CHAPTER 6

CHAPTER – 6

INTUITIONISTIC FUZZY ALMOST SEMI- GENERALIZED CLOSED SETS

In this chapter intuitionistic fuzzy almost semi- Generalized closed sets, intuitionistic fuzzy almost semi-generalized closed mappings due to Shanthi et al. [62] are studied. Properties, characterization and implications of these sets with other sets are discussed.

Section 6.1

Preliminary Definitions on Intuitionistic Fuzzy Almost Semi-Generalized Closed Sets

Definition: 6.1.1

Let $\alpha, \beta \in [0, 1]$ with $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP), written as $\rho_{(\alpha,\beta)}$ is defined to be an IFS(X) given by

$$\rho_{(\alpha,\beta)} = \begin{cases} (\alpha,\beta) & \text{if } x=\rho \\ (0, 1) & \text{otherwise} \end{cases}$$

Definition: 6.1.2

Let A= {(x, μ_A(x), γ_A (x)) / x∈X} be a IFS in an IFTS (X, T) is called intuitionistic fuzzy semi open set (IFSOS) if A ⊆ cl(int(A)).

Definition: 6.1.3

Let A= {(x, μ_A(x), γ_A (x)) / x∈X} be a IFS in an IFTS (X, T) is called intuitionistic fuzzy α-open set (IFαOS) if A ⊆ int(cl(int(A))).

Definition: 6.1.4

Let A= {(x, μ_A(x), γ_A (x)) / x∈X} be a IFS in an IFTS (X, T) is called intuitionistic fuzzy pre open set (IFPOS) if A ⊆ int(cl((A)).

Definition: 6.1.5

Let A= {(x, μ_A(x), γ_A (x)) / x∈X} be a IFS in an IFTS (X, T) is called intuitionistic fuzzy regular open set (IFROS) if int(cl(A))=A.

Definition: 6.1.6

Let A= {(x, μ_A(x), γ_A (x)) / x∈X} be a IFS in an IFTS (X, T) is called intuitionistic fuzzy semi- pre open set (IFSPOS) if there exists B ∈ IFPO(X) such that B ⊆ A ⊆ cl(B).

An IFS A is called an intuitionistic fuzzy semi closed set, intuitionistic fuzzy α- closed set, intuitionistic fuzzy preclosed set, intuitionistic fuzzy regular closed set and intuitionistic fuzzy semi- preclosed set, respectively (IFSCS,IFαCS,IFPCS, IFRCS and IFSPCS respectively), if the complement $\overline{A}$ is an IFSCS,IFαCS,IFPCS, IFRCS and IFSPCS respectively. The family of all intuitionistic fuzzy semi open (respectively intuitionistic fuzzy α- open, intuitionistic fuzzy preopen, intuitionistic fuzzy regular open and intuitionistic fuzzy semipreopen) sets of an IFTS(X,T) is denoted by IFSO(X) (respectively IF α (X), IFPO(X), IFRO(X) and IFSPO(X)).

Definition: 6.1.7

An IFS A of an IFTS(X,T) is called intuitionistic fuzzy semi- generalized closed (intuitionistic fuzzy sg- closed) set (IFSGCS) if scl(A)⊆U, whenever A ⊆ U and U is an IFSOS.

The complement $\overline{A}$ of an intuitionistic fuzzy semi- generalized closed set A is called an intuitionistic fuzzy semi- generalized open (intuitionistic fuzzy sg- open) set (IFSGOS).

Definition: 6.1.8

An IFTS(X, T) is said to be an intuitionistic fuzzy semi- $T_{1/2}$ space if every intuitionistic fuzzy sg- closed set in X is an intuitionistic fuzzy semi- closed in X.

Definition: 6.1.9

A mapping f: (X, τ) →(Y, κ) from an IFTS (X, τ) into an IFTS (Y, κ) is said to be an intuitionistic fuzzy closed mapping if f (A) is an IFCS in Y, for every IFCS A in X.

Definition: 6.1.10

A mapping f: $(X,\tau) \to (Y,\kappa)$ from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy semi closed mapping if f(A) is an IFSCS in Y, for every IFCS A in X.

Definition: 6.1.11

A mapping f: $(X,\tau) \to (Y,\kappa)$ from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy pre- closed mapping if f (A) is an IFPCS in Y, for every IFCS A in X.

Definition: 6.1.12

A mapping f: $(X,\tau) \to (Y,\kappa)$ from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy α- closed mapping if f (A) is an IFαCS in Y, for every IFCS A in X.

Definition: 6.1.13

A mapping f: $(X,\tau) \to (Y,\kappa)$ from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy sg- closed mapping if f (A) is an IFSGCS in Y, foe every IFCS in X.

Definition: 6.1.14

A mapping f: $(X,\tau) \to (Y,\kappa)$ from an IFTS (X,τ) into an IFTS (Y,κ) is said to be an intuitionistic fuzzy sg^*- closed mapping if f (A) is an IFSGCS in Y, for every IFSGCS A in X.

Definition: 6.1.15

A mapping f: X $\to$ Y from an IFTS X into an IFTS Y is called an intuitionistic fuzzy almost sg- continuous mapping if f^{-1}(B) is an IFSGCS in X, for each IFRCS B in X.

Definition: 6.1.16

A mapping f: X → Y from an IFTS X into an IFTS Y is called an intuitionistic fuzzy quasi- sg- closed mapping if f (B) is an IFCS in Y, for each IFSGCS B in X.

Section 6.2

Intuitionistic Fuzzy Almost Semi-Generalized Closed Mappings

In this section characterization of intuitioinistic fuzzy almost semi-generalized closed mappings and intuitioinistic fuzzy almost semi-generalized open mappings are studied.

Definition: 6.2.1

A mapping f: X → Y is said to be an intuitionistic fuzzy almost semi-generalized closed (intuitionistic fuzzy almost sg-closed) mapping if f (A) is an IFSGCS in Y for every IFRCS A in X.

Example: 6.2.2 Let X = {a, b}, Y = {u, v). Let

$$A = < x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.1}, \frac{b}{0.3}\right) >,$$

$$B = < y, \left(\frac{u}{0.4}, \frac{v}{0.3}\right), \left(\frac{u}{0.6}, \frac{v}{0.7}\right) >,$$

Then τ = {0~, 1~, A} and k = {0~, 1~, B} are IFTSs on X and Y respectively. Define a mapping f: (X,τ) → (Y, k) by f (a) = u, f (b) = v. Clearly 0~, 1~ are the only IFRCS in X. Now f (0~) = 0~ and f (1~) are IFSGCS in an IFSGCS. Therefore f (B) is an IFSGCS in Y. Hence f is an intutionistic fuzzy almost sg-closed mapping.

Theorem: 6.2.3

Every intuitionistic fuzzy closed mapping is an intuitionistic fuzzy almost sg-closed mapping.

Proof

Let f: X → Y be an intuitionistic fuzzy closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFCS, B is an IFCS in X. By our assumption f (B) is an IFCS in Y. By the known result every IFCS is an

IFSGCS. Therefore f(B) is an IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-mapping.

The converse of the above theorem is not true as seen from the following example.

Example: 6.2.4

Let X = {a,b}, Y = {u,v}. Let

$$A = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}\right), \left(\frac{a}{0.4}, \frac{b}{0.3}\right) \right\rangle,$$

$$B = \left\langle y, \left(\frac{u}{0.3}, \frac{v}{0.1}\right), \left(\frac{u}{0.5}, \frac{v}{0.7}\right) \right\rangle$$

Then τ = {0~, 1~, A} and k = {0~, 1~, B} are IFTSs on X and Y respectively. Define a mapping f: (X, τ) → (Y, k) by f(a) = u, f (b) = v. Clearly 0~, 1~ are IFSGCS in X. Now f (0~) = 0~ and f (1~) = 1~ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. But f (A) is not an IFCS in Y, where A is an IFCS in X. Therefore f is not an intuitionistic fuzzy closed mapping.

Theorem: 6.2.5

Every intuitionistic fuzzy semi-closed mapping is an intuitionistic fuzzy almost sg-closed mapping.

Proof

Let f: X → Y be an intuitionistic fuzzy semi-closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFCS, B is an IFSCS in X. By our assumption f (B) is an IFSCS in Y. As every IFSCS is an IFSGCS, f(B) is an IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping.

The converse of the above theorem is not true as seen from the following example.

Example: 6.2.6

Let X = {a,b}, Y = {u,v}. Let

$A = < x, \left(\frac{a}{0.4}, \frac{b}{0.5}\right), \left(\frac{a}{0.1}, \frac{b}{0.1}\right) >,$

$B = < y, \left(\frac{u}{0.4}, \frac{v}{0.4}\right), \left(\frac{u}{0.6}, \frac{v}{0.5}\right) >$

Then τ = {0~, 1~, A, B} and k = {0~1~, C} are IFTSs on X and Y respectively. Define a mapping f: (X,τ) → (Y, k) by f(a) = u, f(b) = v. Clearly 0~1~ are the only IFRCS in X. Now f(0~) = 0~ and f(1~) = 1~ are IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping. Now

$\overline{F}(A) = < x, \left(\frac{u}{0.1}, \frac{v}{0.3}\right), \left(\frac{u}{0.3}, \frac{v}{0.6}\right) >,$ $cl(f(\overline{A})) = 1\sim,$

$Int(cl(f(\overline{A}))) = int(1\sim) = 1\sim, \overline{cl(int(cl(f(A))))} = 1\sim, \not\subseteq f(A).$

Therefore $f(\overline{A})$ is not an intuitionistic fuzzy α-closed mapping.

Theorem: 6.2.7

Every intutionistic fuzzy sg-closed mapping is an intutionistic fuzzy almost sg-closed mapping.

Proof

Let f: X → Y be an intutionistic fuzzy sg-closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFCS, B is an IFCS in X. By our assumption f (B) is an IFSGCS in Y. Hence f is an intutitonistic fuzzy almost sg-closed mapping.

The converse of the above theorem is not true as seen from the following example.

Example: 6.2.8

Let X = {a,b}, Y = {u,v}. Let

$A = < x, \left(\frac{a}{0.2}, \frac{b}{0.2}\right), \left(\frac{a}{0.4}, \frac{b}{0.4}\right) >,$

$B = < y, \left(\frac{a}{0.2}, \frac{b}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}\right) >$

$C = < x, \left(\frac{u}{0.5}, \frac{v}{0.6}\right), \left(\frac{u}{0.2}, \frac{v}{0.1}\right) >.$

Then τ = {0~, 1~, A, B} and k = {0~, 1~, C} are IFTSs on X and Y respectively. Define a mapping f: (X,τ) → (Y, κ) by f (a) = u, f (b) = v. Clearly 0~1~ are the only IFRCS in X. Now f(0~) = 0~ and f(1~) = 1~ are IFSGCS in Y. Hence f is an intutionistic fuzzy almost sg-closed mapping.

IFSOS(Y) = {0~,1~, $G_{u,v}^{(l1,m1)(l2,m2)}$; $l_1 \in [0.5,1]$, $l_2 \in [0.6,1]$, $m_1 \in [0,0.2]$, $m_2 \in [0,0.1]$, $l_i + m_i \le l$, i = 1,2}

Where $G_{u,v}^{(l_1,m_1)(l_2,m_2), (l_2,m_2)}$ = $< y,\left(\frac{u}{l_1},\frac{v}{l_2}\right),\left(\frac{u}{m_1},\frac{v}{m_2}\right)>$,

IFSCS(Y) = {0~,1~, $H_{u,v}^{(a_1,b_1)(a_2,b_2)}$ $a_1 \in [0,0.2]$,$a_2 \in [0,0.1]$, $b_1 \in [0.5,1]$, $b_2 \in [0,6.1]$, $a_i + b_i \le 1$, i = 1,2}

Where $H_{u,v}^{(1_1,m_1)(l_2,m_2)}$ =. $< y,\left(\frac{u}{l_1},\frac{v}{l_2}\right),\left(\frac{u}{m_1},\frac{v}{m_2}\right)>$,Now

$$f(\overline{A}) = < y, < x,\left(\frac{u}{0.4},\frac{v}{0.4}\right),\left(\frac{u}{0.2},\frac{v}{0.2}\right)> \text{and } scl(\overline{f(A)}) = 1\sim.$$

Then f $(\overline{A}) \subseteq$ C, but scl(f($\overline{A}$)) $\subseteq$ C. Therefore f $\overline{(A)}$ is not an IFSGCS in Y. Hence f is not an intuitionistic fuzzy sg-closed mapping.

Theorem: 6.2.9

Every intuitionistic fuzzy sg*-closed mapping is an intutionistic fuzzy almost sg-closed mapping.

Proof

Let f : X → Y be an intutionistic fuzzy sg*-closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFSGCS, B is an IFSGCS in X. By our assumption f(B) is an IFSGCS in Y. Hence f is an intutionistic fuzzy almost sg-closed mapping.

The converse of the above theorem is not true as seen from the following example.

Example: 6.2.10

Let X = {a,b}, Y = {u,v}. Let

$$A = < x,\left(\frac{a}{0.2},\frac{b}{0.6}\right),\left(\frac{a}{0.1},\frac{b}{0.3}\right)>,$$

B = $< x, \left(\dfrac{a}{0.4}, \dfrac{b}{0.3}\right), \left(\dfrac{a}{0.3}, \dfrac{b}{0.7}\right) >$,

Then τ = {0~, 1~, A} and k = {0~, 1~, B} are IFTSs on X and Y respectively. Define a mapping f: $(X,\tau) \rightarrow (Y,k)$ by f(a) = u, f(b) = v. Clearly 0~, 1~ are the only IFRCS in X. Now f (0~) = 0~ and f (1~) = 1~ are IFSGCS in Y. Hence f is an intutionistic fuzzy almost sg-closed mapping.

Let C = $< x, \left(\dfrac{a}{0.1}, \dfrac{b}{0.3}\right), \left(\dfrac{a}{0.2}, \dfrac{b}{0.7}\right) >$ be an IFSGCS in X.

IFSOS(X) = {0~,1~, $G_{u,v}^{(l_1,m_1)(l_2,m_2)}$ $l_1 \in [0.2,1]$, $l_2 \in [0.6,1]$, $m_1 \in [0,0.1]$, $m_2 \in [0,0.3]$, $l_i + m_i \leq 1$, i = 1,2}

Where $G_{a,b}^{(l1,m1),(l2,m2)}$ = $< x, \left(\dfrac{a}{l_1}, \dfrac{b}{l_2}\right), \left(\dfrac{a}{m_1}, \dfrac{b}{m_2}\right) >$,

IFSCS(X) = $H_{u,v}^{(a_1,b_1)(a_2,b_2)}$ $a_1 \in [0,0.1]$, $a_2 \in [0,0.3]$, $b_1 \in [0.2,1]$, $b_2 \in [0,6.1]$,$a_i + b_i \leq 1$, i = 1,2}

Where $H_{a,b}^{(a_1,b_1)(a_2,b_2)}$ = $< x, \left(\dfrac{a}{a_1}, \dfrac{b}{a_2}\right), \left(\dfrac{a}{b_1}, \dfrac{b}{b_2}\right) >$.

IFSOS(Y) = {0~,1~, $K_{u,v}^{(\alpha_1,\beta_1)(\alpha_2,\beta_2)}$ $\alpha_1 \in [0.4,1]$, $\alpha_2 \in [0.3,1]$, $\beta_1 \in [0,0.3]$, $\beta_2 \in [0.6,1]$ $\alpha_i + \beta_i \leq 1$, i = 1,2}

Where $H_{a,b}^{(a_1,b_1)(a_2,b_2)}$ = $< x, \left(\dfrac{a}{a_1}, \dfrac{b}{a_2}\right), \left(\dfrac{a}{b_1}, \dfrac{b}{b_2}\right) >$.

IFSOS(Y) = {0~,1~, $M_{u,v}^{(\gamma_1,\delta_1)(\gamma_1,\delta_2)}$ $\gamma_1 \in [0,0.3]$, $\gamma_2 \in [0,0.7]$, $\delta_1 \in [0.4,1]$, $\delta_2 \in [0.3,1]$, $\gamma_i + \delta_i \leq 1$, i = 1,2}

Where $M_{u,v}^{(\gamma_1,\delta_1)(\gamma_1,\delta_2)}$ = $< y, \left(\dfrac{u}{\gamma_1}, \dfrac{v}{\delta_2}\right), \left(\dfrac{u}{\gamma_1}, \dfrac{v}{\delta_2}\right) >$. Now scl(f(C)) = 1~. Therefore f(C) is not an IFSGCS in Y. Hence f is not an intuitionistic fuzzy sg*-closed mapping.

Theorem: 6.2.12

Every intuitionistic fuzzy quasi sg-closed mapping is an intuitionistic fuzzy almost sg-closed mapping.

Proof

Let f: X → Y be an intuitionistic fuzzy quasi sg-closed mapping and let B be an IFRCS in X. Since every IFRCS is an IFSGCS, B is an IFSGCS in X. By our assumption f (B) is an IFCS in Y. As every IFCS is an IFSGCS, f (B) is an IFSGCS in Y. Hence f is an intutionistic fuzzy almost sg-closed mapping.

The converse of the above theorem is not true as seen from the following example.

Example: 6.2.13

Let X = {a,b}, Y = {u,v}. Let

A = $< x, \left(\frac{a}{0.3}, \frac{b}{0.4}\right), \left(\frac{a}{0.1}, \frac{b}{0.3}\right) >$,

B = $< x, \left(\frac{a}{0.4}, \frac{b}{0.3}\right), \left(\frac{a}{0.3}, \frac{b}{0.7}\right) >$,

Then τ = {0~, 1~, A} and k = {0~, 1~, B} are IFTSs on X and Y respectively. Define a mapping f: (X,τ) → (Y,k) by f(a) = u, f(b) = v. Clearly 0~, 1~ are the only IFRCS in X. Now f (0~) = 0~ and f(1~) = 1~ are IFSGCS in Y. Hence f is an intutionistic fuzzy almost sg-closed mapping. Now A is an IFCS in Y. Hence f is not an intutionistic fuzzy quasi sg-closed mapping.

The relation between various types of intutionistic fuzzy closed mappings is given in the Figure 4.2.1. The reverse implications in the Figure4.2.1 are not true in general.

Definition: 6.2.14

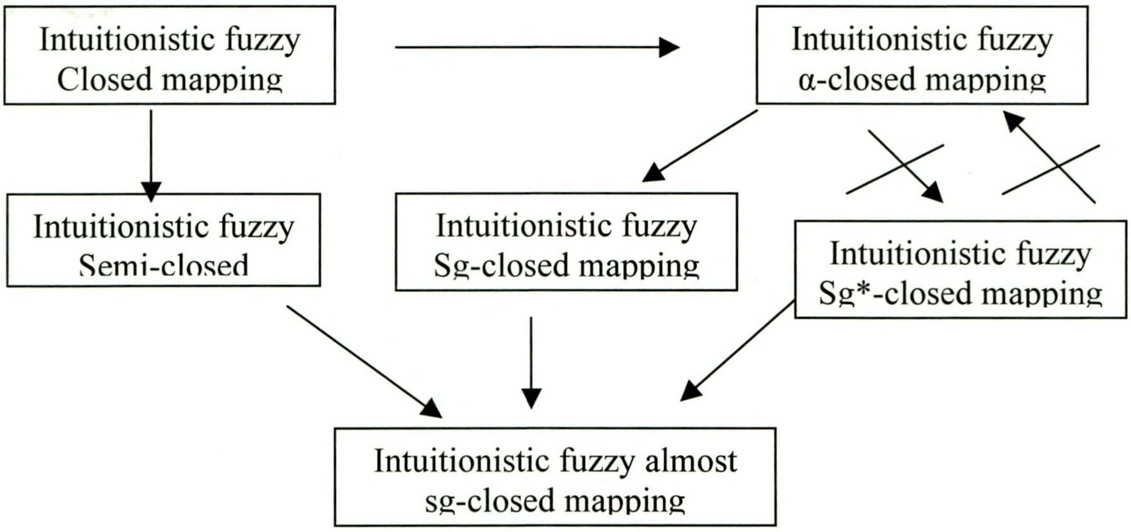
A mapping f: X → Y is said to be an intutionistic fuzzy almost semi-generalized open (intuitionistic fuzzy almost sg-open) mapping if f (A) is an IFSGOS in Y for every IFROS A in X.

Theorem: 6.2.15

Let f: X → Y be a mapping. Then the following are equivalent.

(i) f is an intuitionistic fuzzy almost sg-closed mapping;

(ii) f is an intutionistic fuzzy almost sg-open mapping.



Theorem: 6.2.16

Let f: X → Y be a mapping where Y is an intuitionistic fuzzy semi-T½ space. Then the following are equivalent.

(i) f is an intutionistic fuzzy almost sg-closed mapping;

(ii) scl(f(A)) ⊆ f(cl(A)) for every IFSPOS A in X;

(iii) scl(f(A)) ⊆f(cl(A)) for every IFSOS A in X;

(iv) f(A) ⊆ sint(f(cl(int(A))) for every IFPOS A in X.

Proof

(i) ⇒ (ii) Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFSGCS in Y. Since Y is an intutionistic fuzzy semi-T½ space, f(cl(A)) is an IFSCS in Y. Then scl(f(cl(A))) = f(cl(A)). Now scl(f(A)) ⊆ scl(f(cl(A)) = f (cl(A)). Thus scl(f(A)) ⊆ f(cl(A)).

(ii) ⇒ (iii) Since every IFSOS is an IFSPOS,

(iii) ⇒ (i) Let A be an IFRCS in X. Then A = f(cl(A)), which implies A is an IFSOS in X. By hypothesis, scl(f(A)) ⊆ f(cl(A)) = f(A) ⊆ scl(f(A)). Thus f(A) is an IFSCS and hence f(A) is an IFSGCS in Y. Therefore f is an intuitionistic fuzzy almost sg-closed mapping.

(i) ⇒ (iv) Let A be an IFPOS in X. Then A ⊆ int(cl(A)). Since int(cl(A)) is an IFROS in X, by our assumption f(int(cl(A)) is an IFSGOS in Y. Since Y is an intutionistic fuzzy semi-T½ space, f(int(cl(A)) is an IFSOS in Y. Therefore f (A) ⊆ f (int(cl(A)) = sint(f(int(clA)))).

(iv) ⇒ (i) Let A be an IFRCS in X. Since every IFRCS is an IFPCS, A is an IFPCS in X. By hypothesis f(A) ⊆ sint(f(cl(A)))) = sint(f(A)) ⊆ f(A). This implies f(A) is an IFSOS in Y and hence f(A) is an IFSGOS in Y. Therefore f is an intuitionistic fuzzy almost sg-open mapping. By Theorem 4.2.16, f is an intuitionistic fuzzy almost sg-closed mapping.

Definition: 6.2.17

Let p(α,β) be an IFP of an IFTS (X, τ). An IFSA of X is called an intuitionistic fuzzy semi-neighborhood (IFSN) of p(α,β), if there exists an IFSOS B in X such that p(α,β) ∈ B ⊆ A.

Theorem: 6.2.18

Let f: X → Y be a mapping. Then f is an intuitionistic fuzzy almost sg-closed mapping if for each IFP p (α,β) ∈ Y and for each IFSOS B in X such that f^{-1}(p(α,β))ε B, scl(f(B)) is an intuitionistic fuzzy semi-neighborhood of (p(α,β)) ε Y.

Proof

Let (p (α,β)) ε Y and let A be an IFOS in X. Then A is an IFSOS in X such that f^{-1} (p(α,β)) ∈ A. Then (p(α,β)) ∈ f(A) in Y and scl(f(A)) is an intuitionistic fuzzy semi-neighborhood of (p(α,β)) in Y. Therefore there exists an IFSOS B in Y such that (p(α,β)) ∈ B ⊆ scl(f(A))). We have (p(α,β)) ∈ B ⊆ scl(f(A))). Now B = U { p(α,β)/ p(α,β) ∈ B} = f (A). Therefore f(A) is an IFSOS in Y and hence f(A) is an IFSGOS in Y. Therefore f is an intutionistic fuzzy almost sg-open mapping and by Theorem 4.2.16, f is an intutionistic fuzzy almost sg-closed mapping.

Theorem: 6.2.19

Let f: X → Y be a mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then sgcl(f(A)) ⊆ f(cl(A)) for every IFSPOS A in X.

Proof

Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFSGCS in Y. Then sgcl(f(cl(A))) = f(cl(A)). Now sgcl(f(A)) ⊆ sgcl(f(cl(A))) = f(cl(A)).

Corollary: 6.2.20

Let f: X → Y be a mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then sgcl(f(A)) ⊆ f(cl(A)) for every IFSOS A in X.

Proof

Since every IFSOS is an IFSPOS, the proof follows from Theorem 6.2.19.

Corollary: 6.2.21

Let f: X → Y be a mapping. If f is an intutionistic fuzzy almost sg-closed mapping, then sgcl(f(A)) ⊆ f(cl(A)) for every IFPOS A in X.

Proof

Since every IFSOS is an IFPOS, the proof follows from Theorem 6.2.20.

Theorem: 6.2.22

Let f: → Y be a mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then sgcl(f(cl(A))) ⊆ f(cl(spint(A))) for every IFSPOS A in X.

Proof

Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis, f (cl(A)) is an IFSGCS in Y. Then sgcl(f(cl(A))) = f(cl(A) ⊆ f(cl(A)).

Corollary: 6.2.23

Let f: X → Y be mapping. If f is an intuitionistic fuzzy almost sg-closed mapping, then sgcl(f(cl(A)) ⊆ f(cl(spint(A))) for every IFSOS A in X.

Proof

Since every IFSOS is an IFSPOS, the proof follows from theorem 6.4.22.

Theorem: 6.2.24

Let f: X → Y be a mapping. If f(sint(B) ⊆ sint(f(B)) for every IFS B in X, then f is an intuitionistic fuzzy almost sg-closed mapping.

Proof

Let Be an IFROS in X. By hypothesis f(sint(B)) ⊆ sint(f(B)). Since every IFROS is an IFSOS, B is an IFSOS in X. Therefore sint(B) = B. Hence f(B) = f(sint(B)) ⊆ sint(f(B)) ⊆ f(B). This implies f(B) is an IFSOS in Y. Since every IFSOS is an IFSGOS, f(B) is an IFSGOS in Y. Hence f is an intuitionistic fuzzy almost sg-closed mapping.

Theorem: 6.2.25

Let f: → Y be a mapping. If scl(f(B)) ⊆ f(scl(B)) for every IFS B in X, then f is an intuitionistic fuzzy almost sg-closed mapping.

Proof

Let B be an IFRCS in X. By hypothesis scl(f(B)) ⊆ f(scl(B)). Since every IFRCS is an IFSCS, B is an IFSCS in X. Therefore scl(B) = B. Hence f(B) = f(scl(B)) ⊇ f(B). This imples f(B) is an IFSCS in Y and hence f(B) is an IFSGCS in Y. Thus f is an intuititionistic fuzzy almost sg-closed mapping.

Theorem: 6.2.26

Let f: X → Y be a mapping, where Y is an intuititionistic fuzzy semi-T½ space. Then the following are equivalent.

(i) f is an intuitionistic fuzzy almost sg-open mapping.

(ii) for each IFP p(α,β) in Y and each IFROS B in X such that f^{-1} (p(α,β) ∈ B, cl(f(cl(B))) is an intuititionistic fuzzy semi-neighborhood of p(α,β) inY.

Proof

(i) ⇒ (ii) Let p(α,β) ε f(B). By hypothesis f (B) is an IFSGOS in Y. Since Y is an intuitionistic fuzzy semi-T½ space, f (B) is an IFSOS in Y. Now p(α,β) ∈ f(B) ⊆ f(cl(B)) ⊆ cl(f(cl(B))). Hence cl(f(cl(B))) is an intuititonistic fuzzy semi-neighborhood of p(α,β) in Y.

(ii) ⇒ (i) Let B be an IFOS in X and f^{-1} (p(α,β) εB. This implies p(α,β) ∈ f(B). By hypothesis cl(f(cl(B))) is an intuitionistic fuzzy semi-neighborhood of p(α,β). Therefore there exists an IFSGOS A in Y such that p(α,β) ∈ A ⊆ cl(fcl(B)). Now A = U {p(α,β)/p(α,β) ∈A} = f(B). Therefore f(B) is an IFSOS and hence f(B) is an IFSGOS in Y. Thus f is an intuitionstic fuzzy almot sg-open mapping.

Theorem: 6.2.27

Let f: X → Y be a mapping, where Y is an intuitionistic fuzzy semi-T½ space. Then the following statements are equivalent:

(i) f is an intuitionistic fuzzy almost sg-closed mapping,

(ii) scl(f(A)) ⊆ f (αcl(A)) for every IFSPOS A in X,

(iii) scl(f(A)) ⊆ f(αcl(A)) for every IFSOS A in X,

(iv) f(A) ⊆ sint (f(scl(A))) for every IFPOS A in X.

Proof

(i) ⇒ (ii) Let A be an IFSPOS in X. Then cl(A) is an IFRCS in X. By hypothesis f(cl(A)) is an IFSGCS in Y and hence f(cl(A)) is an IFSCS in Y, since Y is an intuitionistic fuzzy semi-T½ space. This implies scl(cl(A))) = f(cl(A)). Now scl(f(A)) ⊆ scl(f(cl(A))) = f(cl(A)). Since cl(A) is an IFRCS, we have cl(int(cl(A))) = cl(A). Therefore

scl(f(A)) ⊆ f(cl(A)) = f(cl(cl(A)))) ⊆ f(A) U cl(int(cl(A)))) ⊆ f (αcl(A)).

Hence scl(f(A)) ⊆ f(αcl(A)).

(ii) ⇒ (iii) Let A be an IFSOS in X. Since every IFSOS is an IFSPOS, the proof is obvious.

(iii) ⇒ (i) Let A be an IFRCS in X. Then A = cl(int(A)). Therefore A is an IFSOS in X. By hypothesis, scl(f(A)) ⊆ (αcl(A)) ⊆ f(αcl(A)) = f(A) ⊆ scl(f(A)). Hence scl(f(A)) = f (A). Therefore f(A) is an IFSCS in Y and hence f(A) is an IFSGCS in Y. Thus f is an intuitionistic fuzzy almost sg-closed mapping.

(i) ⇒ (iv) Let A be an IFPOS in X. Then A ⊆ int(cl(A)). Since int(cl(A) is an IFROS in X. By hypothesis f(int(cl(A))) is an IFSGOS in Y. Since Y is an intuitoinistic fuzzy semi-T½ space, f(int(cl(A)))) is an IFSOS in Y. Therefore

f(A) ⊆ f(cl(int(A)) = sint(f(int(cl(A)))))

= sint(f(A ∪ int(cl(A)))) = sint(f(scl(A))).

(iv) ⇒ (i) Let A be an IFROS in X. Then A is an IFPOS in X. By hypothesis f(A) ⊆ sint(f(scl(A))). This implies that

f(A) ⊆ sint(f(A ∪ int(cl(A)))) ⊆ sint (f(A ∪ A) = sint (f(A)) ⊆ f(A).

Therefore f(A) is an IFSOS in Y and hence f(A) is an IFSGOS in Y. Thus f is an intuitionistic fuzzy almost sg-closed mapping.

Theorem: 6.2.28

Let f: X → Y be a mapping, where Y is an intuitionistic fuzzy semi- T½ space. If f is an intuitionistic fuzzy almost sg-closed mapping, then int(cl(f(B))) ⊆ f(scl(B) for every IFRCS B in X.

Proof

Let B be an IFRCS in X. Since f is an intuitionistic fuzzy almost sg-closed mapping, f(B) is an IFSGCS in Y. Since Y is an intuitionistic fuzzy semi -T½ space, f (B) is an IFSCS in Y. Therefore scl(f(B))) = f(B). Now int(cl(B))) ⊆ f(B) U int(cl(f(B))) ⊆ scl(f(B)) = f(B) = f(scl(B)).

Hence int(cl(f(B))) ⊆ f (scl(B)).

Theorem: 6.2.29

Let f: X → Y be a mapping, where Y is an intuitionistic fuzzy semi-T½ space. If f is an intutioinistic fuzzy almost sg-closed mapping, then f (sint(B))) ⊆ cl(int(f(B)) for every IFRO B in X.

Proof

The proof follows from theorem 6.2.28 by taking complement.

Theorem: 6.2.30

Let f: X → Y be a bijective mapping. Then the following statements are equivalent:

(i) f is an intuitionistic fuzzy almost sg-open mapping.

(ii) f is an intuitionistic fuzzy almost sg-closed mapping

(iii) f^{-1} is an intuitoinistic fuzzy almost sg-continuous mapping.

Proof

(i) ⇒ (ii) Obvious.

(ii) ⇒ (iii) Let A be an IFRCS in X. By assumption f (A) is an IFSGCS in Y. That is $(f^{-1})^{-1}$ (A) = f (A) is an IFSGCS in Y. Hence f^{-1} is an intuitionistic fuzzy almost sg-continuous mapping.

(iii) ⇒ (i) Let A be an IFRCS in X. By hypothesis $(f^{-1})^{-1}$ (A) = f (A) is an IFSGCS in Y. Hence f is an intuitionistic fuzzy almost sg-open mapping.