CHAPTER 2

CHAPTER 2

STRONGLY AND WEAKLY GENERALIZED CLOSED SETS AND CONTINUOUS MAPS IN MINIMAL STRUCTURES

In this chapter smg-closed sets due to Pushpalatha and Subha [60] and mg-continuous maps in minimal structure due to Parimelazhagan, Nagaveni and Sai Sundara Krishnan [54] are studied. mwg-closed sets and continuous maps due to Parimelazhagan, Balachandaran, Nagaveni [55] in minimal structures are studied. Properties and characterization of these concepts are analyzed.

SECTION: 2.1
PRELIMINARES

Definition: 2.1.1

Let X be a non empty set and P(X) the power set of X. A subfamily m_X of P(X) is called a **minimal structure** (m-structure) on X if $\phi \in m_X$ and $X \in m_X$. The pair (X, m_X) is called a **minimal space** (or m-space).

Definition: 2.1.2

Let X be a non-empty set and m_X is an m-structure on X. For a subset A of X, the **m_X-closure** of A and the **m_X-interior** of A are defined as follows:

(i) m_X-cl (A) = $\cap\{F : A \subseteq F, X - F \in m_X\}$

(ii) m_X-int (A) = $\cup\{U : U \subseteq A, U \in m_X\}$

Lemma: 2.1.3

Let X be a non-empty set and m_X is a minimal structure on X.

For subsets A and B of X the following hold.

(i) m_X-cl(X − A)=X − (m_X-int (A)) and m_X-int(X − A)= X − (m_X–cl (A)).

(ii)If (X − A) $\in$ m_X then m_X-cl (A) = A and if A$\in$ m_X then m_X-int (A) = A.

(iii) m_X-cl (ϕ) = ϕ, m_X-cl(X) = X, m_X-int (ϕ) = ϕ and m_X-int(X) = X.

(iv) If A $\subseteq$ B then m_X- cl (A)$\subseteq$ m_X-cl(B) and m_X-int(A) $\subseteq$ m_X-int(B).

(v) A$\subseteq$$m_X$-cl(A) and m_X-int(A)$\subseteq$ A.

(vi) m_X-cl(m_X-cl (A)) = m_X-cl(A) and m_X-int((m_X-int(A)) = m_X-int(A).

Lemma: 2.1.4

Let X be a non-empty set with minimal structure m_X and A be a subset of X. Then x$\in$$m_X$-cl (A) if and only if U$\cap$A $\neq \phi$ for every U$\in$$m_X$ containing x.

Proof:

Necessary:

Let x $\in$ m_X-cl (A). Assume there exist a U$\in$$m_X$ containing x such that U$\cap$A = ϕ. Then A $\subseteq$ X − U and m_X-cl(A) $\subseteq$ X − U. Since x$\in$U we have x$\notin$$m_X$-cl(A).

Sufficiency:

Let U $\cap$A $\neq \phi$ for every U$\in$$m_X$ containing x. Let x$\notin$$m_X$-cl (A).

Then there exist a subset F of X such that $X - F \in m_X$, $A \subseteq F$ and $x \notin F$ Then there exist $X - F \in m_X$ containing x such that $(X - F) \cap A = \phi$.

Definition: 2.1.5

Let (X,m_X) be an m-space, A subset A of X is said to be **mg-closed** if m_X-cl(A)$\subseteq$G whenever A$\subseteq$G and G is m_X-open.

The complement of an mg-closed set is said to be **mg-open.**

Definition: 2.1.6

A minimal structure m_X on a nonempty set X is said to have **property B** if the union of any family of subsets belong to m_X.

Lemma: 2.1.7

Let X be a nonempty set and m_X a minimal structure on X satisfying property B. For a subset A of X, the following properties hold:

(i) $A \in m_X$ if and only if m_X-int(A) = A.

(ii) A is m_X-closed if and only if m_X-cl (A) = A.

(iii)m_X-int (A)$\in m_X$ and m_X-cl(A) is m_X-closed.

Theorem: 2.1.8

The union of two mg-closed sets is mg-closed.

Proof:

Let A and B be any two mg-closed sets in X. Let G be an m_X-open set in X, such that $A \cup B \subseteq G$, then $A \subseteq G$ and $B \subseteq G$. Since A and B are mg-closed, m_X-cl(A)$\subseteq$G and m_X-cl(B)$\subseteq$G. Hence m_X-cl($A \cup B$)

$\subseteq m_X$-cl(A) $\cup$ m_X-cl(B)$\subseteq$G.Hence m_X-cl(A$\cup$B) $\subseteq$G. Therefore A$\cup$B is mg-closed.

Remark: 2.1.9

The intersection of two mg-closed sets need not mg-closed set.

Example: 2.1.10

Consider the m-space X = {a, b, c}, m_X = {ϕ, X, {a}}. Then the setA = {a, b} and B = {a, c} are mg-closed sets. But the intersection {a} is not mg-closed.

SECTION: 2.2

smg-CLOSED SETS

Definition: 2.2.1

Let (X,m_X) be an m-space. A subset A of X is said to be **strongly minimal generalized closed (smg-closed)** if m_X-cl(A)$\subseteq$G whenever A$\subseteq$G and G is mg-open.

The complement of smg-closed set is called a **smg-open** set in (X,m_X).

Remark: 2.2.2

Let (X,τ) be a topological space and m_X be minimal structure on X. If m_X = τ, then a smg-closed set is a strongly g-closed set in X.

Theorem: 2.2.3

If A is m_X-closed, then A is smg-closed.

Proof:

Let A be an m_X-closed set in (X, m_X). Let $A \subseteq G$, where G is mg-openin (X,m_X). Since A is m_X-closed, m_X-cl(A) = A and so m_X-cl(A)$\subseteq$G.Therefore A is smg-closed.

Remark: 2.2.4

The converse of the above theorem [2.2.3] need not be true.

Example: 2.2.5

Consider the m-space X = {a, b, c} with minimal structure m_X = {ϕ, X, {a}, {a, b}}. The set {a, c} is smg-closed but not m_X-closed set.

Theorem: 2.2.6

Union of two smg-closed sets is smg-closed.

Proof:

Let A and B be any two smg-closed sets in X. Let G be an mg-open set in X such that $A \cup B \subseteq G$, then $A \subseteq G$ and $B \subseteq G$. Since A and B are smg-closed, m_X-cl(A)$\subseteq$G and m_X-cl(B)$\subseteq$G. Hence m_X-cl($A \cup B$)$\subseteq m_X$-cl(A)$\cup m_X$-cl(B)$\subseteq$G and hence m_X-cl($A \cup B$)$\subseteq$G. Therefore $A \cup B$ is smg-closed.

Theorem: 2.2.7

A subset A of X is smg-closed in (X,m_X) if and only if m_X-cl(A) − A contains no non-empty mg-closed set in X.

Proof:

Let A be a smg-closed in (X,m_X) and let F be a non-empty mg-closed subset of m_X-cl(A) − A. Then $F \subseteq m_X$-cl(A) − A and $F \subseteq m_X$-cl(A)$\cap A^C$. Therefore $F \subseteq m_X$-cl(A) and $F \subseteq A^C$.Since F^C is mg-open and A is smg-closed, m_X-cl(A)$\subseteq F^C$.That is $F \subseteq m_X$-cl(A)^C. Hence $F \subseteq$ m_X-cl(A) $\cap$ m_X-cl(A)$^C = \phi$. That is $F = \phi$. Thus m_X-cl(A) − A contains no non-empty mg-closed set.

Conversely assume that m_X-cl(A) − A contains no non-empty mg-closed set. Let $A \subseteq G$, G is mg-open. Suppose that m_X-cl(A) is not contained in G. Then m_X-cl(A)$\cap G^C$ is a non-empty mg-closed set of m_X-cl(A) − A. This is a contradiction. Therefore m_X-cl (A) $\subseteq$G and hence A is smg-closed.

Theorem: 2.2.8

If A is smg-closed and mg-open then A is m_X-closed.

Proof:

Let A be smg-closed and mg-open. Since A is mg-open and smg-closed, we have m_X-cl(A)$\subseteq$A and hence m_X-cl(A) = A. Therefore A is m_X-closed.

Theorem: 2.2.9

If A is smg-closed and $A \subseteq B \subseteq m_X$-cl(A), then B is smg-closed.

Proof:

Let A be smg-closed and $A \subseteq B \subseteq m_X$-cl(A). Let $B \subseteq G$, where G is m_X-g-open. Since A is smg-closed and $A \subseteq B \subseteq G$, m_X-cl(A)$\subseteq$G.

But $B \subseteq m_X\text{-cl}(A) \Rightarrow m_X\text{-cl}(B) \subseteq m_X\text{-cl}(A) \subseteq G$. Therefore B is smg-closed.

Theorem: 2.2.10

Every smg-closed set in X is a mg-closed set in X.

Proof:

Let A be smg-closed set. Let $A \subseteq G$ where G is m_X-open. Since every m_X-open set is mg-open and A is smg-closed, $m_X\text{-cl}(A) \subseteq G$. Therefore A is mg-closed.

Remark: 2.2.11

The converse of the above theorem [2.2.10] need not be true.

Example: 2.2.12

Consider the m-space X = {a, b, c} with m-structure $m_X = \{\phi, X, \{a\}\}$. The set {b} is mg-closed but not smg-closed.

Theorem: 2.2.13

For each $x \in X$, {x} is mg-closed in X or $\{x\}^C$ is smg-closed set in X.

Proof:

Let {x} be not mg-closed. Then the only mg-open set containing $\{x\}^C$ is X and the $m_X\text{-cl}(\{x\})^C$ is contained in X. Hence $\{x\}^C$ is smg-closed in X.

Theorem: 2.2.14

A subset A of X is smg-open in X if and only if $F \subseteq m_X\text{-int}(A)$ whenever $F \subseteq A$ and F is mg-closed in X.

Proof:

Let A be smg-open set in X. Then A^C is smg-closed set in X. Let F be a mg-closed set such that $F \subseteq A$. Then F^C is mg-open and $A^C \subseteq F^C$. So $[m_X\text{-cl}(A^C)] \subseteq F^C$, $[m_X\text{-cl}(A)]^C \subseteq F^C$. Thus $F \subseteq m_X\text{-int}(A)$.

Conversely assume that $F \subseteq m_X\text{-int}$ (A) whenever F is mg-closed and $F \subseteq A$. Let G be an mg-open set such that $A^C \subseteq G$. Than G^C is mg-closed, $G^C \subseteq (A^C)^C = A$. By hypothesis: $G^C \subseteq [m_X\text{-int}(A)]$, $G \supseteq [m_X\text{-int}(A)]^C$. That is $G \supseteq [m_X\text{-cl}(A^C)]$. Therefore A^C is smg-closed. Hence A is smg-open.

SECTION: 2.3
mg-CONTINUOUS MAPS

Definition: 2.3.1

A map $f : (X,m_X) \rightarrow (Y,m_Y)$ is said to be **g-continuous** if $f^{-1}(O)$ is m_X-g-closed in (X,m_X) for every m_Y-closed set O of (Y,m_Y).

Definition: 2.3.2

A function f: $(X,m_X) \rightarrow (Y,m_Y)$is said to be **mg-continuous** if $f^{-1}(V)$ is mg-closed in(X,m_X) for every m-closed V in (Y,m_Y).

Theorem: 2.3.3

If a map f: $(X,m_X) \rightarrow (Y,m_Y)$ from a minimal space (X,m_X) into a minimal space (Y,m_Y) is m-continuous then it is mg-continuous.

Proof:

Let f: $(X,m_X) \rightarrow (Y,m_Y)$ be m-continuous. Let F be any closed set in (Y,m_Y). Then the inverse image $f^{-1}(F)$ is closed in (Y,m_Y). Since every closed set is mg-closed $f^{-1}(F)$ is mg-closed in (X,m_X). Therefore f is mg-continuous.

Remark: 2.3.4

The converse of the above theorem [2.3.3] need not be true.

Example: 2.3.5

Let X = {a, b, c}, τ = {ϕ, {a}, X}, Y = {p, q} and σ = {ϕ, {p}, Y}. Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be defined by f (a) = f(c) = q, f (b) = p. Then f is m-continuous. But f is not mg-continuous. Since for the open set G = {p} in (Y,m_Y). $f^{-1}(a)$ = {b} is not open in (X,m_X). Therefore f is mg-continuous.

Theorem: 2.3.6

Let f: $(X,m_X) \rightarrow (Y,m_Y)$ be a mapping from a minimal space (X,m_X) into a minimal space(Y,m_Y) The following statements are equivalent:

(i) f is mg-continuous.

(ii) The inverse image of each open set in(Y,m_Y) is g-open in (X,m_X).

Proof:

(i) $\Rightarrow$ (ii)

Let f: $(X,m_X) \rightarrow (Y,m_Y)$is mg-continuous and let G be open in (Y,m_Y). Then G^C is closed in (Y,m_Y). Since f is mg-continuous $f^{-1}(G^C)$ is mg-closed in (X,m_X). But $f^{-1}(G^C) = X - f^{-1}(G)$. Thus $X - f^{-1}(G)$ is mg-closed in X and so $f^{-1}(G)$ is mg-open in (X,m_X).

(ii) $\Rightarrow$ (i)

Let the inverse image of each open set in (Y,m_Y) is mg-open in (X,m_X). Let F be any closed set in (Y,m_Y). Then F^C is open in (Y,m_Y). By assumption, $f^{-1}(F^C)$ is mg-open in (X,m_X). But $f^{-1}(F^C) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is mg-open in (X,m_X).Therefore f is mg-continuous.

Definition: 2.3.7

A minimal space (X,m_X) is said to be **mg-T_2** if for each pair of distinct points x, $y \in X$ there exist $U,V \in m_X$ containing x and y respectively such that $U \cap V = \phi$.

Theorem: 2.3.8

If f: $(X,m_X) \rightarrow (Y,m_Y)$ is a m-continuous injection and (Y,m_Y) is mg-T_2 then (X,m_X) is mg-T_2.

Proof:

Let x, y be any distinct points of X. Then $f(x) \neq f(y)$. Since (Y,m_Y) is mg-T_2, there exist a disjoint sets U,V in m_Y containing f(x) and f(y) respectively. Since f is m-continuous there exist G, $H \in m_X$ containing x, y respectively such that f $(G) \subseteq U$ and f $(H) \subseteq U$. This implies that $G \cap H = \phi$. Hence X is mg-T_2.

Theorem: 2.3.9

Let X and Z be any minimal spaces and Y be $T_{1/2}$ space. Then the composition $g \circ f : (X,m_X) \rightarrow (Z,m_Z)$ of the mg-continuous maps $f : (X,m_X) \rightarrow (Y,m_Y)$ and $g : (Y,m_Y) \rightarrow (Z,m_Z)$ is also mg-continuous.

Proof:

Let F be closed in (Z,m_Z). Since g is mg-continuous, $g^{-1}(F)$ is mg-closed in (Y,m_Y). But (Y,m_Y) is $T_{1/2}$ and so $g^{-1}(F)$ is closed. Since f is mg-continuous, $f^{-1}(g^{-1}(F))$ is mg-closed in (X,m_X).But $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$. Therefore $g \circ f$ is mg-continuous.

Remark: 2.3.10

The converse of the above theorem [2.3.9] need not be true.

Example: 2.3.11

Let X = Y = Z = {a, b, c}. Let $\tau = \{\phi, a, \{a, b\}, X\}$, $\sigma = \{\phi, \{a\}, \{b, c\}, X\}$, $\eta = \{\phi, \{a, c\}, Z\}$. Let f: $(X,m_X) \rightarrow (Y,m_Y)$ be defined by f(a) = c, f(b) = b and f(c) = c. Let g: $(Y,m_Y) \rightarrow (Z,m_Z)$ be the identity map. Then f and g are mg-continuous. But $g \circ f$ is not mg-continuous. Since F = {b} is closed in (Z,m_Z). $g^{-1}(F) = F$ and $f^{-1}(F) = F$ is not mg-closed in (X,m_X). Therefore $g \circ f$ is not mg-continuous.

Definition: 2.3.12

Let (X,m_X) be a minimal space. Let A be a subset of X. Consider the minimal structure μ_A of A.Then the space (A, μ_A) is called the **subminimal space** of (X,m_X).

Theorem: 2.3.13

Let f: $(X,m_X) \rightarrow (Y,m_Y)$ be a mg-continuous map from minimal space (X,m_X) into a minimal space (Y,m_Y) and H be a closed subset of X. Then the restriction *f/H*: H $\rightarrow (Y,m_Y)$ is mg-continuous. Where H is endowed with sub minimal structure.

Proof:

Let H be a closed subset of (X,m_X) and let F be any closed subset in (Y,m_Y). Since f is mg-continuous, $f^{-1}(F)$ is mg-closed in (X,m_X), since the intersection of a closed set and a mg-closed is mg-closed set, $f^{-1}(F)\cap H = H_1$ is a mg-closed set in (X,m_X). It is sufficient to show that $H_1$is mg-closed in H. Let G_1 be any open set of H such that $G_1 \supseteq H_1$. Let $G_1 = G\cap H$, where G is open in (X,m_X). Since H_1 is mg-closed in (X, m_X). $H_1 \subseteq G$, Now $cl_H(H_1) = H_1 \cap H \subseteq G\cap H=G_1$. Therefore f/H is mg-continuous.

Remark: 2.3.14

In the above theorem [2.3.13], the assumption of the closed ness of H cannot be removed as seen from the following example.

Example: 2.3.15

Let (X,m_X) and (Y,m_Y) and f be as in Y = {p, q}, X = {a, b, c}, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{p\}, Y\}$. Let f : $(X,m_X) \rightarrow (Y,m_Y)$ be defined by f(a) = f(c) = {q},f(b) = {p}. Now H = {a, b} is not closed in X. Then f is mg-continuous but the restriction f/H is not mg-continuous. Since for the closed set F = {q} in (y,m_Y) $f^{-1}(F)$ = {a, c} and $f^{-1}(F)\cap H$ = {a} is not mg-closed in H.

Definition: 2.3.16

A non-empty set X with a minimal structure m_X is said to be **mg-compact** if for every cover of X by sets of m_X has a finite sub cover.

Theorem: 2.3.17

If f: $(X,m_X) \rightarrow (Y,m_Y)$ is a mg-continuous function and K is an mg-compact set of (X,m_X) then f(K) is mg-compact.

Proof:

Let $\{v_i : i \in I\}$ be any cover of f(K) by sets of m_Y For each $x \in X$, there exist $i(x) \in I$ such that $f(x) \in v_i(x)$since f is mg-continuous there exist $U(x) \in m_X$ containing x such that $f(U(x)) \subseteq v_i(x)$. The family $\{v_i(x): x \in k\}$ is a cover of k by sets of m_X. Since K is mg-compact, there exist a finite number of points say $x_1, x_2....x_n$ in k such that $K \subseteq \cup \{U(x_k) : x_k, 1 \leq k \leq n\}$ Therefore we obtain $f(K) \subseteq \cup \{f(U(x_k) : x_k \in K, 1 \leq k \leq n\} \subseteq \cup \{v_{ix}(k) : x_k \in K, 1 \leq k \leq n\}$. Therefore f (K) is mg-compact.

SECTION: 2.4
mwg-CLOSED SETS

Definition: 2.4.1

Let (X,m_X) be an m-space. A subset A of X is said to be **mwg-closed** if $m_X\text{-cl}(m_X\text{-int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U \in m_X$.

Theorem: 2.4.2

If A is mg-closed in X, then it is mwg-closed.

Proof:

Let $A \subseteq G$ and $G \in m_X$. Since A is mg-closed, m_X-cl(A) $\subseteq G$. Therefore m_X-cl(m_X-int(A))$\subseteq G$. Hence A is mwg-closed.

Remark: 2.4.3

The converse of the above theorem [2.4.2] need not be true.

Example: 2.4.4

Consider X = {a, b, c} with τ = {X, ϕ, {a}, {a, b}}. In this m_X-closed set, {b} is mwg-closed but not mg-closed set.

Lemma: 2.4.5

For subsets A, B of X, the following properties hold:

(i) If A is m_X-closed then A is mwg-closed.

(ii) If m_X has property B and A is mwg-closed and m_X-open then A is m_X-closed.

(iii) If A is mwg-closed and $A \subseteq B \subseteq m_X$-cl($m_X$-int(A)) then B is mwg-closed.

Proof:

(i) Let $A \subseteq U$, where U is m_X-open. Since A is m_X-closed, A=m_X-cl(A)$\subseteq$U and hence m_X-cl(m_X-int(A))$\subseteq$ U. Therefore A is mwg-closed.

(ii) Let m_X have property B and A is mwg-closed and m_X-open. It is obvious that $A \subseteq m_X$-cl (A). To prove m_X-cl(A) $\subseteq$A. As $A \subseteq A$ and A is mwg-closed and m_X-open, m_X-cl(m_X-int(A))$\subseteq$A. m_X-cl(A)$\subseteq$A. m_X-cl(A) =A and hence A is m_X-closed.

(iii) Let A be mwg-closed and $A \subseteq B \subseteq m_X$-cl ($m_X$-int(A)).

$[m_X\text{-cl}(m_X\text{-int}(B))] - B \subseteq [m_X\text{-cl}(m_X\text{-int}(A))] - A$.

Since m_X-cl (m_X-int(A)) − A contains no nonempty mwg-closed set. m_X-cl(m_X-int(B)) − B does not contain any non-empty mwg-closed set. Therefore B is mwg-closed.

Lemma: 2.4.6

A subset A of X is mwg-open if and only if $F \subseteq m_X$-cl (m_X-int (A)) whenever $F \subseteq A$ and F is m_X-closed.

Proof:

Let A be a mwg-open set in X. Then A^C is mwg-closed set in X. Let F be mwg-closed set such that $F \subseteq A$. Then F^C is mwg-open and $A^C \subseteq F^C$. So $[m_X\text{-cl}(A^C)] \subseteq F^C$, $[m_X\text{-cl}(A)]^C \subseteq F^C$.Thus $F \subseteq m_X$-cl(m_X-int(A)).

Conversely assume that $F \subseteq m_X$-cl (m_X-int(A)) whenever F is m_X-closed and $F \subseteq A$. Let G be an mwg-open set such that $A^C \subseteq G$. Then G^C is mwg-closed. $G^C \subseteq (A^C)^C = A$. By hypothesis $G^C \subseteq [m_X\text{-int (A)}]$, $G \supseteq [m_X\text{-cl (A)}]^C$ and therefore A^C are mwg-closed. Hence A is mwg-open.

Theorem: 2.4.7

For each $x \in X$ either $\{x\}$ is m_X-closed or $\{x\}^C$ is mwg-open.

Proof:

Let $\{x\}$ be not mwg-closed. Then the only mwg-open set containing$\{x\}^C$ is X and the m_X-cl ($\{x\}^C$) is contained in X, hence $\{x\}^C$ is mwg-closed.

Corollary: 2.4.8

For subsets A, B of X the following properties hold:

(i) If A is m_X-open then A is mwg-open.

(ii) If m_X has property B and A is mwg-open and m_X-closed then A is m_X-open.

(iii) If A is mwg-open and m_X-int(m_X-cl(A))$\subseteq$B$\subseteq$A then B is mwg-open.

Theorem: 2.4.9

Let m_X have property B. Then for a subset A of X the following properties are equivalent. However (i) $\Rightarrow$ (ii) and (ii) $\Rightarrow$ (iii) hold even if m_X does not have property B.

(i) A is mwg-closed.

(ii) m_X-cl(A) − A does not contain any nonempty m_X-closed set.

(iii) m_X-cl(A) − A is mwg-open.

Proof:

(i) $\Rightarrow$(ii)

Let F be non-empty mwg-closed subset of m_X-cl (int (A)) − A. Then F $\subseteq$ m_X-cl (m_X-int(A)) − A and F$\subseteq$$m_X$-cl ($m_X$-int (A))$\cap$ A^C. F$\subseteq$$m_X$-cl($m_X$-int (A)) and F$\subseteq$$A^C$. Since F^C is mwg-open set, and A is mwg-closed, m_X-cl (m_X-int(A)) $\subseteq$ F^C. F $\subseteq$ $[m_X\text{-cl } (m_X\text{-int(A))}]^C$. Hence F $\subseteq$ [m_X-cl (m_X-int (A))]$\cap$$[m_X\text{-cl}(m_X\text{-int(A))}]^C$ $=\phi$. Thus F $=\phi$. Hence m_X-cl(m_X-int (A)) − A contains no nonempty m_X-closed set.

Conversely assume that m_X-cl (m_X-int (A)) − A contains no nonempty m_X-closed set. Let A $\subseteq$ G and G be mwg-open. Suppose that

m_X-cl (m_X-int(A)) is not contained in G. Then [m_X-cl (m_X-int(A))] $\cap G^C$ is a non-empty mwg-closed set of m_X-cl(m_X-int(A)) − A. This is contradiction. Therefore A is mwg-closed.

(ii) $\Rightarrow$ (iii)

Let m_X-cl (m_X-int(A)) − A contains no nonempty mwg-closed set. Let $A \subseteq G$ is mwg-open. Suppose that m_X-cl (m_X-int(A)) is contained in G. Then [m_X-cl (m_X-int(A))] $\cap G^C$ is empty. Hence m_X is closed set. *This is contradiction.* ie. m_X-cl(m_X-int(A)) − A is open.

Lemma: 2.4.10

A subset A of X is mwg-closed if and only if [m_X-cl (int (A))] $\cap F = \phi$ whenever $A \cap F = \phi$ and F is m_X-closed.

Proof:

Let A is mwg-closed. By definition m_X-cl(m_X-int(A))$\subseteq$U and also $F \subseteq U$. i.e., [m_X-cl(m_X-int(A))] − F $= \phi$ is mwg-closed. Therefore F is m_X-closed.

Conversely assume that m_X-cl (m_X-int(A)) − A is closed. By the above theorem (2.4.10), m_X-cl(m_X-int(A)) − A contains no nonempty closed set. i.e., m_X-cl (m_X-int(A)) − A $= \phi$. Therefore A is mwg-closed.

SECTION: 2.5
mwg-CONTINUOUS MAPS

Definition: 2.5.1

A function f: $(X,m_X) \rightarrow (Y,m_Y)$ is said to be **mwg-continuous** if $f^{-1}(V)$ is mwg-closed in (X,m_X) for every mwg-closed set V in (Y,m_Y).

Theorem: 2.5.2

Let f: $(X,m_X) \rightarrow (Y,m_Y)$ from a minimal space (X,m_X) into a minimal space (Y,m_Y) be m-continuous. Then it is mwg-continuous.

Proof:

Let F be any closed set in (X,m_X). Then the inverse image $f^{-1}(F)$ is closed in (Y,m_Y). Since every closed set in mwg closed set $f^{-1}(F)$ is mwg-closed set in (X,m_X). Therefore f is mwg-continuous.

Remark: 2.5.3

The converse of the above theorem [2.5.2] need not be true.

Example: 2.5.4

Let $X = \{a, b, c\}$, $\tau = \{\phi, a, X\}$, $Y = \{p, q\}$ and $\sigma = \{\phi, p, Y\}$. Let f: $(X, \tau) \rightarrow (Y, \sigma)$ be defined by $f(a) = f(c) = \{q\}$, $f(b) = \{p\}$. Then f is m-continuous but it is not mwg-continuous. Since the open set $G = \{p\}$ in (Y,m_Y) $f(a) = b$ is not open in (X,m_X).

Theorem: 2.5.5

Let f: $(X,m_X) \rightarrow (Y,m_Y)$ be a map from a minimal space (X,m_X) into a minimal space (Y,m_Y). The following statements are equivalent.

(i) f is mwg-continuous.

(ii) The inverse image of each open set in (Y,m_Y) is g-open in (X,m_X).

Proof:

(i)$\Rightarrow$(ii)

Let f: $(X,m_X) \rightarrow (Y,m_Y)$ be mwg-continuous. Let G be open in (Y,m_Y). Then G^C is closed in (Y,m_Y). Since f is mwg-closed in (X,m_X) and

$f^{-1}(G^C)$ is mwg-closed in (X,m_X). But $f^{-1}(G^C) = X - f^{-1}(G)$. Thus $X - f^{-1}(G)$ is mwg-closed in (X,m_X) and so $f^{-1}(G)$ is mwg-open in (X,m_X).

(ii) $\Rightarrow$ (i)

Let the inverse image of each open set in (Y,m_Y) is mwg-open in (X,m_X). Let F be any closed set in (Y,m_Y). By assumption $f^{-1}(F^C)$ is mwg-open. But $f^{-1}(F^C) = X - f^{-1}(F)$. Therefore $X - f^{-1}(F)$ is mwg-open in (X,m_X) and $f^{-1}(F)$ is mwg-closed. Hence f is mwg-continuous.