On Regular Generalized Semi continuous mappings in Intuitionistic fuzzy topological spaces

Anitha, R

(13PMA002)

Thesis submitted to

Avinashilingam Institute for Home Science and Higher Education for Women,

Coimbatore - 641 043

In Partial Fulfilment of the Requirements for the

Degree of Master of Science in Mathematics

March, 2015

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Signature of the Supervisor

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INTRODUCTION

INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [1965] which was followed by the introduction of fuzzy topology by Chang [1968]. Later Attanassov [1986] introduced the idea of intuitionistic fuzzy sets using the notion of fuzzy sets. Intuitionistic fuzzy set is the extension of the standard fuzzy sets. Any research based on fuzzy sets can be described in terms of intuitionistic fuzzy sets. Intuitionistic fuzzy set has many geometrical interpretations.

In the last ten years intuitionistic fuzzy sets were applied in different areas. The intuitionistic fuzzy approach to artificial intelligence includes treatment of decision making and machine learning, neural networks and pattern recognition, expert systems database, machine reasoning, logic programming.

Currently, there are many applications of intuitionistic fuzzy sets in medical diagnosis and chemistry. So intuitionistic fuzzy set plays a vital role in mathematics field.

Using the notion of intuitionistic fuzzy sets, Coker [1997] introduced intuitionistic fuzzy topological spaces which gives many interesting ideas in topological field.

This thesis presents a new class of sets in intuitionistic fuzzy topological spaces called intuitionistic fuzzy regular generalized semiclosed sets, its respective open sets, applications, continuous mappings, almost continuous mappings and contra continuous mappings.

In chapter I the recent development of intuitionistic fuzzy topology contributed by various authors is mentioned and the definitions defined by them are presented.

Chapter II dealt with the discussion of intuitionistic fuzzy regular generalized semiclosed sets and some important theorems.

In chapter III the concept of intuitionistic fuzzy regular generalized semiopen sets and their properties are introduced and investigated. Important theorems using intuitionistic fuzzy regular generalized semi $T_{1/2}$ spaces are obtained.

Chapter IV dealt with the theory of intuitionistic fuzzy regular generalized semi continuous mappings and their properties are obtained.

In chapter V dealt with the theory of intuitionistic fuzzy almost regular generalized semi continuous mappings are introduced and some important theorems are discussed.

Chapter VI dealt with the theory of intuitionistic fuzzy contra regular generalized semi continuous mappings and their properties.

Throughout this thesis (X,τ), (Y,σ) and (Z,γ) denote the intuitionistic fuzzy topological spaces on which no separation axioms are assumed unless otherwise explicitly mentioned.

REVIEW OF LITERATURE

REVIEW OF LITERATURE

The intuitionistic fuzzy topological space is quite interesting and useful in many areas of mathematics. Intuitionistic fuzzy semi open sets, intuitionistic fuzzy pre open sets and intuitionistic fuzzy α closed sets were introduced by Guracy, H., Coker, D., and Haydar, Es. A., [1997] in intuitionistic fuzzy topological spaces. Young Bae Jun and Seok - Zun Song., [2005] introduced intuitionistic fuzzy semi- pre open sets and intuitionistic fuzzy semi- pre closed sets in intuitionistic fuzzy topological spaces. Krsteska, B., E. Ekici., [2007] have introduced intuitionistic fuzzy contra continuous mappings, intuitionistic fuzzy contra pre continuous mappings and intuitionistic fuzzy contra strongly pre continuity mappings.

FUZZY SETS

[Zadeh, L.A., 1965]

In this article, the author has characterized the fuzzy sets by membership function. Further, the notions of union, intersection, complement, relation, convexity are extended to such sets and various properties of these notions of fuzzy sets are established.

INTUITIONISTIC FUZZY SETS

[Krassimir T. Atanassov., 1986]

In this paper he introduced the concepts of intuitionistic fuzzy sets and discussed many of the properties. This paper gives the strong foundation to study intuitionistic fuzzy topology. After he introduced the concept of intuitionistic fuzzy sets, many people found many interesting theorems on this particular set.

THE CATEGORY OF INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Seok Jong and Eun Pyo Lee., 2000]

The author has introduced the concept of intuitionistic fuzzy neighborhood and the properties of continuous, open and closed maps in the intuitionistic fuzzy topological spaces are investigated.

ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Guracy, H., Coker, D., and Haydar, Es. A., 1997]

In this paper, the authors have constructed the definitions of fuzzy continuity, fuzzy compactness, fuzzy connectedness and fuzzy hausdorff spaces and obtained the several preservation properties.

ON INTUITIONISTIC FUZZY POINT

[Coker, D., and Demirci, M., 1995]

In this article, the authors have constructed one of the most important definition namely intuitionistic fuzzy points in intuitionistic fuzzy topological spaces and they have produced any interesting theorems.

GENERALIZED CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Thakur, S.S., and Rekha Chaturvedi., 2006]

The authors have introduced the concept of intuitionistic fuzzy generalized continuous mappings in intuitionistic fuzzy topological spaces and obtained some interesting theorems.

ON INTUITIONISTIC FUZZY SEMI GENERALIZED CLOSED SETS

[R. Santhi and K. Arun prakash., 2010]

In this article, the authors have introduced the concept of intuitionistic fuzzy semi generalized closed sets and intuitionistic fuzzy semi generalized open sets in intuitionistic fuzzy topological spaces. Further they have discussed the application of intuitionistic fuzzy semi generalized closed sets in intuitionistic fuzzy semi $T_{1/2}$ spaces.

CHAPTER I

CHAPTER I

PRELIMINARIES

Definition 1.1: [1] An ***intuitionistic fuzzy set*** (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A (x), \nu_A(x)\rangle$$

where the functions $\mu_A : X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non – membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 1.2: [1] Let A and B be two IFSs of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}.$$

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle / x \in X\}$

(d) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle / x \in X\}$

(e) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle / x \in X\}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A\rangle$ instead of $A = \{\langle x,\mu_A(x), \nu_A(x)\rangle / x \in X\}$. The IFSs $0\sim = \langle x, 0, 1\rangle$ and $1\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 1.3: [3] An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

i. $0\sim, 1\sim \in \tau$,

ii. $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

iii. $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X,τ) is called an ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an ***intuitionistic fuzzy open set*** (IFOS in short) in X. The complement A^c of an IFOS A in (X,τ) is called an ***intuitionistic fuzzy closed set*** (IFCS in short) in X.

Definition 1.4: [3] Let (X,τ) be an IFTS and $A = \langle x, \mu_A, \nu_A\rangle$ be an IFS in X. Then the ***intuitionistic fuzzy interior*** and ***intuitionistic fuzzy closure*** are defined by

$int(A) = \cup \{G / G \text{ is an IFOS in X and } G \subseteq A\}$

$cl(A) = \cap \{K / K \text{ is an IFCS in X and } A \subseteq K\}$

Note that for any IFS A in (X,τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 1.5: [5] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X,τ) is said to be an

(i) ***intuitionistic fuzzy semi closed set*** (IFSCS in short) if $int(cl(A)) \subseteq A$

(ii) ***intuitionistic fuzzy α closed set*** (IFαCS in short) if $cl(int(cl(A))) \subseteq A$

(iii) ***intuitionistic fuzzy regular closed set*** (IFRCS in short) if $A = cl(int(A))$

Definition 1.6: [5] An IFS A in an IFTS (X,τ) is said to be

(i) ***intuitionistic fuzzy semiopen*** if $A \subseteq cl(int(A))$

(ii) ***intuitionistic fuzzy α open*** if $A \subseteq int(cl(int(A)))$

(iii) ***intuitionistic fuzzy regular open*** if $A = int(cl(A))$

Definition 1.7: [11] Two IFSs A and B are said to be ***q- coincident*** ($A_q B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 1.8: [11] Two IFSs are said to be ***not q - coincident*** ($A\, {}_q^c\, B$ in short) if and only if $A \subseteq B^c$.

Definition 1.9: [4] An ***intuitionistic fuzzy point*** (IFP in short), written as $p_{(\alpha,\beta)}$, is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An IFP $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 1.10: [10] Let $p_{(\alpha, \beta)}$ be an IFP of an IFTS (X,τ). An IFS A of X is called an ***intuitionistic fuzzy neighbourhood*** (IFN in short) of $p_{(\alpha, \beta)}$ if there exists an IFOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Definition 1.11: [5] Let A be an IFS in an IFTS (X,τ). Then

(i) $sint(A) = \cup \{ G \,/\, G$ is an IFSOS in X and $G \subseteq A \}$

(ii) $scl(A) = \cap \{ K \,/\, K$ is an IFSCS in X and $A \subseteq K \}$

Result 1.12: [9] Let A be an IFS in (X,τ), Then

(i) $scl(A) = A \cup int(cl(A))$

(ii) $sint(A) = A \cap cl(int(A))$

Definition 1.13: [3] Let X and Y be non empty sets and $f: X \rightarrow Y$ be a function. If $B = \{\langle y, \mu_B(y), \nu_B(y)\rangle \,/\, y \in Y\}$ is an intuitionistic fuzzy set in Y, then the ***preimage*** of B under f is denoted and defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\rangle \,/\, x \in X\}$$

where $f^{-1}(\mu_B)(x) = (\mu_B)(f(x))$ for evey $x \in X$.

Corollary 1.14: [3] Let A, A_i $(i \in J)$ be intuitionistic fuzzy sets in X and B, B_j $(j \in K)$ be intuitionistic fuzzy sets in Y and $f : X \rightarrow Y$ be a function.Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

c) $A \subseteq f^{-1}(f(A))$ [If f is injective , then $A=f^{-1}(f(A))$]

d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective , then $B=f(f^{-1}(B))$]

e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

g) $f^{-1}(0\sim) = 0\sim$

h) $f^{-1}(1\sim) = 1\sim$

i) $f^{-1}(B^c) = (f^{-1}(B))^c$

Result 1.15: [3] Let A, B and C be intuitionistic fuzzy sets in X. Then

i. $(A \subseteq B)$ and $(C \subseteq D) \Rightarrow (A \cup C) \subseteq (B \cup D)$ and $(A \cap C) \subseteq (B \cap D)$

ii. $A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq (B \cap C)$

iii. $A \subseteq C$ and $B \subseteq C \Rightarrow (A \cup B) \subseteq C$

iv. $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$

v. $(A \cup B)^c = A^c \cap B^c$

vi. $(A \cap B)^c = A^c \cup B^c$

vii. $A \subseteq B \Rightarrow B^c \subseteq A^c$

viii. $(A^c)^c = A$

ix. $(0\sim)^c = 1\sim$

x. $(1\sim)^c = 0\sim$

Theorem 1.16: [3] Let (X,τ) be an IFTS and A, B be intuitionistic fuzzy sets in X. Then the following properties hold:

i. $Int(A) \subseteq A$

ii. $A \subseteq cl(A)$

iii. $A \subseteq B \Rightarrow int(A) \subseteq int(B)$

iv. $A \subseteq B \Longrightarrow cl(A) \subseteq cl(B)$

v. $int(int(A)) = int(A)$

vi. $cl(cl(A)) = cl(A)$

vii. $int(A \cap B) = int(A) \cap int(B)$

viii. $cl(A \cup B) = cl(A) \cup cl(B)$

ix. $int(1_\sim) = 1_\sim$

x. $cl(0_\sim) = 0_\sim$

Definition 1.17: [11] An IFS A in an IFTS (X,τ) is said to be an

(i) ***intuitionistic fuzzy generalized closed set*** (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X,

(ii) ***intuitionistic fuzzy regular generalized closed set*** (IFRGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

Definition 1.18: [5] Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an ***intuitionistic fuzzy continuous*** (IF continuous in short) ***mapping*** if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

Definition 1.19: [11] Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an ***intuitionistic fuzzy generalized continuous*** (IFG continuous in short) ***mapping*** if $f^{-1}(B) \in IFGCS(X)$ for every IFCS B in Y.

Definition 1.20: [6] Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an

(i) ***intuitionistic fuzzy semi continuous*** (IFS continuous in short) ***mapping*** if $f^{-1}(B) \in IFSO(X)$ for every $B \in \sigma$

(ii) ***intuitionistic fuzzy α continuous*** (IFα continuous in short) ***mapping*** if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$

Definition 1.21: [7] Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an

(i) ***intuitionistic fuzzy contra continuous*** (IFC continuous in short) ***mapping*** if $f^{-1}(B) \in IFO(X)$ for each IFCS B in Y

(ii) ***intuitionistic fuzzy contra α continuous*** (IFCα continuous in short) ***mapping*** if $f^{-1}(B) \in IF\alpha O(X)$ for each IFCS B in Y

Definition 1.22: [11] An IFTS(X,τ) is said to be an ***intuitionistic fuzzy*** $T_{1/2}$ ($IFT_{1/2}$ in short) ***space*** if every intuitionistic fuzzy generalized closed set (IFGCS in short) in X is an IFCS in X.

CHAPTER II

CHAPTER II

INTUITIONISTIC FUZZY REGULAR GENERALIZED SEMICLOSED SETS

In this section, we have introduced the notion of intuitionistic fuzzy regular generalized semiclosed sets in intuitionistic fuzzy topological spaces and studied some of their properties.

Definition 2.1: An IFS A of an IFTS (X,τ) is said to be an ***intuitionistic fuzzy regular generalized semiclosed set*** (IFRGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X.

Example 2.2: Let $X = \{a, b\}$ and $U = \langle x, (0.4, 0.3), (0.6, 0.7)\rangle$ where $\mu_a = 0.4$, $\mu_b = 0.3$, $\nu_a = 0.6$, $\nu_b = 0.7$ then $\tau = \{0\sim, U, 1\sim\}$ is an IFT in X. Let $A = \langle x, (0.1, 0.2), (0.7, 0.8)\rangle$ be any IFS in X. Here $A \subseteq U$ where U is an IFROS in X. Now, $scl(A) = \langle x, (0.4, 0.3), (0.6, 0.7)\rangle = U \subseteq U$. Therefore A is an IFRGSCS in (X,τ).

Theorem 2.3: Every IFCS in (X,τ) is an IFRGSCS in (X,τ) but not conversely.

Proof: Let $A \subseteq U$ and U be an IFROS in X. Now $scl(A) = A \cup int(cl(A)) = A \cup int(A)$, since by the hypothesis $cl(A) = A$. Therefore $scl(A) \subseteq A \cup A = A \subseteq U$. Hence A is an IFRGSCS in (X,τ).

Example 2.4: In Example 2.2, A is an IFRGSCS, but not an IFCS in X, since $cl(A) = \langle x, (0.6, 0.7), (0.4, 0.3)\rangle \neq A$.

Theorem 2.5: Every IFRCS in (X,τ) is an IFRGSCS in (X,τ) but not conversely.

Proof: Let A be an IFRCS. Since every an IFRCS is an IFCS, by Theorem 2.3, A is an IFRGSCS in (X,τ).

Example 2.6: In Example 2.2, A is an IFRGSCS in X, but not an IFRCS in X, since $cl(int(A)) = 0\sim \neq A$.

Theorem 2.7: Every IFSCS in (X,τ) is an IFRGSCS in (X,τ) but not conversely.

Proof: Let A ⊆ U be an IFSCS and U be an IFROS in X. we have scl(A) = A ∪ int(cl(A)) ⊆ A ∪ A = A ⊆ U, since by hypothesis int(cl(A)) ⊆ A. Hence A is an IFRGSCS in (X,τ).

Example 2.8: In Example 2.2**,** A is an IFRGSCS, but not an IFSCS in X, since int(cl(A)) = ⟨x, (0.4, 0.3), (0.6, 0.7)⟩ = U ⊈ A.

Theorem 2.9: Every IFGCS in (X,τ) is an IFRGSCS in (X,τ) but not conversely.

Proof: Let A ⊆ U be an IFGCS and U be an IFROS in X. We have scl(A) = A ∪ int(cl(A)) ⊆ A ∪ int(U) ⊆ U, by hypothesis cl(A) ⊆ U. Hence A is an IFRGSCS in (X,τ).

Example 2.10: Let X = {a, b} and U = ⟨x, (0.4, 0.3), (0.6, 0.5)⟩. Then τ = {0~, U, 1~} is an IFT on X. Let A = ⟨x, (0.3, 0.3), (0.6, 0.5)⟩ be an IFS in X. Here A ⊆ U where U is an IFROS in X. Now, scl(A) = ⟨x, (0.4, 0.3), (0.6, 0.5)⟩ = U ⊆ U. Therefore A is an IFRGSCS, but it is not an IFGCS in X, since A ⊆ U, but cl(A) = ⟨x, (0.6, 0.5), (0.4, 0.3)⟩ ⊈ U.

Theorem 2.11: Every IFαCS in (X,τ) is an IFRGSCS in (X,τ) but not conversely.

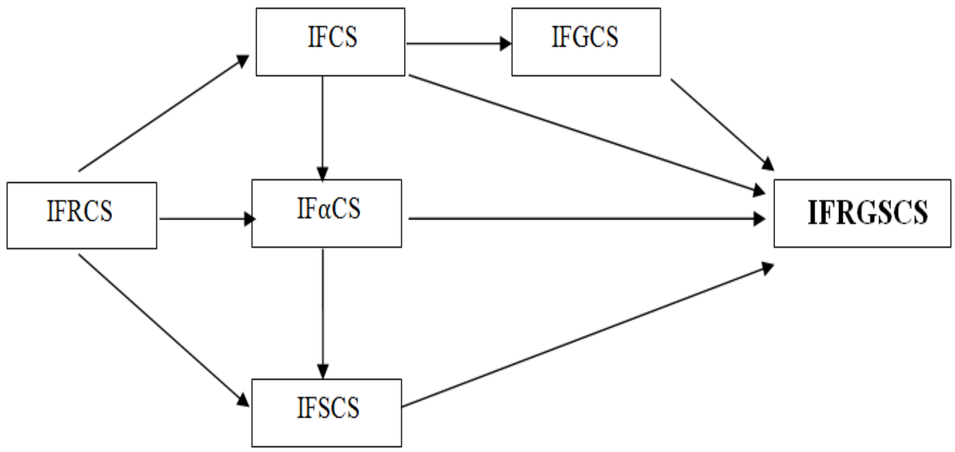
Proof: Let A ⊆ U and U be an IFROS in X. we have scl(A) = A ∪ int(cl(A)) ⊆ A ∪ cl(int(cl(A))) ⊆ A ∪ A, since by hypothesis cl(int(cl(A))) ⊆ A. Therefore scl(A) ⊆ U. Hence A is an IFRGSCS in (X,τ).

Example 2.12: Let X = {a,b} and U = ⟨x, (0.4, 0.2), (0.6, 0.7)⟩ Then τ = {0~, U, 1~} is an IFT on X. Let A = ⟨x, (0.3, 0.2), (0.7, 0.8)⟩ be an IFS in X. Here A ⊆ U where U is an IFROS in X. Now scl(A) = ⟨x, (0.4, 0.2), (0.6, 0.7)⟩ = U. Therefore A is an IFRGSCS, but not an IFαCS in X, since A ⊆ U, but cl(int(cl(A))) = ⟨x, (0.6, 0.7), (0.4, 0.2)⟩ ⊈ A.

Theorem 2.13: If an IFS is both an IFPOS and an IFSCS in (X,τ) then it is an IFRGSCS in (X,τ).

Proof: Let an IFS A be both an IFPOS and IFSCS in (X,τ). Then A ⊆ int(cl(A)) and int(cl(A)) ⊆ A. Therefore A = int(cl(A)). Now let A ⊆ U and U be an IFROS in X. Then scl(A) = A ∪ int(cl(A)) = A ∪ A = A ⊆ U. Hence A is an IFRGSCS in (X,τ).

In the following diagram, we provide the relations between various types of intuitionistic fuzzy closedness.



In the above diagram the reverse implications are not true in general.

Theorem 2.14: Let A ⊆ B ⊆ scl(B) and A be an IFRGSCS in an IFTS (X,τ), then B is an IFRGSCS in (X,τ).

Proof: Let U be an IFROS in X such that B ⊆ U. Then A ⊆ U, as A ⊆ B and since A is an IFRGSCS in X, scl(A) ⊆ U. Now, B ⊆ scl(A) ⇒ scl(B) ⊆ scl(A) ⊆ U. This implies scl(A) ⊆ U. Hence B is an IFRGSCS in (X,τ).

Theorem 2.15: If A is both an IFROS and an IFRGSCS in X. Then A is an IFSCS in (X,τ).

Proof: As A ⊆ A, by hypothesis we have scl(A) ⊆ A. But A ⊆ scl(A). Therefore A = scl(A). Hence A is an IFSCS in (X,τ).

Theorem 2.16: Let (X,τ) be an IFTS and A be an IFSCS in X. Then A is an IFRGSCS if and only if A $_q{}^c$ F ⇒ scl(A) $_q{}^c$ F for every IFRCS F of X.

Proof: Necessity: Let F be an IFRCS of X and A $_q{}^c$ F. Then A ⊆ F^c, by Definition, 1.8 and F^c is IFROS in X. Therefore scl(A) ⊆ F^c because A is an IFRGSCS. Hence scl(A) $_q{}^c$ F.

Sufficiency: Let U be an IFROS of X such that $A \subseteq U$. Then $A\ q^c\ (U^c)$ and (U^c) is an IFRCS in X. Hence by hypothesis, $scl(A)\ q^c\ (U^c)$. Therefore $scl(A) \subseteq (U^c)^c = U$ and A is an IFRGSCS in (X,τ).

Theorem 2.17: Let (X,τ) be an IFTS. Then every IFS in (X,τ) is an IFRGSCS if and only if IFSO(X) = IFSC(X).

Proof: Necessity: Suppose that every IFS in (X,τ) is an IFRGSCS. Let $U \in$ IFRO(X), then $U \in$ IFSO(X) and by hypothesis, $scl(U) \subseteq U \subseteq scl(U)$. This implies scl(U) = U. Therefore $U \in$ IFSC(X). Hence IFSO(X) $\subseteq$ IFSC(X). Let $A \in$ IFSC(X), then $A^c \in$ IFSO(X) $\subseteq$ IFSC(X). that is, $A^c \in$ IFSC(X). Therefore $A \in$ IFSO(X). Hence IFSC(X) $\subseteq$ IFSO(X). Thus, IFSO(X) = IFSC(X).

Sufficiency: Suppose that IFSO(X) = IFSC(X). Let $A \subseteq U$ and U be an IFROS. Then $U \in$ IFSO(X) and $scl(A) \subseteq scl(U) = U$, since $U \in$ IFSC(X), by hypothesis. Therefore A is an IFRGSCS in X.

Theorem 2.18: Let A be an IFRGSCS in (X,τ) and $p_{(\alpha,\beta)}$ be an IFP in X such that $int(p(\alpha,\beta))\ _q\ scl(A)$, then $cl(int(p(\alpha,\beta)))\ _q\ A$.

Proof: Let A be an IFRGSCS and let $int(p(\alpha,\beta))\ _q\ scl(A)$. If $cl(int(p(\alpha, \beta)))\ q^c\ A$ then by Definition 1.8, $A \subseteq [cl(int(p(\alpha,\beta)))]^c$ where $[cl(int(p(\alpha,\beta)))]^c$ is an IFROS. Then by hypothesis, $scl(A) \subseteq [cl(int(p(\alpha, \beta)))]^c = int(cl(p(\alpha, \beta))^c) \subseteq cl(p(\alpha, \beta))^c = (int(p(\alpha, \beta)))^c$. This implies $scl(A)\ q^c\ int(p(\alpha, \beta))$. Therefore by Definition 1.8, $int(p(\alpha, \beta))\ q^c\ scl(A)$, which is a contradiction to the hypothesis. Hence $cl(int(p(\alpha, \beta)))\ _q\ A$.

CHAPTER III

CHAPTER III

REGULAR GENERALIZED SEMIOPEN SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

In this section, we have introduced the notion of intuitionistic fuzzy regular generalized semiopen sets and studied some of their properties.

Definition 3.1: An IFS A of an IFTS (X,τ) is said to be an ***intuitionistic fuzzy regular generalized semiopen set*** (IFRGSOS in short) if $sint(A) \supseteq U$ whenever $A \supseteq U$ and U is an IFRCS in (X,τ).

It is to be noted that the complement A^c of an IFRGSCS A in an IFTS (X,τ) is an IFRGSOS in (X,τ).

Example 3.2: Let $X = \{a, b\}$ and $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$ where $\mu_a = 0.4$, $\mu_b = 0.5$, $\nu_a = 0.6$, $\nu_b = 0.5$ and $G_2 = \langle x, (0.4, 0.4), (0.6, 0.6)\rangle$ where $\mu_a = 0.4$, $\mu_b = 0.4$, $\nu_a = 0.6$, $\nu_b = 0.6$. Let $A = \langle x, (0.7, 0.6), (0.3, 0.3)\rangle$ be any IFS in X. Here $A \supseteq U$ where $U = \langle x, (0.6, 0.5), (0.4, 0.5)\rangle$ is an IFRCS in X. Now, $sint(A) = \langle x, (0.6, 0.5), (0.4, 0.5)\rangle \supseteq U$. Therefore A is an IFRGSOS in (X,τ).

Theorem 3.3: Every IFOS, IFROS, IFSOS, IFGOS and IFαOS is an IFRGSOS but the converses are not true in general.

Proof: Straight forward.

Example 3.4: Let $X = \{a,b\}$ and let $\tau = \{0\sim, G_1, G_2, 1\sim\}$ where $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$. Let $A = \langle x, (0.8, 0.6), (0.2, 0.3)\rangle$ be any IFS in X. Then $A \supseteq U$ where $U = \langle x, (0.6, 0.5), (0.4, 0.5)\rangle$ is an IFRCS in X. Now $sint(A) = \langle x, (0.6, 0.5), (0.4, 0.5)\rangle = U \supseteq U$. Therefore A is an IFRGSOS but not an IFOS in X, as $int(A) = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle \neq A$.

Example 3.5: In Example 3.4, Let $A = \langle x, (0.7, 0.8), (0.3, 0.2)\rangle$ be any IFS in X. Then $A \supseteq U$ where $U = \langle x, (0.6, 0.5), (0.4, 0.5)\rangle$ is an IFRCS in X. Now $sint(A) = \langle x, (0.6, 0.5),$

(0.4, 0.5)⟩ = U ⊇ U. Therefore A is an IFRGSOS but not an IFROS in X, as int(cl(A)) = 1~ ≠ A.

Example 3.6: Let X = {a,b} and let τ = {0~, G_1, G_2, 1~} where G_1 = ⟨x, (0.2, 0.5), (0.8, 0.5)⟩ and G_2 = ⟨x, (0.2, 0.4), (0.8, 0.6)⟩. Let A = ⟨x, (0.8, 0.8), (0.1, 0.2)⟩ be any IFS in X. Then A ⊇ U where U = ⟨x, (0.8, 0.5), (0.2, 0.5)⟩ is an IFRCS in X. Now sint(A) = ⟨x, (0.8, 0.5), (0.2, 0.5)⟩ = U ⊇ U. Therefore A is an IFRGSOS but not an IFSOS in X, as cl(int(A)) = ⟨x, (0.8, 0.5), (0.2, 0.5)⟩ ⊉ A.

Example 3.7: Let X = {a,b} and let τ = {0~, G_1, G_2, 1~} where G_1 = ⟨x, (0.3, 0.5), (0.7, 0.5)⟩ and G_2 = ⟨x, (0.2, 0.5), (0.8, 0.5)⟩. Let A = ⟨x, (0.8, 0.8), (0.1, 0.2)⟩ be any IFS in X. Then A ⊇ U where U = ⟨x, (0.7, 0.5), (0.3, 0.5)⟩ is an IFRCS in X. Now sint(A) = ⟨x, (0.7, 0.5), (0.3, 0.5)⟩ = U ⊇ U. Therefore A is an IFRGSOS but not an IFGOS in X, as int(A) = ⟨x, (0.3, 0.5), (0.7, 0.5)⟩ ⊉ U.

Example 3.8: Let X = {a,b} and let τ = {0~, G_1, G_2, 1~} where G_1 = ⟨x, (0.3, 0.4), (0.7, 0.5)⟩ and G_2 = ⟨x, (0.2, 0.3), (0.8, 0.6)⟩. Let A = ⟨x, (0.8, 0.7), (0.2, 0.3)⟩ be any IFS in X. Then A ⊇ U where U = ⟨x, (0.7, 0.5), (0.3, 0.4)⟩ is an IFRCS in X. Now sint(A) = ⟨x, (0.7, 0.5), (0.3, 0.4)⟩ = U ⊇ U. Therefore A is an IFRGSOS but not an IFαOS in X, as int(cl(int(A))) = ⟨x, (0.3, 0.4), (0.7, 0.5)⟩ ⊉ A.

Theorem 3.9: If A is both an IFRCS and an IFRGSOS in X, then A is an IFSOS in (X,τ).

Proof: As A ⊇ A and A is an IFRCS. We have A ⊇ cl(int(A)) which implies A ∩ A ⊇ A ∩ cl(int(A)). Therefore A ⊇ sint(A).Since A is also an IFRGSOS and A ⊇ A where A is an IFRCS. We get, sint(A) ⊇ A. Hence A = sint(A). Therefore A is an IFSOS in (X,τ).

Theorem 3.10: Let (X,τ) be an IFTS then for every A ∊ IFRGSO(X) and for every B ∊ IFS(X), sint(A) ⊆ B ⊆ A ⇒ B ∊ IFRGSO(X).

Proof: Let A be an IFRGSOS in X and B be an IFS of X. Let sint(A) ⊆ B ⊆ A. Then A^c is an IFRGSCS and A^c ⊆ B^c ⊆ $(sint(A))^c$ = scl(A^c). Therefore B^c is an IFRGSCS, by Theorem 2.14. which implies B is an IFRGSOS in X. Hence B ∊ IFRGSO(X).

3.1 Applications of intuitionistic fuzzy regular generalized semiopen sets

In this section we have provided some applications of intuitionistic fuzzy regular generalized semiopen sets.

Definition 3.1.1: If every IFRGSOS in (X,τ) is an IFSOS in (X,τ), then the space can be called as an ***intuitionistic fuzzy regular semi $T_{1/2}$ space*** ($IF_{rs}T_{1/2}$ in short).

Theorem 3.1.2: An IFTS (X,τ) is an $IF_{rs}T_{1/2}$ space if and only if IFSC(X) = IFRGSC(X).

Proof: Necessity: Let A be an IFRGSCS in (X,τ). Then A^c is an IFRGSOS in (X,τ). By hypothesis, A^c is an IFSOS in (X,τ) and therefore A is an IFSCS in (X,τ). Hence IFSC(X) = IFRGSC(X).

Sufficiency: Let A be an IFRGSOS in (X,τ). Then A^c is an IFRGSCS in (X,τ). By hypothesis, A^c is an IFSCS in (X,τ) and therefore A is an IFSOS in (X,τ). Hence (X,τ) is an $IF_{rs}T_{1/2}$ space.

Theorem 3.1.3: Let A be an IFS of X then the following properties are equivalent if X is an $IF_{rs}T_{1/2}$ space:

(i) A ∈ IFRGSO(X)
(ii) A ⊆ cl(int(A))
(iii)There exists IFOS G such that G ⊆ A ⊆ cl(G).

Proof: (i) ⟹ (ii): Let A ∈ IFRGSO(X). This implies A is an IFSOS in X, since X is an $IF_{rs}T_{1/2}$ space. Then A^c is an IFSCS in X. Therefore int(cl(A^c)) ⊆ A^c. This implies A ⊆ cl(int(A)).

(ii) ⟹ (iii): Let A ⊆ cl(int(A)). Hence int(A) ⊆ A ⊆ cl(int(A)). Then there exists IFOS G in X such that G ⊆ A ⊆ cl(G) where G = int(A).

(iii) ⟹ (i): Suppose that there exists IFOS G such that G ⊆ A ⊆ cl(G). It is clear that $(cl(G))^c$ ⊆ A^c. That is $(cl(int(A)))^c$ ⊆ A^c, since G = int(A). This implies int(cl(A^c)) ⊆ A^c. That is A^c is an IFSCS in X. Therefore A is an IFSOS in X and hence by Theorem 3.3, Hence A ∈ IFRGSO(X).

Definition 3.1.4: An IFTS (X,τ) is said to be an ***intuitionistic fuzzy regular semi $T^{*}_{1/2}$ space*** ($IF_{rs}T^{*}_{1/2}$ space in short) if every IFRGSCS is an IFCS in (X,τ).

Remark 3.1.5: Every $IF_{rs}T^{*}_{1/2}$ space is an $IF_{rs}T_{1/2}$ space but not conversely.

Proof: Let (X,τ) be an $IF_{rs}T^{*}_{1/2}$ space. Let A be an IFRGSCS in (X,τ). By hypothesis, A is an IFCS. Since every IFCS is an IFSCS [6], A is an IFSCS in (X,τ). Hence (X,τ) is an $IF_{rs}T_{1/2}$ space.

Example 3.1.6: Let $X = \{a, b\}$ and $\tau = \{0\sim, G_1, G_2, G_3, 1\sim\}$ where, $G_1 = \langle x, (0.5, 0.5), (0.3, 0.1)\rangle$, $G_2 = \langle x, (0.1, 0.1), (0.7, 0.7)\rangle$ and $G_3 = \langle x, (0.5, 0.5), (0.4, 0.2)\rangle$. Let $A = \langle x, (0.4, 0.1), (0.5, 0.5)\rangle$ be any IFS in X. Then $A \subseteq G_1$, where G_1 is an IFROS. Now $scl(A) = A \subseteq G_1$. This implies A is an IFRGSCS. Since every IFRGSCS is an IFSCS, we get (X,τ) is an $IF_{rs}T_{1/2}$ space. Now $cl(A) = \langle x, (0.4, 0.2), (0.5, 0.5)\rangle \neq A$. This implies A is not an IFCS. Hence (X,τ) is not an $IF_{rs}T^{*}_{1/2}$ space.

Theorem 3.1.7: For any IFS A in (X,τ) where X is an $IF_{rs}T^{*}_{1/2}$ space, $A \in IFRGSO(X)$ if and only if every IFP $p_{(\alpha,\beta)} \in A$, there exist an IFRGSOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Proof: Necessity: If $A \in IFRGSO(X)$, then we can take $B = A$ so that $p_{(\alpha,\beta)} \in B \subseteq A$. For every IFP $p_{(\alpha,\beta)} \in A$.

Sufficiency: Let A be an IFS in (X,τ) and assume that there exist $B \in IFRGSO(X)$ such that $p_{(\alpha,\beta)} \in B \subseteq A$. Since X is an $IF_{rs}T^{*}_{1/2}$ space, B is an IFOS. Then $A = \bigcup_{p(\alpha,\beta) \in A} \{p_{(\alpha,\beta)}\} \subseteq \bigcup_{p(\alpha,\beta) \in A} B \subseteq A$. Therefore $A = \bigcup_{p(\alpha,\beta) \in A} B$, which is an IFOS in X, by Theorem 3.3. Hence A is an IFRGSOS in X.

Definition 3.1.8: An IFTS (X,τ) is said to be an ***intuitionistic fuzzy regular generalized semi $T_{1/2}$*** ($IF_{rgs}T_{1/2}$ in short) ***space*** if every IFRGSCS in X is an IFGSCS in X.

Theorem 3.1.9: If an IFTS (X,τ) is an $IF_{rgs}T_{1/2}$ space, then every IFRGSOS is an IFGSOS.

Proof: Let A be an IFRGSOS in X. This implies A^c is an IFRGSCS in X. Since X is an $IF_{rgs}T_{1/2}$ space, A^c is an IFGSCS in X. Hence A is an IFGSOS in X.

Theorem 3.1.10: Let an $IF_{rgs}T_{1/2}$ space be an IFTS. If A is an IFS of X then the following conditions are equivalent:

(i) $A \in IFRGSO(X)$

(ii) $U \subseteq cl(int(A))$ whenever $U \subseteq A$ and U is an IFCS in X

(iii) There exists IFOSs G and G_1 such that $G_1 \subseteq U \subseteq cl(G)$

Proof: (i) $\Rightarrow$ (ii): Let $A \in IFRGSO(X)$. This implies A is an IFGSOS in X, since X is an $IF_{rgs}T_{1/2}$ space. Then A^c is an IFGSCS in X. Therefore $scl(A^c) \subseteq V$ whenever $A^c \subseteq V$ and V is an IFOS in X. That is $int(cl(A^c)) \subseteq V$. This implies $V^c \subseteq cl(int(A))$ whenever $V^c \subseteq A$ and V^c is an IFCS in X. Replacing V^c by U, $U \subseteq cl(int(A))$ whenever $U \subseteq A$ and U is an IFCS in X.

(ii) $\Rightarrow$ (iii): Let $U \subseteq cl(int(A))$ whenever $U \subseteq A$ and U is an IFCS in X. Hence $int(U) \subseteq U \subseteq cl(int(A))$. Then there exists IFOSs G and G_1 such that $G_1 \subseteq U \subseteq cl(G)$ where $G = int(A)$ and $G_1 = int(U)$.

(iii) $\Rightarrow$ (i): Suppose that there exists IFOSs G and G_1 such that $G_1 \subseteq U \subseteq cl(G)$. It is clear that $(cl(G))^c \subseteq U^c$. That is $(cl(int(A)))^c \subseteq U^c$. This implies $int(cl(A^c)) \subseteq U^c$, $A^c \subseteq U^c$ and U^c is an IFOS in X. This implies $scl(A^c) \subseteq U^c$. That is A^c is an IFGSCS in X. This implies A is an IFGSOS in X. Therefore $A \in IFRGSO(X)$.

CHAPTER IV

CHAPTER IV

INTUITIONISTIC FUZZY REGULAR GENERALIZED SEMI CONTINUOUS MAPPINGS

In this section, we have introduced the notion of intuitionistic fuzzy regular generalized semi continuous mappings and studied some of their properties.

Definition 4.1: A mapping f : (X,τ) → (Y,σ) is called an ***intuitionistic fuzzy regular generalized semi continuous*** (IFRGS continuous in short) ***mapping*** if $f^{-1}(B)$ is an IFRGSCS in (X,τ) for every IFCS B of (Y,σ).

Example 4.2: Let X = {a,b}, Y = {u,v} and G_1 = ⟨x, (0.5, 0.6), (0.3, 0.4)⟩ where μ_a=0.5, μ_b=0.6, ν_a=0.3, ν_b=0.4 and G_2 = ⟨x, (0.2, 0.3), (0.7, 0.6)⟩ where μ_a=0.2, μ_b=0.3 ν_a=0.7, ν_b=0.6 and G_3 = ⟨y, (0.6, 0.6), (0.3, 0.4)⟩ where μ_u=0.6, μ_v=0.6, ν_u=0.3, ν_v=0.4. Then τ = {0~, G_1, G_2, 1~} and σ = {0~, G_3, 1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y,σ) by f(a) = u and f(b) = v. The IFS G_3^c = ⟨y, (0.3, 0.4), (0.6, 0.6)⟩ is an IFCS in Y. Then $f^{-1}(G_3^c)$ = ⟨x, (0.3, 0.4), (0.6, 0.6)⟩ where μ_a=0.3, μ_b=0.4, ν_a=0.6, ν_b=0.6 is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = (f^{-1}(G_3^c)) \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X.

Theorem 4.3: Every IF continuous mapping is an IFRGS continuous mapping but not conversely.

Proof: Let f : (X,τ) → (Y,σ) be an IF continuous mapping. Let A be an IFCS in Y. Since f is IF continuous mapping, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFRGSCS, by Theorem 2.3. $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFRGS continuous mapping.

Example 4.4: Let X = {a,b}, Y = {u,v} and G_1 = ⟨x, (0.5, 0.3), (0.4, 0.2)⟩ and G_2 = ⟨x, (0.1, 0.2), (0.9, 0.8)⟩ and G_3 = ⟨y, (0.5, 0.4), (0.4, 0.2)⟩**.** Then τ = {0~, G_1, G_2, 1~} and σ = {0~, G_3, 1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y,σ) by f(a) = u and f(b) = v. The IFS G_3^c = ⟨y, (0.4, 0.2), (0.5, 0.4)⟩ is an IFCS in Y. Then $f^{-1}(G_3^c)$ = ⟨x, (0.4, 0.2), (0.5, 0.4)⟩ is an IFS in X. Then

$f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = (f^{-1}(G_3^c)) \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFCS in X, since $cl(f^{-1}(G_3^c)) = G_1^c \neq f^{-1}(G_3^c)$. Therefore f is an IFRGS continuous mapping but not an IF continuous mapping.

Theorem 4.5: Every IFS continuous mapping is an IFRGS continuous mapping but not conversely.

Proof: Let $f : (X,\tau) \to (Y,\sigma)$ be an IFS continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFSCS in X. Since every IFSCS is an IFRGSCS, by Theorem 2.7. $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFRGS continuous mapping.

Example 4.6: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.2, 0.5), (0.7, 0.4)\rangle$, $G_2 = \langle x, (0.2, 0.4), (0.7, 0.6)\rangle$ and $G_3 = \langle y, (0.8, 0.6), (0.2, 0.4)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.2, 0.4), (0.8, 0.6)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.2, 0.4), (0.8, 0.6)\rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_2 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFSCS in X, since $int(cl(f^{-1}(G_3^c))) = G_2 \not\subseteq f^{-1}(G_3^c)$. Therefore f is an IFRGS continuous mapping but not an IFS continuous mapping.

Theorem 4.7: Every IFG continuous mapping is an IFRGS continuous mapping but not conversely.

Proof: Let $f : (X,\tau) \to (Y,\sigma)$ be an IFS continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFGCS in X. Since every IFGCS is an IFRGSCS, by Theorem 2.9. $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFRGS continuous mapping.

Example 4.8: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$, $G_2 = \langle x, (0.2, 0.5), (0.8, 0.5)\rangle$ and $G_3 = \langle y, (0.7, 0.8), (0.2, 0.2)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.2, 0.2), (0.7, 0.8)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.2, 0.2), (0.7, 0.8)\rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an

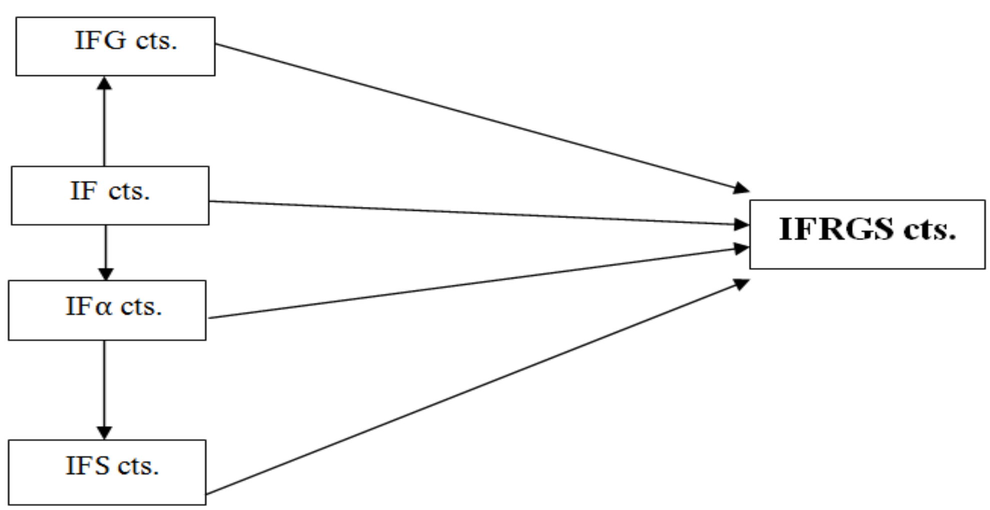
IFRGSCS in X but not an IFGCS in X, since $cl(f^{-1}(G_3^c))) = G_1^c \nsubseteq G_1$. Therefore f is an IFRGS continuous mapping but not an IFG continuous mapping.

Theorem 4.9: Every IFα continuous mapping is an IFRGS continuous mapping but not conversely.

Proof: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFα continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFαCS in X. Since every IFαCS is an IFRGSCS, by Theorem 2.11. $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFRGS continuous mapping.

Example 4.10: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.4, 0.5), (0.6, 0.5)\rangle$, $G_2 = \langle x, (0.3, 0.4), (0.7, 0.6)\rangle$ and $G_3 = \langle y, (0.6, 0.6), (0.4, 0.3)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.4, 0.3), (0.6, 0.6)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.4, 0.3), (0.6, 0.6)\rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFαCS in X, since $cl(int(cl(f^{-1}(G_3^c)))) = G_1^c \nsubseteq f^{-1}(G_3^c)$. Therefore f is an IFRGS continuous mapping but not an IFα continuous mapping.

The relation between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram 'cts' means continuous.



In the above diagram the reverse implications are not true in general.

Theorem 4.11: A mapping f : X → Y is an IFRGS continuous mapping if and only if the inverse image of each IFOS in Y is an IFRGSOS in X.

Proof: Necessity: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. Since f is an IFRGS continuous mapping, $f^{-1}(A^c)$ is an IFRGSCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRGSOS in X.

Sufficiency: Let A be an IFCS in Y. This implies A^c is an IFOS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, where $(f^{-1}(A))^c$ is an IFRGSOS in X. This implies $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFRGS continuous mapping.

Theorem 4.12: Let f : (X,τ) → (Y,σ) be a mapping and let $f^{-1}(A)$ be an IFRCS in X for every IFCS A in Y. Then f is an IFRGS continuous mapping.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFRCS in X, by hypothesis. Since every IFRCS is an IFRGSCS, by Theorem 2.5. $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFRGS continuous mapping.

Theorem 4.13: Let f : (X,τ) → (Y,σ) be an IFRGS continuous mapping, then

(i) f is an IF continuous mapping if X is an $IF_{rs}T^*_{1/2}$ space
(ii) f is an IFGS continuous mapping if X is an $IF_{rgs}T_{1/2}$ space
(iii) f is an IFS continuous mapping if X is an $IF_{rs}T_{1/2}$ space

Proof: (i) Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFRSGCS in X, by hypothesis. Since X is an $IF_{rs}T^*_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

(ii) Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFRGSCS in X, by hypothesis. Since X is an $IF_{rgs}T_{1/2}$ space, $f^{-1}(A)$ is an IFGSCS in X. Hence f is an IFGS continuous mapping.

(iii) Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFRGSCS in X, by hypothesis. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(A)$ is an IFSCS in X. Hence f is an IFS continuous mapping.

Theorem 4.14: Let f : (X,τ) → (Y,σ) be an IFRGS continuous mapping and g : (Y,σ) → (Z,γ) be an IF continuous mapping, then g∘f : (X,τ) → (Z,γ) is an IFRGS continuous mapping.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IFRGS continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSCS in X. Hence g∘f is an IFRGS continuous mapping.

Theorem 4.15: Let f : (X,τ) → (Y,σ) be an IFRGS continuous mapping and g : (Y,σ) → (Z,γ) is an IFG continuous mapping and Y is an $IFT_{1/2}$ space, then g∘f : (X,τ) → (Z,γ) is an IFRGS continuous mapping.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFGCS in Y, by hypothesis. Since Y is an $IFT_{1/2}$ space, $g^{-1}(A)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(A))$ is an IFRGSCS in X, by hypothesis. Hence g∘f is an IFRGS continuous mapping.

Theorem 4.16: Let f : (X,τ) → (Y,σ) be an IFRGS continuous mapping, then for each IFP $p_{(\alpha,\beta)}$ of X and each A ∊ σ such that $f(p_{(\alpha,\beta)})$ ∊ A, there exists an IFRGSOS B of X such that $p_{(\alpha,\beta)}$ ∊ B and f(B) ⊆ A.

Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and A ∊ σ such that $f(p_{(\alpha,\beta)})$ ∊ A. Put B = $f^{-1}(A)$. Then by hypothesis B is an IFRGSOS in X such that $p_{(\alpha,\beta)}$ ∊ B and f(B) = $f(f^{-1}(A))$ ⊆ A.

Theorem 4.17: Let f : (X,τ) → (Y,σ) be an IFRGS continuous mapping, then for each IFP $p_{(\alpha,\beta)}$ of X and each A ∊ σ such that $f(p_{(\alpha,\beta)})$ $_q$ A, there exists an IFRGSOS B of X such that $p_{(\alpha,\beta)}$ $_q$ B and f(B) ⊆ A.

Proof: Let $p_{(\alpha,\beta)}$ be an IFP of X and A ∊ σ such that $f(p_{(\alpha,\beta)})$ $_q$ A. Put B = $f^{-1}(A)$. Then by hypothesis B is an IFRGSOS in X such that $p_{(\alpha,\beta)}$ $_q$ B and f(B) = $f(f^{-1}(A))$ ⊆ A.

Theorem 4.18: Let f : (X,τ) → (Y,σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IF_{rs}T_{1/2}$ spaces:

(i) f is an IFRGS continuous mapping
(ii) $f^{-1}(B)$ is an IFRGSOS in X for each IFOS B in Y
(iii) for every IFP $p_{(\alpha,\beta)}$ in X and for every IFOS B inY such that $f(p_{(\alpha,\beta)})$ ∊ B, there exists an IFRGSOS A in X such that $p_{(\alpha,\beta)}$ ∊ A and f(A) ⊆ B.

Proof: (i) ⇒ (ii) is obvious from the Theorem 4.11.

(ii) ⇒ (iii) Let B be any IFOS in Y and let $p_{(\alpha,\beta)} \in X$. Given $f(p_{(\alpha,\beta)}) \in B$. By hypothesis $f^{-1}(B)$ is an IFRGSOS in X. Take $A = f^{-1}(B)$. Now $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)}))$. Therefore $f^{-1}((f(p_{(\alpha,\beta)}))) \subseteq f^{-1}(B) = A$. This implies $p_{(\alpha,\beta)} \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let A be an IFCS in Y. Then its complement, $A^c = B$ (say) is an IFOS in Y. Let $p_{(\alpha,\beta)} \in X$ and $f(p_{(\alpha,\beta)}) \in B$. Then there exists an IFRGSOS, say $C = f^{-1}(B)$ in X such that $p_{(\alpha,\beta)} \in C$ and $f(C) \subseteq B$. Therefore $f^{-1}(B)$ is an IFRGSOS in X. That is $f^{-1}(A^c)$ is an IFRGSOS in X and hence $f^{-1}(A)$ is an IFRGSCS in X as $f^{-1}(A^c) = (f^{-1}(A))^c$. Thus f is an IFRGS continuous mapping.

Theorem 4.19: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IF_{rs}T_{1/2}$ spaces:

(i) f is an IFRGS continuous mapping

(ii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFRGSOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$

(iii) for each IFP $p_{(\alpha,\beta)}$ in X and for every IFN A of $f(p_{(\alpha,\beta)})$, there exists an IFRGSOS B in X such that $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq A$

Proof: (i) ⇒ (ii) Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFOS C in Y such that $f(p_{(\alpha,\beta)}) \in C \subseteq A$. Since f is an IFRGS continuous mapping, $f^{-1}(C) = B$ (say), is an IFRGSOS in X and $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(C) \subseteq f^{-1}(A)$. Therefore $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in X$ and let A be an IFN of $f(p_{(\alpha,\beta)})$. Then there exists an IFRGSOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq f^{-1}(A)$, by hypothesis. Therefore $p_{(\alpha,\beta)} \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(iii) ⇒ (i) Let B be an IFOS in Y and let $p_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(p_{(\alpha,\beta)}) \in B$. Therefore B is an IFN of $f(p_{(\alpha,\beta)})$. Then by hypothesis there exists an IFRGSOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B)$ is an IFRGSOS in X, by Theorem 3.3. Hence f is an IFRGS continuous mapping.

Theorem 4.20: Let f : (X,τ) → (Y,σ) be an mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an $IF_{rs}T_{1/2}$ spaces:

(i) f is an IFRGS continuous mapping
(ii) $f^{-1}(B)$ is an IFRGSOS in X forevery IFOS B in Y
(iii) $f^{-1}(int(B)) \subseteq cl(int(f^{1}(B)))$ for every IFS B in Y

Proof: (i) ⇒ (ii) is obviously true by Theorem 4.11

(ii) ⇒ (iii) Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFRGSOS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(int(B))$ is an IFSOS in X. Then $f^{-1}(int(B)) \subseteq cl(int(f^{-1}(int(B)))) \subseteq cl(int(f^{-1}(B)))$ as $int(B) \subseteq B$.

(iii) ⇒ (i) Let B be an IFOS in Y. Then int(B) = B. By hypothesis $f^{-1}(B) \subseteq cl(int(f^{-1}(B)))$. This implies $f^{-1}(B)$ is an IFSOS in X. Therefore it is an IFRGSOS in X, by Theorem 3.3 and hence f is an IFRGS continuous mapping.

Theorem 4.21: Let f : (X,τ) → (Y,σ) be an mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an $IF_{rs}T_{1/2}$ spaces:

(i) f is an IFRGS continuous mapping
(ii) $f^{-1}(B)$ is an IFRGSCS in X for every IFCS B in Y
(iii) $int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y

Proof: (i) ⇒ (ii) is obvious from the Definition 4.1

(ii) ⇒ (iii) Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFRGSCS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(cl(A))$ is an IFSCS. Then $int(cl(f^{-1}(A))) \subseteq int(cl(f^{-1}(cl(A)))) \subseteq f^{-1}(cl(A))$. Thus $int(cl(f^{-1}(A))) \subseteq f^{-1}(cl(A))$.

(iii) ⇒ (i) Let A be an IFCS in Y. By hypothesis $int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFSCS in X and hence it is an IFRGSCS, by Theorem 2.7. Thus f is an IFRGS continuous mapping.

Definition 4.22: Let (X,τ) be an IFTS. The ***regular generalized semi closure*** (rgscl(A) in short) for any IFS A is defined as follows.

$rgscl(A) = \cap \{K / K$ is an IFRGSCS in X and $A \subseteq K\}$

It is to be noted that A is an IFRGSCS, then rgscl(A) = A.

Theorem 4.23: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFRGS continuous mapping. Then the following conditions hold:

(i) $f(rgscl(A)) \subseteq cl(f(A))$, for every IFS A in X.
(ii) $rgscl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B in Y.

Proof: (i) Let A be any IFS A in Y. Since cl(f(A)) is an IFCS in Y and f is an IFRGS continuous mapping, then $f^{-1}(cl(f(A)))$ is an IFRGSCS in X. That is rgscl(A) $\subseteq f^{-1}(cl(f(A)))$. Therefore $f(rgscl(A)) \subseteq f(f^{-1}(cl(f(A)))) \subseteq cl(f(A))$, for every IFS A in X.

(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(rgscl(f^{-1}(B))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. Hence $rgscl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B in Y.

CHAPTER V

CHAPTER V

INTUITIONISTIC FUZZY ALMOST REGULAR GENERALIZED SEMI CONTINUOUS MAPPINGS

In this section, we have introduced the notion of intuitionistic fuzzy almost regular generalized semi continuous mapping and studied some of their properties.

Definition 5.1: A mapping f : X → Y is said to be an ***intuitionistic fuzzy almost regular generalized semi continuous*** (IFaRGS continuous in short) ***mapping*** if $f^{-1}(A)$ is an IFRGSCS in X for every IFRCS A in Y.

Example 5.2: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.5, 0.6), (0.3, 0.3)\rangle$ where μ_a=0.5, μ_b=0.6, ν_a=0.3, ν_b=0.3 and $G_2 = \langle x, (0.3, 0.2), (0.7, 0.6)\rangle$ where μ_a=0.3, μ_b=0.2, ν_a=0.7, ν_b=0.6 and $G_3 = \langle y, (0.4,0.3), (0.4,0.5)\rangle$ where μ_u=0.4, μ_v=0.3, ν_u=0.4, ν_v=0.5. Then τ = {0~, G_1, G_2, 1~} and σ = {0~, G_3, 1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y,σ) by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.4, 0.5), (0.4, 0.3)\rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.4, 0.5), (0.4, 0.3)\rangle$ where μ_a=0.4, μ_b=0.5, ν_a=0.4, ν_b=0.3 is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X. Thus f is an IFaRGS continuous mapping.

Theorem 5.3: Every IF continuous mapping is an IFaRGS continuous mapping but not conversely.

Proof: Let f : (X,τ) → (Y,σ) be an IF continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X, by hypothesis. Since every IFCS is an IFRGSCS, by Theorem 2.3. $f^{-1}(V)$ is an IFRGSCS in X. Hence f is an IFaRGS continuous mapping.

Example 5.4: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.6, 0.4), (0.4, 0.2)\rangle$, $G_2 = \langle x, (0.1, 0.2), (0.8, 0.8)\rangle$, $G_3 = \langle y, (0.4, 0.3), (0.6, 0.3)\rangle$. Then τ = {0~, G_1, G_2, 1~} and σ = {0~, G_3, 1~} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y,σ) by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.6, 0.3), (0.4, 0.3)\rangle$ is an IFRCS in Y. Then

$f^{-1}(G_3^c) = \langle x, (0.6, 0.3), (0.4, 0.3)\rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFCS in X, since $cl(f^{-1}(G_3^c)) = G_1^c \neq f^{-1}(G_3^c)$. Therefore f is an IFaRGS continuous mapping but not an IF continuous mapping.

Theorem 5.5: Every IFG continuous mapping is an IFaRGS continuous mapping but not conversely.

Proof: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFG continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFGCS in X, by hypothesis. Since every IFGCS is an IFRGSCS, by Theorem 2.9. $f^{-1}(V)$ is an IFRGSCS in X. Hence f is an IFaRGS continuous mapping.

Example 5.6: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.6, 0.5), (0.2, 0.4)\rangle$, $G_2 = \langle x, (0.2, 0.4), (0.8, 0.5)\rangle$, $G_3 = \langle y, (0.4, 0.4), (0.5, 0.4)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.5, 0.4), (0.4, 0.4)\rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.5, 0.4), (0.4, 0.4)\rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFGCS in X, since $cl(f^{-1}(G_3^c)) = G_2^c \not\subseteq G_1$. Therefore f is an IFaRGS continuous mapping but not an IFG continuous mapping.

Theorem 5.7: Every IFS continuous mapping is an IFaRGS continuous mapping but not conversely.

Proof: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFS continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^1(V)$ is an IFSCS in X, by hypothesis. Since every IFSCS is an IFRGSCS, by Theorem 2.7. $f^{-1}(V)$ is an IFRGSCS in X. Hence f is an IFaRGS continuous mapping.

Example 5.8: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.4, 0.5), (0.3, 0.2)\rangle$ $G_2 = \langle x, (0.2,0.2), (0.8, 0.6)\rangle$, $G_3 = \langle y, (0.3, 0.4), (0.4, 0.4)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.4, 0.4), (0.3, 0.4)\rangle$ is an IFRCS in Y. Then

$f^{-1}(G_3^c) = \langle x, (0.4, 0.4), (0.3, 0.4)\rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFSCS in X, since $int(cl(f^{-1}(G_3^c))) = G_1 \nsubseteq f^{-1}(G_3^c)$. Therefore f is an IFaRGS continuous mapping but not an IFS continuous mapping.

Theorem 5.9: Every IFα continuous mapping is an IFaRGS continuous mapping but not conversely.

Proof: Let $f : (X,\tau) \to (Y,\sigma)$ be an IFα continuous mapping. Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFαCS in X, by hypothesis. Since every IFαCS is an IFRGSCS, by Theorem 2.11. We get, $f^{-1}(V)$ is an IFRGSCS in X. Hence f is an IFaRGS continuous mapping.

Example 5.10: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.3, 0.2)\rangle$ $G_2 = \langle x, (0.3, 0.2), (0.6, 0.5)\rangle$, $G_3 = \langle y, (0.4, 0.4), (0.5, 0.4)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3^c = \langle y, (0.5, 0.4), (0.4, 0.4)\rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.5, 0.4), (0.4, 0.4)\rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFαCS in X, since $cl(int(cl(f^{-1}(G_3^c))) = G_2^c \nsubseteq f^{-1}(G_3^c)$. Therefore f is an IFaRGS continuous mapping but not an IFα continuous mapping.

Theorem 5.11: A mapping $f : X \to Y$ is an IFaRGS continuous mapping if and only if the inverse image of each IFROS in Y is an IFRGSOS in X.

Proof: Necessity: Let A be an IFROS in Y. This implies A^c is an IFRCS in Y. Since f is an IFaRGS continuous mapping, $f^{-1}(A^c)$ is an IFRGSCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRGSOS in X.

Sufficiency: Let A be an IFRCS in Y. This implies A^c is an IFROS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, where $(f^{-1}(A))^c$ is an IFRGSOS in X. This implies $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFaRGS continuous mapping.

Theorem 5.12: Let $f : X \to Y$ be a mapping where $f^{-1}(V)$ is an IFRCS in X for every IFCS in Y. Then f is an IFaRGS continuous mapping but not conversely.

Proof: Let V be an IFRCS in Y. Since every IFRCS is an IFCS, V is an IFCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IFRGSCS, by Theorem 2.5. $f^{-1}(V)$ is an IFRGSCS in X. Hence f is an IFaRGS continuous mapping.

Example 5.13: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.5, 0.6), (0.3, 0.4)\rangle$ $G_2 = \langle x, (0.3, 0.2), (0.6, 0.6)\rangle$, $G_3 = \langle y, (0.4, 0.4), (0.4, 0.5)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3^c = \langle y, (0.4, 0.5), (0.4, 0.4)\rangle$ is an IFRCS in Y. Then $f^{-1}(G_3^c) = \langle x, (0.4, 0.5), (0.4, 0.4)\rangle$ is an IFS in X. Then $f^{-1}(G_3^c) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3^c)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3^c)$ is an IFRGSCS in X but not an IFRCS in X, since $cl(int(f^{-1}(G_3^c))) = G_2^c \neq f^{-1}(G_3^c)$. Therefore f is an IFaRGS continuous mapping, but not the mappings as in Theorem 5.12.

Theorem 5.14: Let $f : X \to Y$ be a mapping. If $f^{-1}(sint(B)) \subseteq sint(f^{-1}(B))$ for every IFS B in Y, then f is an IFaRGS continuous mapping.

Proof: Let B be an IFROS in Y. By hypothesis, $f^{-1}(sint(B)) \subseteq sint(f^{-1}(B))$. Since B is an IFROS, it is an IFSOS in Y. Therefore sint(B) = B. Hence $f^{-1}(B) = f^{-1}(sint(B)) \subseteq sint(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B) = sint(f^{-1}(B))$. This implies $f^{-1}(B)$ is an IFSOS in X and hence $f^{-1}(B)$ is an IFRGSOS in X, by Theorem 3.3. Thus f is an IFaRGS continuous mapping.

Remark 5.15: The converse of the above Theorem 5.14 is true if B is an IFROS in Y and X is an $IF_{rs}T_{1/2}$ space.

Proof: Let f be an IFaRGS continuous mapping. Let B be an IFROS in Y. Then $f^{-1}(B)$ is an IFRGSOS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(B)$ is an IFSOS in X. This implies $f^{-1}(B) = sint(f^{-1}(B))$. Now $f^{-1}(sint(B)) = sint(f^{-1}(sint(B))) \subseteq sint(f^{-1}(B))$. Therefore $f^{-1}(sint(B)) \subseteq sint(f^{-1}(B))$.

Theorem 5.16: Let $f : X \to Y$ be a mapping. If $scl(f^{-1}(B)) \subseteq f^{-1}(scl(B))$ for every IFS B in Y, then f is an IFaRGS continuous mapping.

Proof: Let B be an IFRCS in Y. By hypothesis, $scl(f^{-1}(B)) \subseteq f^{-1}(scl(B))$. Since B is an IFRCS, it is an IFSCS in Y. Therefore $scl(B) = B$. Hence $f^{-1}(B) = f^{-1}(scl(B)) \supseteq scl(f^{-1}(B)) \supseteq f^{-1}(B)$. Therefore $f^{-1}(B) = scl(f^{-1}(B))$. This implies $f^{-1}(B)$ is an IFSCS in X and hence $f^{-1}(B)$ is an IFRGSCS in X, by Theorem 2.7. Thus f is an IFaRGS continuous mapping.

Remark 5.17: The converse of the above Theorem 5.16 is true if B is an IFRCS in Y and X is an $IF_{rs}T_{1/2}$ space.

Proof: Let f be an IFaRGS continuous mapping. Let B be an IFRCS in Y. Then $f^{-1}(B)$ is an IFRGSCS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(B)$ is an IFSCS in X. This implies $scl(f^{-1}(B)) = f^{-1}(B)$. Now $f^{-1}(scl(B)) = scl(f^{-1}(scl(B))) \supseteq scl(f^{-1}(B))$. Therefore $f^{-1}(scl(B)) \supseteq scl(f^{-1}(B))$.

Theorem 5.18: Let $f : X \to Y$ be a mapping where X is an $IF_{rs}T_{1/2}$ space. If f is an IFaRGS continuous mapping, then $int(cl(f^{-1}(B))) \subseteq f^{-1}(scl(B))$ for every IFRCS B in Y.

Proof: Let B be an IFRCS in Y. By hypothesis, $f^{-1}(B)$ is an IFRGSCS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(B)$ is an IFSCS in X. This implies $scl(f^{-1}(B)) = f^{-1}(B)$. Now $int(cl(f^{-1}(B))) \subseteq f^{-1}(B) \cup int(cl(f^{-1}(B))) \subseteq scl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(scl(B))$, as every IFRCS is an IFSCS. Hence $int(cl(f^{-1}(B))) \subseteq f^{-1}(scl(B))$.

Theorem 5.19: Let $f : X \to Y$ be a mapping where X is an $IF_{rs}T_{1/2}$ space. If f is an IFaRGS continuous mapping, then $f^{-1}(sint(B)) \subseteq cl(int(f^{-1}(B)))$ for every IFROS B in Y.

Proof: This theorem can be easily proved by taking complement in Theorem 5.18.

CHAPTER VI

CHAPTER VI

INTUITIONISTIC FUZZY CONTRA REGULAR GENERALIZED SEMI CONTINUOUS MAPPINGS

In this section, we have introduced the notion of intuitionistic fuzzy contra regular generalized semi continuous mappings and studied some of their properties.

Definition 6.1: A mapping $f : X \to Y$ is said to be an ***intuitionistic fuzzy contra regular generalized semi continuous*** (IFCRGS continuous in short) ***mapping*** if $f^{-1}(A)$ is an IFRGSCS in (X,τ) for every IFOS A in (Y,σ).

Example 6.2: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.4, 0.6), (0.3, 0.4)\rangle$ where μ_a=0.4, μ_b=0.6, ν_a=0.3, ν_b=0.4 and $G_2 = \langle x, (0.2, 0.4), (0.7, 0.6)\rangle$ where μ_a=0.2, μ_b=0.4, ν_a=0.7, ν_b=0.6 and $G_3 = \langle y, (0.3, 0.5), (0.5, 0.4)\rangle$ where μ_u=0.3, μ_v=0.5, ν_u=0.5, ν_v=0.4. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3 = \langle y, (0.3, 0.5), (0.5, 0.4)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.3, 0.5), (0.5, 0.4)\rangle$ where μ_a=0.3, μ_b=0.5, ν_a=0.5, ν_b=0.4 is an IFS in X. Then $f^{-1}(G_3) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3)$ is an IFRGSCS in X. Thus f is an IFCRGS continuous mapping.

Theorem 6.3: Every IFC continuous mapping is an IFCRGS continuous mapping but not conversely.

Proof: Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFCS in X, by hypothesis. Since every IFCS is an IFRGSCS, by Theorem 2.3. $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFCRGS continuous mapping.

Example 6.4: Let $X = \{a,b\}$, $Y = \{u,v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.4, 0.2)\rangle$ $G_2 = \langle x, (0.1, 0.2), (0.9, 0.7)\rangle$ and $G_3 = \langle y, (0.5, 0.3), (0.4, 0.2)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $G_3 = \langle y, (0.5, 0.3), (0.4, 0.2)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.5, 0.3), (0.4, 0.2)\rangle$ is an IFS in X. Then $f^{-1}(G_3) \subseteq G_1$

where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3)$ is an IFRGSCS in X but not an IFCS in X, since $cl(f^{-1}(G_3)) = G_2^c \neq f^{-1}(G_3)$. Therefore f is an IFCRGS continuous mapping but not an IFC continuous mapping.

Theorem 6.5: Every IFCα continuous mapping is an IFCRGS continuous mapping but not conversely.

Proof: Let A be an IFOS in Y. Then $f^{-1}(A)$ is an IFCS in X, by hypothesis. Since every IFαCS is an IFRGSCS, by Theorem 2.11. $f^{-1}(A)$ is an IFRGSCS in X. Therefore f is an IFCRGS continuous mapping.

Example 6.6: Let X = {a,b}, Y = {u,v} and $G_1 = \langle x, (0.5, 0.5), (0.5, 0.4)\rangle$ $G_2 = \langle x, (0.4, 0.3), (0.5, 0.5)\rangle$ and $G_3 = \langle y, (0.3, 0.2), (0.5, 0.5)\rangle$. Then $\tau = \{0\sim, G_1, G_2, 1\sim\}$ and $\sigma = \{0\sim, G_3, 1\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \to (Y,\sigma)$ by f(a) = u and f(b) = v. The IFS $G_3 = \langle y, (0.3, 0.2), (0.5, 0.5)\rangle$ is an IFOS in Y. Then $f^{-1}(G_3) = \langle x, (0.3, 0.2), (0.5, 0.5)\rangle$ is an IFS in X. Then $f^{-1}(G_3) \subseteq G_1$ where G_1 is an IFROS in X. Now $scl(f^{-1}(G_3)) = G_1 \subseteq G_1$. Therefore $f^{-1}(G_3)$ is an IFRGSCS in X but not an IFαCS in X, since $cl(int(cl(f^{-1}(G_3)))) = G_2^c \not\subseteq f^{-1}(G_1)$. Therefore f is an IFCRGS continuous mapping but not an IFCα continuous mapping.

Theorem 6.7: A mapping $f : X \to Y$ is an IFCRGS continuous mapping if and only if the inverse image of each IFCS in Y is an IFRGSOS in X.

Proof: Necessity: Let A be an IFCS in Y. This implies A^c is an IFOS in Y. Since f is an IFCRGS continuous mapping, $f^{-1}(A^c)$ is an IFRGSCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRGSOS in X.

Sufficiency: Let A be an IFOS in Y. This implies A^c is an IFCS in Y. By hypothesis, $f^{-1}(A^c)$ is an IFRGSOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, where $(f^{-1}(A))^c$ is an IFRGSOS in X, $f^{-1}(A)$ is an IFRGSCS in X. Hence f is an IFCRGS continuous mapping.

Theorem 6.8: Let $f : (X,\tau) \to (Y,\sigma)$ be a mapping and let $f^{-1}(A)$ be an IFROS in X for every IFCS A in Y. Then f is an IFCRGS continuous mapping.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFROS in X, by hypothesis. Since every IFROS is an IFRGSOS, by Theorem 3.3. $f^{-1}(A)$ is an IFRGSOS in X. Hence f is an IFCRGS continuous mapping.

Theorem 6.9: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFCRGS continuous mapping and $g : (Y,\sigma) \rightarrow (Z,\gamma)$ be an IF continuous mapping, then $g \circ f : (X,\tau) \rightarrow (Z,\gamma)$ is an IFCRGS continuous mapping.

Proof: Let A be an IFOS in Z. Then $g^{-1}(A)$ is an IFOS in Y, by hypothesis. Since f is an IFCRGS continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSCS in X. Hence $g \circ f$ is an IFCRGS continuous mapping.

Theorem 6.10: Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFCRGS continuous mapping and $g : (Y,\sigma) \rightarrow (Z,\gamma)$ be an IFG continuous mapping and Y is an $IFT_{1/2}$ space, then $g \circ f : (X,\tau) \rightarrow (Z,\gamma)$ is an IFCRGS continuous mapping.

Proof: Let A be an IFOS in Z. Then $g^{-1}(A)$ is an IFGOS in Y, by hypothesis. Since Y is an $IFT_{1/2}$ space, $g^{-1}(A)$ is an IFOS in Y. Therefore $f^{-1}(g^{-1}(A))$ is an IFRGSCS in X, by hypothesis. Hence $g \circ f$ is an IFCRGS continuous mapping.

Theorem 6.11: Let $f : X \rightarrow Y$ be a mapping. Suppose that one of the following properties hold:

(i) $f(scl(A)) \subseteq int(f(A))$ for each IFS A in X
(ii) $scl(f^{-1}(B)) \subseteq f^{-1}(int(B))$ for each IFS B in Y
(iii) $f^{-1}(cl(B)) \subseteq sint(f^{-1}(B))$ for each IFS B in Y

Then f is an IFCRGS continuous mapping.

Proof: (i) ⇒ (ii) Let B be an IFS in Y. Then $f^{-1}(B)$ is an IFS in X. By hypothesis, $f(scl(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. Now $scl(f^{-1}(B)) \subseteq f^{-1}(f(scl(f^{-1}(B)))) \subseteq f^{-1}(int(B))$

(ii) ⇒ (iii) is obvious by taking complement in (ii).

Suppose (iii) holds: Let A be an IFCS in Y. Then cl(A) = A and $f^{-1}(A)$ is an IFS in X. Now $f^{-1}(A) = f^{-1}(cl(A)) \subseteq sint(f^{-1}(A)) \subseteq f^{-1}(A)$, by hypothesis. This implies $f^{-1}(A)$ is an

IFSOS in X and hence an IFRGSOS in X, by Theorem 3.3. Thus f is an IFCRGS continuous mapping.

Theorem 6.12: Let $f : X \to Y$ be a bijective mapping. Then f is an IFCRGS continuous mapping if $cl(f(A)) \subseteq f(sint(A))$ for every IFS A in X.

Proof: Let A be an IFCS in Y. Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X. By hypothesis $cl(f(f^{-1}(A))) \subseteq f(sint(f^{-1}(A)))$. Since f is an onto, $f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl(f(f^{-1}(A))) \subseteq f(sint(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(sint(f^{-1}(A)))) = sint(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFSOS in X and hence an IFRGSOS in X, by Theorem 3.3. Thus f is an IFCRGS continuous mapping.

Theorem 6.13: If $f : X \to Y$ is an IFCRGS continuous mapping, where X is an $IF_{rs}T_{1/2}$ space, then the following conditions hold:

(i) $scl(f^{-1}(B)) \subseteq f^{-1}(int(scl(B)))$ for every IFOS B in Y

(ii) $f^{-1}(cl(sint(B))) \subseteq sint(f^{-1}(B))$ for every IFCS B in Y

Proof: (i) Let B be an IFOS in Y. By hypothesis $f^{-1}(B)$ is an IFRGSCS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(B)$ is an IFSCS in X. This implies $scl(f^{-1}(B)) = f^{-1}(B) = f^{-1}(int(B)) \subseteq f^{-1}(int(scl(B)))$.

(ii) can be proved easily by taking the complement of (i).

Theorem 6.14: If $f : X \to Y$ is a mapping where X is an $IF_{rs}T_{1/2}$ space, then the following are equivalent:

(i) f is an IFCRGS continuous mapping

(ii) for each IFP $p_{(\alpha,\beta)} \in X$ and for each IFCS B containing $f(p_{(\alpha,\beta)})$, there exists an IFSOS $A \subseteq X$ and $p_{(\alpha,\beta)} \in A$ such that $A \subseteq f^{-1}(B)$

(iii) for each IFP $p_{(\alpha,\beta)} \in X$ and for each IFCS B containing $f(p_{(\alpha,\beta)})$, there exists an IFSOS $A \subseteq X$ and $p_{(\alpha,\beta)} \in A$ such that $f(A) \subseteq B$

Proof: (i) $\Rightarrow$ (ii) Let B be an IFCS in Y. Let $p_{(\alpha,\beta)}$ be an IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is an IFRGSOS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(B)$ is an IFSOS in X. Now let $A = sint(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $A \subseteq f^{-1}(B)$.

(ii) ⇒ (iii) Let B be IFCS in Y. Let $p_{(\alpha,\beta)}$ be an IFP in X such that $f(p_{(\alpha,\beta)}) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(f(p_{(\alpha,\beta)})) \in f^{-1}(B)$. By hypothesis $f^{-1}(B)$ is an IFSOS in X and $A \subseteq f^{-1}(B)$. This implies $f(A) \subseteq f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let B be any IFCS in Y and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)}) \in B$. By hypothesis there exists an IFSOS A in X such that $p_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$. This implies $p_{(\alpha,\beta)} \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. That is $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since A is an IFSOS, $A = sint(A) \subseteq sint(f^{-1}(B))$. Therefore $p_{(\alpha,\beta)} \in sint(f^{-1}(B))$. But $f^{-1}(B) = \cup_{p_{(\alpha,\beta)} \in f^{-1}(B)} p_{(\alpha,\beta)} \subseteq sint(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is an IFSOS in X and hence $f^{-1}(B)$ is an IFRGSOS in X, by Theorem 3.3. Thus f is an IFCRGS continuous mapping.

Theorem 6.15: For a mapping $f : X \to Y$, the following are equivalent, where X is an $IF_{rs}T_{1/2}$ space:

(i) f is an IFCRGS continuous mapping
(ii) for every IFCS A in Y, $f^{-1}(A)$ is an IFRGSOS in X
(iii) for every IFOS B in Y, $f^{-1}(B)$ is an IFRGSCS in X
(iv) for any IFCS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$, then $p_{(\alpha,\beta)}\ _q\ sint(f^{-1}(A))$.
(v) for any IFCS A in Y and for any IFP $p_{(\alpha,\beta)} \in X$, if $f(p_{(\alpha,\beta)})\ _q\ A$, then there exists an IFRGSOS B such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$

Proof: (i) ⇔ (ii) and (ii) ⇔ (iii) are obvious.

(ii) ⇒ (iv) Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. By hypothesis, $f^{-1}(A)$ is an IFRGSOS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(A)$ is an IFSOS in X. This implies $sint(f^{-1}(A)) = f^{-1}(A)$. Hence $p_{(\alpha,\beta)}\ _q\ sint(f^{-1}(A))$.

(iv) ⇒ (ii) Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. By hypothesis, $p_{(\alpha,\beta)}\ _q\ sint(f^{-1}(A))$. That is $f^{-1}(A) \subseteq sint(f^{-1}(A))$. But $sint(f^{-1}(A)) \subseteq f^{-1}(A)$.

Therefore $f^{-1}(A) = \mathrm{sint}(f^{-1}(A))$. Thus $f^{-1}(A)$ is an IFSOS in X and hence $f^{-1}(A)$ is an IFRGSOS in X, by Theorem 3.3.

(iv) ⇒ (v) Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. By hypothesis, $p_{(\alpha,\beta)}\ _q\ \mathrm{sint}(f^{-1}(A))$. Thus $f^{-1}(A)$ is an IFSOS in X and hence $f^{-1}(A)$ is an IFRGSOS in X, by Theorem 3.3. Let $f^{-1}(A) = B$. Therefore $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

(v) ⇒ (iv) Let $A \subseteq Y$ be an IFCS and let $p_{(\alpha,\beta)} \in X$. Let $f(p_{(\alpha,\beta)})\ _q\ A$. Then $p_{(\alpha,\beta)}\ _q\ f^{-1}(A)$. By hypothesis, there exists an IFRGSOS B in X such that $p_{(\alpha,\beta)}\ _q\ B$ and $f(B) \subseteq A$. Let $B = f^{-1}(A)$. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(A)$ is an IFSOS in X. Therefore $p_{(\alpha,\beta)}\ _q\ \mathrm{sint}(f^{-1}(A))$.

Theorem 6.16: A mapping $f : X \to Y$ is an IFCRGS continuous mapping if $f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{int}(f^{-1}(B))$ for every IFS B in Y.

Proof: Let $B \subseteq Y$ be an IFCS. Then $\mathrm{cl}(B) = B$. Since every IFCS is an IFSCS [6], $\mathrm{scl}(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{int}(f^{-1}(B)) \subseteq (f^{-1}(B)$. This implies $f^{-1}(B) = \mathrm{int}(f^{-1}(B))$. Therefore $f^{-1}(B)$ is an IFOS in X. Therefore f is an IFC continuous mapping. Then by Theorem 3.3, f is an IFCRGS continuous mapping.

Theorem 6.17: A mapping $f : X \to Y$ is an IFCRGS continuous mapping, where X is an $IF_{rs}T_{1/2}$ space if and only if $f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{sint}(f^{-1}(\mathrm{cl}(B)))$ for every IFS B in Y.

Proof: Necessity: Let $B \subseteq Y$ be an IFS. Then cl(B) is an IFCS in Y. By hypothesis $f^{-1}(\mathrm{cl}(B))$ is an IFRGSOS in X. Since X is an $IF_{rs}T_{1/2}$ space, $f^{-1}(\mathrm{cl}(B))$ is an IFSOS in X. This implies $f^{-1}(\mathrm{cl}(B)) = \mathrm{sint}(f^{-1}(\mathrm{cl}(B)))$. Therefore $f^{-1}(\mathrm{scl}(B)) \subseteq f^{-1}(\mathrm{cl}(B)) = \mathrm{sint}(f^{-1}(\mathrm{cl}(B)))$.

Sufficiency: Let $B \subseteq Y$ be an IFS. Then cl(B) is an IFCS in Y. By hypothesis, $f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{sint}(f^{-1}(\mathrm{cl}(B))) = \mathrm{sint}(f^{-1}(B))$. But $\mathrm{scl}(B) = B$. Therefore $f^{-1}(B) = f^{-1}(\mathrm{scl}(B)) \subseteq \mathrm{sint}(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFSOS in X and hence $f^{-1}(B)$ is an IFRGSOS in X, by Theorem 3.3. Hence f is an IFCRGS continuous mapping.

Theorem 6.18: An IF continuous mapping $f : X \rightarrow Y$ is an IFCRGS continuous mapping if IFRGSO(X) = IFRGSC(X).

Proof: Let $A \subseteq Y$ be an IFOS. By hypothesis, $f^{-1}(A)$ is an IFOS in X and hence $f^{-1}(A)$ is an IFRGSOS in X, by Theorem 3.3. Thus $f^{-1}(A)$ is an IFRGSCS in X, as IFRGSO(X) = IFRGSC(X). Therefore f is an IFCRGS continuous mapping.

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

In this thesis we have studied the concept of intuitionistic fuzzy regular generalized semiclosed sets, intuitionistic fuzzy regular generalized semiopen sets, intuitionistic fuzzy regular generalized semi continuous mappings, intuitionistic fuzzy almost regular generalized semi continuous mappings and intuitionistic fuzzy contra regular generalized semi continuous mappings. We have discussed many interesting theorems on them.

In chapter I we have discussed basic definition and concepts. In chapter II we have discussed intuitionistic fuzzy regular generalized semiclosed sets. In chapter III we have discussed intuitionistic fuzzy regular generalized semiopen sets. In chapter IV we have discussed intuitionistic fuzzy regular generalized semi continuous mappings. In chapter V we have discussed intuitionistic fuzzy almost regular generalized semi continuous mappings. In chapter VI we have discussed intuitionistic fuzzy contra regular generalized semi continuous mappings.

Thus we conclude that an intuitionistic fuzzy regular generalized semiclosed sets does not imply intuitionistic fuzzy closed sets, intuitionistic fuzzy semiclosed sets, etc in general and we have proved them by suitable examples. Also we have enjoyed by proving some interesting theorems based on intuitionistic fuzzy regular generalized semiclosed sets and intuitionistic fuzzy regular generalized semi continuous mappings.

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] **Atanassov, K.T,** Intuitionistic fuzzy sets, Fuzzy sets and systems, (1986), 87-96.

[2] **Chang, C.L.,** Fuzzy Topological Spaces , J.Math. Anal. Appl. 24, (1968), 182-190.

[3] **Coker, D.,** An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems, (1997), 81-89.

[4] **Coker, D.,** and **Demirci, M.,** On Intuitionistic fuzzy points, Notes on Intuitionistic Fuzzy Sets (1995), 79-84.

[5] **Gurcay, H., Coker, D.,** and **Haydar, Es. A.,** On fuzzy continuity in intuitionistic fuzzy topological spaces, The Jour. of Fuzzy Math., (1997), 365-378.

[6] **Joung kon Jeon, Young Bae Jun** and **Jin Han Park.,** Intuitionistic fuzzy alpha continuity and Intuitionistic fuzzy pre continuity, International Journal of Mathematics and Mathematical Sciences, (2005), 3091-3101.

[7] **Krsteska, B.,** and **E. Ekici,** Intuitionistic fuzzy contra strong precontinuity, Faculty of Sciences and Mathematics, University of Nis, Siberia, (2007), 273–284.

[8] **Sakthivel, K.,** Intuitionistic Fuzzy Alpha Generalized Continuous Mappings and Intuitionistic Fuzzy Alpha Generalized Irresolute Mappings, Applied Mathematical Sciences, (2010), 1831 – 1842.

[9] **Santhi, R.,** and **Arun prakash, K.,** On Intuitionistic Fuzzy Semi Generalized Closed set and its Applications, Int.J.Contemp.Math. Sciences., (2010), 1677-1688.

[10] **Seok Jong Lee** and **Pyo Lee,** The category of Intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc. (2000), 63-76.

[11] **Thakur, S. S.,** and **Rekha Chaturvedi.,** Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si Cercetari Stiintifice, Seria; Mathematica, (2006), 257-272,

[12] **Young Bae Jun** and **Seok- Zun Song,** Intuitionistic fuzzy semi-pre open sets and Intuitionistic semi-pre continuous mappings, Jour. of Appl. Math and computing, (2005), 467-474.

[13] **Zadeh, L.A.,** Fuzzy Sets, Information and Control, (1965), 338-353.

LIST OF PUBLICATIONS

PUBLICATIONS

[1] **Anitha, R.,** and **Jayanthi, D.,** On Intuitionistic Fuzzy Regular Generalized Semiclosed sets, International Journal of Advance Foundation and Research In Science & Engineering, Vol 1, Issue 9, 38-42, 2015.

[2] **Anitha, R.,** and **Jayanthi, D.,** Regular Generalized Semiopen sets in Intuitionistic Fuzzy Topological Spaces, (To appear).

[3] **Anitha, R.,** and **Jayanthi, D.,** On intuitionistic fuzzy Regular Generalized semi continuous mappings, International Journal of Science and Research, Vol 4, Issue 3, 1022 – 1026, 2015.

[4] **Anitha, R.,** and **Jayanthi, D.,** On intuitionistic fuzzy contra Regular Generalized semi continuous mappings, (Submitted).

[5] **Anitha, R.,** and **Jayanthi, D.,** On intuitionistic fuzzy almost Regular Generalized semi continuous mappings, International Journal of Engineering Science & Management Research, Vol 2, Issue 4, 11-17, 2015.