$ (d) $(0,1/2]$

3. Limit points of the set of positive integers is _____

(a) not a real number (b)R (c) Q (d)Z

4. If $C=\{0\}\cup(1,2)$ then $\overline{C}$ =

(a) $\{0\}\cap(1,2)$ (b) $(1,2)$ (c) $\{0\}\cup[1,2]$ (d) $\{0\}\cup(1,2]$

5.If the open ball $B(X,\varepsilon)=(a,b)$ *then* ε =then ϵ =_____

(a) $(a-b)/2$ (b) $(a+b)/2$ (c) $(b-a)/2$ (d) $(ab)/2$

6. The metric $d(x,y)=1$ *if* $x\neq y$ *and* $d(x,y)=0$ *if* $x=y$ induces the _____ topology

(a)Discrete (b)indiscrete (c)order (d)finite complement

7. Which of the following identity function is not continuous?

(a) $f:R_l\to R$ (b) $f:R\to R_l$ (c) $f:R_l\to R_l$ (d) $f:R\to R$

8. If X *and* Y are topological spaces and the function $f:X\to Y$ defined as $f(x)=2x$ is

(a)identity (b)constant (c)continuous (d)not continuous.

9. A two point space X is connected in the _____ topology

(a) indiscrete (b) discrete (c) order (d) standard

10. The space R^{ω} is not connected in the _____ topology

(a)compact open (b)product (c)box (d)metric

Part – B

Answer all questions **5x4=20**

11.a. Define basis of a topology and prove that the collection B of all circular regions in the Plane generates the same topology as the collection B' of all rectangular regions.

(Or)

11.b. Prove that the topologies R_l and R_k are strictly finer than the standard topology on R.

12.a. Define the subspace topology and prove that if A is a subspace of X and B is a subspace of Y, then the product topology on $A \times B$ is the same as the topology $A \times B$ inherits as a subspace of $X \times Y$. **(Or)**

12.b. Let Y be a subspace of X then prove that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

13.a. Let $\{X_\alpha\}$ be a indexed family of spaces. Let $A_\alpha \subseteq X_\alpha$ for each α then prove that $\prod \overline{A_\alpha} = \overline{\prod A_\alpha}$ **(Or)**

13.b. Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))\alpha \in J$, where $f_\alpha : A \to X_\alpha$ for eachα. Let $\prod_{\alpha \in J} X_\alpha$ have the product topology. Then prove that the function f is continuous if and only if each function f_α is continuous.

14.a. Define the box and product topologies on $X \times Y$ and compare them.

(Or)

14.b. Prove that the uniform topology on R^J is finer than the product topology and coarser than the box topology.

15.a. Prove that a path connected space is connected.

(Or)

15.b. State and prove intermediate value theorem.

Part-C

Answer all questions **5 x 7=35**

16.a. If X is any set, prove that the collection of all one-point subsets of X is a basis for the discrete topology on X **(Or)**

16.b.Prove that the collection of all circular regions in the plane form a basis for a topology and prove that the collection of all rectangular regions and circular regions generate the same topology.

17.a. Define order topology and subspace topology. If Y be a subset of X which is an ordered set in the order topology, prove that the order topology on Y is the same as the topology in Y inherits as a subspace of X.

(Or)

17. b. Prove the following:

i. The subspace of a Hausdorff space is a Hausdorff space.

ii. The product of Hausdorff spaces is a Hausdorff space.

18.a. Let X and Y be topological spaces and f is a mapping from X into Y then prove that the following are equivalent:

a. f is continuous

b. For every subset A of X f(closure(A) is contained in the closure of $f(A)$

c. For every subset B of Y the set $f^{-1}(B)$ is closed in X

d. For each point x in X and each neighbourhood U of $f(x)$, there is a neighborhood U of x such that $f(U)$ is contained in U

(Or)

18.b. Let X and Y be topological spaces and f is a continuous function from X into Y then prove that for every convergent sequence x_n that converges to x in X the sequence $f(x_n)$ converges to $f(x)$ and also prove the converse is also true when X is metrizable.

19.a. State and prove Uniform limit theorem.

(Or)

19.b. State and prove any four rules for constructing continuous

20.a. Prove that a finite Cartesian product of connected spaces is connected.

(Or)

20.b.i .Prove that a space X is locally connected if and only if for every open set U of X each component of U is open in X.

ii. Define a path in a topological space X and prove that the components of X form equivalence classes

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