Design of Life Test Group Acceptance Sampling Plans

Prema, M
(12PMA014)

Thesis Submitted to
Avinashilingam Institute for Home Science and Higher Education for Women,
Coimbatore-641 043

In Partial Fulfilment of the Requirements for the

Degree of Master of Science in Mathematics

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Signature of the Supervisor

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Introduction

INTRODUCTION

Life time of a component measures the operational success of the system and it is concerned not only with the assessment but also helps in the improvement of system performance. The cost of unreliability is not only the cost of failing item but that of the associated equipment which is damaged as a result of failure due to the interdependency between components in complex system. For example the failure of the circuit breaker can lead to complete disaster in the power system and failure of breaks in the transportation system can lead to accidents.

The life time of the equipment depend on quality of materials used in manufacturing, the technology of production, the condition under which they operate and so on. Product quality has become one of the most important issues that distinguish different commodities in a global business market. Two important techniques for ensuring quality are the statistical process control and statistical product control.

The acceptance sampling plans are concerned with accepting or rejecting a submitted lot of products on the basis of the quality of products inspected in a sample taken from the lot(where lot of products may be incoming materials or finished goods, purchased material, components etc). Acceptance sampling is an audit tool to ensure that the output of a process conforms to requirements. The acceptance sampling plans are classified on the basis of the number of samples involved in making decision on the submitted lot such as single, double, multiple, sequential and group acceptance sampling plans. There are always possible to face two risks associated with acceptance sampling plan due to the inference on population units with only randomly selected small number of sample units. The probability of rejection of a good lot known as producer's risk and the probability of acceptance of bad lot called the consumer's risk. An acceptance sampling plan is always designed so that both risks are at the required minimum level.

An acceptance sampling plan is a scheme that establishes the minimum sample size to be used for testing. This become particularly important if the quality of product is defined by its lifetime. Often, it is implicitly assumed when designing a sampling plan that only a single item is put in a tester. However, in practice testers accommodating a multiple number of items at a time are used because testing time and cost can be saved by testing items simultaneously. The items in a tester can be regarded as a group and the number of items in a group is called the group size. An acceptance sampling plan based on such groups of items in a group is called a group acceptance sampling plan(GASP). If the group acceptance sampling plan is used in conjunction with truncated life tests, it is called a group acceptance sampling plan based on the truncated life test assuming that the lifetime of product follows a certain probability distribution. For such a type of test, the determination of sample size is equivalent to determine the number of groups.

In the literature several sampling plans are available for life testing in terms of mean. Acceptance sampling plans for truncated life test based on mean may not satisfy the engineering requirement on the specific quality like strength, breaking stress and so on of the product. This gives way to significant deterioration in quality and may not meet the consumer's expectation. Most of the probability distributions employed for life testing are not symmetric. In such distributions the mean value may not be adequate to describe the central tendency of the distribution. Under these circumstances sampling plans for life testing based on median seems more appropriate. These reasons motivated to develop acceptance sampling plan for truncated life test based on the median.

The lifetime variations in products can be modelled using a statistical probability distribution. The design parameters of the acceptance sampling plan for life testing depend on the underlying statistical distribution.

The generalized exponential distribution is an important lifetime distribution in survival analysis. It is observed that the two-parameter generalized exponential distribution can be used quiet effectively to analyze positive lifetime data.

Kumaraswamy-log-logistic accommodates several important distributions as sub-models. This motivated the researcher to use Kumaraswamy-log-logistic distribution as the distribution of the life time. This assumption on the life time of the product paves way to have wider applicability.

Profile of the study

This dissertation is the result of review of following research articles and development of certain other group acceptance sampling plan for truncated life test.

i. The researcher reviewed a research article, a group acceptance sampling plan for lifetimes following a generalized exponential distribution of Srinivasa Rao (2009).

ii. The researcher proposed Group acceptance sampling plan for truncated life test based on median using Kumaraswamy log logistic distribution under binomial and Poisson model.

Synopsis

SYNOPSIS

This dissertation concentrates on the development of designing of the group acceptance sampling plan for truncated life test based on the population median of generalized exponential distribution and Kumaraswamy-log-logistic distribution to meet the specified consumer's confidence level.

The relevant literature essential for the preparation of this dissertation on designing life test group acceptance sampling plans based on different life time distribution is given in the review of literature.

The basic terms and the distribution which are relevant for the preparation of this dissertation are presented in **Basic concepts**.

The content of the dissertation is presented in two chapters.

In Chapter I a group acceptance sampling plan for truncated life testing when the life time of the product following generalized exponential distribution is presented. Minimum number of groups to meet the consumer's confidence level, operating characteristic values and minimum median ratio to meet the specified producer's risk are derived and analyzed. Tables are constructed and numerical examples are provided to explain the applicability.

The designing of a group acceptance sampling plan based on the life time of the product following Kumaraswamy-log-logistic distribution is given in chapter II. For the developed group sampling plan the minimum number of groups to meet the consumer's confidence level, operating characteristic values and minimum median ratio to meet the specified producer's risk are calculated and examined. Tables are constructed and numerical examples are also given to enhance the applicability of the test plan.

Review of Literature

REVIEW OF LITERATURE

This dissertation focuses on the development of group acceptance sampling plan for truncated life test based on the life time of product quality following generalized exponential distribution and Kumaraswamy-log-logistic distribution.

Life test acceptance sampling plans are essential to decide on the acceptance or rejection of the lot of products or items. This initiated many researchers to work on the development of acceptance sampling plans for truncated life tests using different life time distributions for the products.

Several research papers have been published for designing group sampling plans for the truncated life test when life time of the product following various statistical distributions. Epstein (1954) first introduced acceptance sampling plans for the truncated life test when the life time of the product follows exponential distribution. Sobel and Tishendrof (1959) pointed out certain new life test objectives for using acceptance sampling plans. Goode and Kao (1961) developed an acceptance sampling plan using the Weibull distribution as a life time distribution. Gupta and Groll (1961) derived the acceptance sampling plan for the gamma distribution and discussed the problem of acceptance sampling when the life time is truncated at a pre assigned time and Gupta (1962) designed the acceptance sampling test plan for the life time of the product having log-normal distribution. Fertig and Mann (1980) introduced life test sampling plan for two-parameter Weibull population.

Kumaraswamy (1980) studied the generalized probability density function for double bounded random processes. Cordeiro and de Castro (2011) studied the mathematical properties of a new family of generalized distributions. Combined works of Kumaraswamy (1980) and Cordeiro and de Castro (2011) resulted in a new model known as Kumaraswamy log logistic distribution with four parameters. The new distribution is more suitable for testing goodness of fit of these sub models and defining regression model.

Kantam and Rosaiah (1998) derived an acceptance sampling plan for truncated life test using half logistic distribution. Kantam, Rosaiah and Srinivasa Rao (2001) developed acceptance sampling plan for life test based on log-logistic model. Gupta and Kundu (2001) introduced the generalized exponential distribution and suggested that generalized exponential distribution can be used as an alternative to gamma and Weibull distribution in many real life situations.

Baklizi (2003) introduced acceptance sampling based on truncated life tests with Pareto distribution of the second kind. Baklizi and Masri (2004) developed acceptance sampling plan for truncated life test based on Birnbaum Saunder's model using average assuming that the life test is truncated at a pre assigned time. Gupta and Kundu (2003a) discussed the closeness between the gamma and generalized distributions. Gupta and Kundu (2003b) studied the discrimination between gamma and generalized exponential distribution. Balakrishnan et.al. (2007) proposed an acceptance sampling plan from truncated life test based on the Birnbaum-Saunders distribution.

Rosaiah and Kantam (2005) developed an acceptance sampling procedure for truncated life test based on mean using the inverse Rayleigh distribution. Tsai and Wu (2006) developed the test sampling plan using generalized exponential distribution. Rosaiah et al (2006) proposed acceptance sampling plan for life testing using the exponentiated log-logistic distribution. Kantam et.al. (2006) proposed an economic reliability test plan by assuming the life time of the product as log-logistic distribution. Variable acceptance sampling plan for Weibull distributed items are studied by Jun et.al. (2006). Kantam et.al. (2007) proposed economic reliability test plan with inverse Rayleigh variate.

Rao et.al. (2008) proposed the acceptance sampling plans for truncated life test assuming Marshall- Olkin extended Lomax distribution as a life time distribution. Srinivasa Rao (2009a) developed a group acceptance sampling for the truncated life test when the life time of the item follows Marshall-Olkin extended Lomax distribution. Rao et.al. (2009b) developed reliability test plans using Marshall-Olkin extended Lomax distribution.

Aslam (2008) introduced economic reliability acceptance sampling plans for generalized Rayleigh distribution. Aslam and Jun (2009a) proposed group acceptance sampling plans for truncated life test based on the inverse Rayleigh and log-logistic distributions. Aslam and Jun (2009b) introduced group acceptance sampling plans for truncated life test having Weibull distribution. Aslam et.al. (2009) considered group acceptance sampling plan based on truncated life test for gamma distributed items. Srinivasa Rao (2009) developed a group acceptance sampling for the truncated life test when the life time of the item follows Marshall-Olkin extended Lomax distribution. A double acceptance sampling plan for generalized log-logistic distributions with known shape parameters were derived by Aslam et.al. Aslam et.al. (2010) introduced time truncated acceptance sampling plan when the life time of the item follows the generalized exponential distribution. Aslam et.al. (2011) derived improved group sampling plans based on time-truncated life tests. Radhakrishnan and Alagirisamy (2011) described the construction of group acceptance sampling plan using weighted binomial distribution.

Basic concepts

BASIC CONCEPTS

The basic terms, definitions and notations required for the development of this dissertation are given under the heading Basic concepts.

ACCEPTANCE SAMPLING

Acceptance sampling is the methodology that deals with procedures by which decisions to accept or not to accept are based on the result of the inspection of samples.

ACCEPTANCE SAMPLING PLAN

An acceptance sampling plan is “a specific plan that states the sample size or sizes to be used and the associated acceptance and non-acceptance criteria”.

INSPECTION

Inspection is the process of measuring, examining, testing or otherwise comparing the unit of product with the requirements.

INSPECTION BY ATTRIBUTES

Inspection by attributes is inspection where by certain characteristics of units of products are inspected and classified simply as conforming or not-conforming to the specified requirements.

100% INSPECTION

100% inspection means the inspection of every unit of product for the defects concerned listed for an inspection station. The two terms screening and 100% inspection are used interchangeably in this dissertation.

SAMPLING INSPECTION

Sampling inspection means the inspection for the defects concerned where the units selected for inspection are selected by random sampling.

CONFORMING UNIT

Conforming unit is one which meets the acceptance criteria established for the characteristic being considered.

NON-CONFORMING UNIT

Non-conforming unit is one which does not meet the acceptance criteria established for the characteristic being considered.

CLEARENCE NUMBER

The number of consecutive conforming (defect free) units required to adjust inspection action is clearance number. It is denoted by ‘i’.

SAMPLING FREQUENCY

The desired ratio between the numbers of units of product randomly selected and inspected at a inspection station during period of sampling inspection. It is denoted by ‘f’. The procedure used in selecting the sampling units should give each unit of product presented during periods of sampling inspection an equal chance of being selected and inspected.

HYPOTHESIS

A hypothesis is a proposed statement regarding a phenomenon.

NULL HYPOTHESIS

A null hypothesis is a statement of no significant difference between the actual and observed values of the parameter. The null hypothesis assumes that any kind of difference or significance in a set of data is due to choice. It is denoted by H_0.

ALTERNATIVE HYPOTHESIS

Alternative hypothesis is made against null hypothesis. It is denoted by H_1. This hypothesis implies that sample observations are influenced by some non-random cause.

PROCESS QUALITY

A statistical measure for the quality of product from a given process is procees quality. The most commonly used measure of process quality is the percentage or proportion of non-conforming units in the process.

ACCEPTABLE QUALITY LEVEL (AQL)

The maximum percentage or proportion of variant units in a lot or batch that , for the purpose of acceptance sampling can be considered satisfactory as a process average.

LIMITING QUALITY LEVEL (LQL)

The percentage or proportion of variant units in a batch or lot which , for the purpose of acceptance sampling, the consumer wishes the probability of acceptance to be restricted to a specific low value .

PRODUCER'S RISK

For a given sampling plan , the probability of not accepting quality of which has a designated numerical value representing a level which it is generally desiered to accept. It is denoted by α .

CONSUMER'S RISK

For a given sampling plan, the probability of acceptance of a lot quality of which has a designated numerical value representing a level which it is seldom desired to accept. It is denoted by β .

OPERATING CHARECTERISTIC (OC) CURVE

Associated with each sampling plan there is an OC curve which portrays the performance of the sampling plan against good and poor quality. OC curve shows graphically the interrelationship of risks, probability and qualities for a given sampling plan.

AVERAGE SAMPLE NUMBER (ASN)

The average number of sample units per lot used for making decisions - acceptance or non-acceptance. A plot of ASN against p is called the ASN curve.

These definitions are from American National Standards Institute.

BINOMIAL DISTRIBUTION

A random variable is said to have the binomial distribution if its probability mass function is

$$p(x) = \left(C_x^n\right) p^x (1-p)^{n-x}$$

where n=sample size, n>0

p=proportion defective, 0≤p≤1

q = proportion defective, q=1-p

x=number of occurrence of defectives, x=0, 1, 2, . . ., n

POISSON DISTRIBUTION

A random variable is said to have Poisson distribution, if its probability mass function is

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \text{ , x=0, 1, 2 , ... } \infty$$

where λ = mean number of defective, λ>0

x = number of occurrence of defectives, x = 0,1,2,…, . ∞

GENERALIZED EXPONENTIAL DISTRIBUTION

Let $X_1, X_2, X_3, ..., X_n$ be independently identically distributed random variable with the shape parameter θ and scale parameter λ, its probability distribution function is given by

$$f_x(x;\theta;\lambda) = \theta\lambda(1-e^{-\lambda x})^{\theta-1} e^{-\lambda x}, x > 0, \theta > 0, \lambda > 0$$

KUMARASWAMY – LOG – LOGISTIC DISTRIBUTION

The probability density function of the Kumaraswamy – log – logistic distribution is given by

$$f(a,b,c,\gamma) = \frac{ab\gamma}{c^{a\gamma}} t^{a\gamma-1} \left(1+(t/c)^{\gamma}\right)^{-(a+1)} \left[1-\left(1-\left(1+(t/c)^{\gamma}\right)^{-1}\right)^{a}\right]^{b}, \text{ t>0}$$

where c is the scale parameter and a, b and γ are shape parameters.

NOTATIONS

GASP - Group Acceptance Sampling Plan

p - the proportion of defective units submitted from production process

q - 1-p ,the probability of rejecting a lot

P - the probability of accepting a lot for a given value of process quality, p.

α - producer's risk

β - consumer's risk

N - lot size, the number of units in the lot

r - number of items in each g

g - number of groups

n - sample size, the number of units in the sample

i	-	number of consecutive lots that must be accepted under normal inspection, in order to permit switching to skipping inspection.
c	-	acceptance number
f	-	fraction of lots inspected during a period of skipping inspection.
AOQ	-	average outgoing quality after inspection
AOQL	-	the maximum of AOQ over all possible values of p
ASN	-	average sample number
ATI	-	average total inspection
μ	-	median life time
μ_0	-	specified median life
μ/μ_0	-	median ratio to the specified life
t_0	-	pre assigned experimental time
KLLD	-	Kumaraswamy- Log –Logistic Distribution

Chapter - I

Chapter I

Life Test Group Acceptance Sampling Plan Based On Generalized Exponential Distributions

In this chapter, a truncated life test group acceptance sampling plan using median is developed based on truncated lifetimes when the lifetime of an item follows a generalized exponential distribution. For a given group size, the minimum number of groups and the acceptance number required are determined for specified consumer confidence level and the test termination time. The operating characteristic values for various quality levels are calculated and analysed. The minimum ratios of true median life to the specified life at given producer's risk are obtained. Tables are constructed and the results are illustrated through various numerical examples.

Generalized Exponential Distribution

The lifetime of a product is assumed to have the generalized exponential distribution. Let $T \mid \{(\delta, \lambda)\}$ with $\delta > 0$ and $\lambda > 0$ be a lifetime that is distributed according to a two parameter generalized exponential distribution. Its density function and distribution function are given respectively as

$$g_{T \mid \{(\delta,\lambda)\}}(t) = \frac{\delta}{\lambda} e^{-\frac{t}{\lambda}} \left(1 - e^{-\frac{t}{\lambda}} \right) \qquad \text{for } t > 0, \delta > 0, \lambda > 0 \tag{1.1}$$

$$G_{T \mid \{(\delta,\lambda)\}}(t) = \left(1 - e^{-\frac{t}{\lambda}} \right)^{\delta} \qquad \text{for } t > 0, \delta > 0, \lambda > 0 \tag{1.2}$$

where λ and δ are scale and shape parameters respectively.

The median, μ is defined by

$$P(X \geq \mu) = P(X \leq \mu) = \frac{1}{2}$$

The median of this distribution for a specified value of $\delta = 2$ is given by

$$\mu = 1.2279\lambda \tag{1.3}$$

The Group Acceptance Sampling Plan (GASP)

The Operating Procedure of Group acceptance sampling plan for the truncated life test is

i. Select the number of groups g and allocate pre-defined r items to each group so that the sample size for a lot will be n = r.g.

ii. Select the acceptance number c for a group and the experiment time t_0 .

iii. Perform the experiment for the g groups simultaneously and record the number of failures for each group.

iv. Accept the lot if at most c failures occur in each group.

v. Reject the lot if more than c failures occur in any group and terminate the experiment.

The proposed sampling plan becomes the ordinary sampling plan for r = 1 and therefore n = g.

Let μ be the true value of the median of the lifetime distribution of a product, and μ_0 denote the specified median, under the assumption that the lifetime of an item follows a generalized exponential distribution. A product is assumed as good and acceptable for consumer's use, if the actual value μ is not smaller than the specified value μ_0, otherwise the lot of the product will be rejected. In acceptance sampling schemes the hypothesis $\mathrm{H}_0 : \mu \geq \mu_0$ is tested based on the number of failures from a sample in a pre-fixed time. If the number of failures exceeds the acceptance number c, the lot is rejected otherwise it is accepted. The sample size and the acceptance number have to be determined so that there is enough evidence for $\mu \geq \mu_0$ at a given level of the consumer's risk.

For the generalized exponential distribution with various values of the acceptance number c, group size r and the termination time t_0 the number of group g is to be estimated. Since it is convenient to set the termination time as a multiple of the specified value μ_0 of the median, therefore consider $t_0 = a\mu_0$ for a given constant, a termination ratio.

The probability of rejecting a good lot is called the producer's risk, and is denoted by α whereas the probability of accepting a bad lot is known as the consumer's risk and is denoted by β. The parameter value g of the proposed sampling plan is determined for ensuring the consumer's risk β. Often the, consumer's risk β is expressed as by the consumer's confidence level. If the confidence level is P^*, then the consumer's risk, β will be $1-\mathrm{P}^*$. The number of groups, g is determined in the proposed sampling plan so that the consumer's risk does not exceed a given value β.

To develop the Group Acceptance Sampling Plan, we can use the binomial distribution if the lot size, n is large and the probability of success, p is same in each trial and Poisson distribution if the lot size, n is very large and the probability of the success, p is very small such that np is finite.

According to the GASP the lot of products is accepted only if there are at most c failures observed in each of the g groups. So, the lot acceptance probability using binomial and Poisson model are given by

$$\left.\begin{aligned} L(p) &= \left(\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i} \right)^g \\ L(p) &= \left(\sum_{i=0}^{c} e^{-rp} \frac{(rp)^i}{i!} \right)^g \end{aligned}\right\} \quad (1.4)$$

where p is the probability that an item in a group fails before the termination time $t_0 = a\mu_0$. The probability p for the generalized exponential distribution with $\delta = 2$ is given by,

$$p = G_{T|\{(2,\lambda)\}}(t_0) = \left(1 - e^{-\frac{1.2279 a\mu}{\mu_0}} \right)^{\delta} \qquad \text{for } t > 0 \qquad (1.5)$$

The minimum number of groups g required can be determined by considering the consumer's risk when the true median life equals the specified median life by the following inequality,

$$L(p_0) \le \beta \tag{1.6}$$

where p_0 is the failure probability at $\mu = \mu_0$ and it is given by

$$p_0 = \left(1 - e^{-1.2279a}\right)^{\delta} \tag{1.7}$$

Particularly for c = 0 known as zero failure test, the number of groups g can be determined by the minimum integer satisfying the following inequalities,

$$\left.\begin{array}{ll} g \ge \dfrac{\ln \beta}{r \ln(1 - p_0)} & \text{under binomial model} \\ \\ g \le \dfrac{-\ln \beta}{rp} & \text{under Poisson model} \end{array}\right\} \tag{1.8}$$

The minimum number of groups required for the proposed sampling plan in case of the generalized exponential distributions for the special case $\delta = 2$ are calculated and displayed in Tables 1.1.to 1.4. for the specified values of group size, acceptance number time multiplier and consumer's risk using equations (1.6), (1.7) and (1.8).

Numerical values in Tables1.1. to 1.4. reveal that

i. increase in consumer's risk increases the number of groups for fixed termination ratio.
ii. increase in termination ratio decreases the number of groups for fixed consumer's risk.
iii. increase in acceptance number increases the number of groups for fixed consumer's risk and termination ratio.
iv. increase in number of items in groups decreases the number of groups for fixed consumer's risk and acceptance number.

Table1.1. Minimum number of groups, g in Zero failure test under binomial model

β	r	a					
		0.6	0.8	1.0	1.5	2.0	2.5
0.25	2	3	2	2	1	1	1
	4	2	2	1	1	1	1
	6	1	1	1	1	1	1
	8	1	1	1	1	1	1
0.10	2	4	3	2	1	1	1
	4	2	1	1	1	1	1
	6	1	1	1	1	1	1
	8	1	1	1	1	1	1
0.05	2	5	4	3	2	1	1
	4	3	2	2	1	1	1
	6	2	2	1	1	1	1
	8	2	1	1	1	1	1
0.01	2	2	2	1	1	1	1
	4	4	3	2	1	1	1
	6	3	2	2	1	1	1
	8	2	2	1	1	1	1

Table1.2. Minimum number of groups, g in Zero failure test under Poisson model

β	r	a					
		0.6	0.8	1.0	1.5	2.0	2.5
0.25	2	3	2	2	1	1	1
	4	2	2	1	1	1	1
	6	1	1	1	1	1	1
	8	1	1	1	1	1	1
0.10	2	4	3	2	1	1	1
	4	3	2	1	1	1	1
	6	2	1	1	1	1	1
	8	1	1	1	1	1	1
0.05	2	6	4	3	2	1	1
	4	4	3	2	1	1	1
	6	3	2	1	1	1	1
	8	1	1	1	1	1	1
0.01	2	3	2	2	1	1	1
	4	2	2	1	1	1	1
	6	2	2	1	1	1	1
	8	1	1	1	1	1	1

Table1.3. Minimum number of groups (g) for the proposed plan with $\delta = 2$ under binomial model

β	r	c	a					
			0.6	0.8	1.0	1.5	2.0	2.5
0.25	2	1	19	10	4	2	1	1
	4	1	4	2	1	1	1	1
		2	23	8	4	2	1	1
	6	1	2	1	1	1	1	1
		2	7	3	2	1	1	1
		3	27	8	4	1	1	1
	8	1	2	1	1	1	1	1
		2	3	2	1	1	1	1
		3	9	3	2	1	1	1
		4	36	8	4	1	1	1
		5	208	31	9	2	1	1

β	r	c	a					
			0.6	0.8	1.0	1.5	2.0	2.5
0.10	2	1	31	14	9	4	2	2
	4	1	7	4	2	1	1	1
		2	35	13	7	3	2	1
	6	1	4	2	2	1	1	1
		2	11	4	3	1	1	1
		3	45	13	6	2	1	1
	8	1	3	2	1	1	1	1
		2	5	3	2	1	1	1
		3	15	5	3	1	1	1
		4	59	14	6	2	1	1
		5	345	51	15	3	2	1

Continuation of Table 1.3

β	r	c	a 0.6	0.8	1.0	1.5	2.0	2.5
0.05	2	1	40	27	11	5	3	2
	4	1	9	5	3	2	1	1
		2	46	17	8	3	2	1
	6	1	5	3	2	1	1	1
		2	14	6	3	2	1	1
		3	59	17	8	3	2	1
	8	1	3	2	1	1	1	1
		2	7	3	2	1	1	1
		3	19	7	3	2	1	1
		4	76	18	7	2	1	1
		5	449	66	20	4	2	1

β	r	c	a 0.6	0.8	1.0	1.5	2.0	2.5
0.01	2	1	61	28	17	7	4	3
	4	1	13	7	4	2	2	1
		2	70	25	13	5	3	2
	6	1	7	4	3	1	1	1
		2	21	8	5	2	1	1
		3	90	26	11	4	2	2
	8	1	4	3	2	1	1	1
		2	10	5	3	1	1	1
		3	30	10	5	2	1	1
		4	117	27	11	3	2	1
		5	690	101	30	6	3	2

Table1.4. Minimum number of groups (g) for the proposed plan with $\delta = 2$ under Poisson model

β	r	c	a: 0.6	0.8	1.0	1.5	2.0	2.5
0.25	2	1	13	7	5	3	3	1
	4	1	4	3	2	1	1	1
		2	14	6	4	2	1	1
	6	1	3	2	1	1	1	1
		2	6	3	2	1	1	1
		3	16	6	4	2	2	1
	8	1	2	1	1	1	1	1
		2	3	2	1	1	1	1
		3	8	3	2	1	1	1
		4	20	6	3	2	1	1
		5	58	14	6	3	2	2
0.10	2	1	22	12	8	5	4	3
	4	1	7	4	3	2	2	2
		2	23	10	6	3	3	2
	6	1	4	3	2	1	1	1
		2	10	5	3	2	2	1
		3	27	10	6	3	2	2
	8	1	3	2	1	1	1	1
		2	5	3	2	1	1	1
		3	12	5	3	2	1	1
		4	32	10	5	3	2	2
		5	96	23	10	4	3	2

Continuation of Table 1.4.

β	r	c	a 0.6	0.8	1.0	1.5	2.0	2.5
0.05	2	1	28	15	10	6	5	4
	4	1	9	5	4	3	2	2
		2	30	13	8	4	3	3
	6	1	5	3	2	2	1	1
		2	12	6	4	2	2	2
		3	35	13	7	4	3	2
	8	1	3	2	2	1	1	1
		2	7	4	3	2	1	1
		3	16	7	4	2	1	2
		4	42	13	7	3	2	2
		5	125	30	13	5	2	2

β	r	c	a 0.6	0.8	1.0	1.5	2.0	2.5
0.01	2	1	43	23	16	9	7	6
	4	1	14	8	6	4	3	3
		2	46	20	12	6	5	4
	6	1	7	5	3	2	2	2
		2	19	9	6	3	3	2
		3	54	20	11	5	4	3
	8	1	5	3	2	2	1	1
		2	10	5	4	2	2	2
		3	24	10	6	3	2	2
		4	64	20	10	5	3	3
		5	192	45	20	7	5	4

Operating Characteristic Values

Operating characteristic values display the discrimination power of the sampling plan. Once the minimum number of groups g is obtained, one may be interested to find the probability of acceptance of a lot when the quality of the product is sufficiently good. The product is considered to be good if $\mu \geq \mu_0$. For $\delta = 2$ the probabilities of acceptance obtained from equations (1.5) and (1.5) are given in Tables 1.5 and 1.6 based on various selected values of the median ratio, consumer's risk, termination ratio and the time multiplier.

The following observations are observed from the computed numerical values presented in Tables 1.5 and 1.6.

i. increase in the consumer's risk increases the value of operating characteristics for fixed termination ratio and acceptance number.

ii. increase in termination ratio decreases the value of operating characteristic for fixed consumer's risk and number of items in group.

iii. increase in the median ratio increases the operating characteristic values for fixed termination ratio, consumer's risk and number of items in a group.

iv. increases in number of items in a group increases the operating characteristic values for fixed acceptance number, consumer's risk and termination ratio.

v. increases in groups increases the operating characteristic values for fixed acceptance number, consumer's risk and termination ratio.

Table.1.5. Operating characteristics value of group sampling plan with c =1 under binomial model

β	r	g	a	μ / μ_0					
				2	4	6	8	10	12
0.25	2	4	1.0	0.834211	0.980615	0.995383	0.998361	0.999287	0.999647
		2	1.5	0.754804	0.963261	0.990392	0.996426	0.998399	0.999192
		1	2.0	0.750020	0.955694	0.987436	0.995118	0.997752	0.998844
	4	4	1.0	0.880140	0.994838	0.999387	0.999869	0.999962	0.999987
		2	1.5	0.742255	0.981947	0.997467	0.999414	0.999823	0.999936
		1	2.0	0.687529	0.968585	0.994840	0.998707	0.999589	0.999847
	6	2	1.0	0.788210	0.988417	0.998545	0.999638	0.999908	0.999968
		1	1.5	0.622620	0.963354	0.994303	0.998626	0.999575	0.999845
		1	2.0	0.343787	0.887812	0.978321	0.994192	0.998087	0.999272
	8	1	1.0	0.773385	0.985375	0.998064	0.999569	0.999873	0.999955
		1	1.5	0.398997	0.916764	0.985646	0.996392	0.998860	0.999579
		1	2.0	0.144557	0.773385	0.948870	0.985375	0.995015	0.998064
0.10	2	9	1.0	0.665075	0.956911	0.989642	0.996317	0.998397	0.999207
		4	1.5	0.569729	0.927871	0.980877	0.992865	0.996801	0.998386
		2	2.0	0.562529	0.913352	0.97503	0.99026	0.995509	0.997689
	4	2	1.0	0.938158	0.997416	0.999694	0.999935	0.999981	0.999993
		1	1.5	0.861542	0.990932	0.998733	0.999707	0.999911	0.999968
		1	2.0	0.687529	0.968585	0.99484	0.998707	0.999589	0.999847
	6	2	1.0	0.788210	0.988417	0.998545	0.999683	0.999908	0.999968
		1	1.5	0.622620	0.963354	0.994303	0.998626	0.999575	0.999845
		1	2.0	0.343787	0.887812	0.978321	0.994192	0.998087	0.999272
	8	2	1.0	0.598124	0.970964	0.996132	0.999139	0.999746	0.999911
		1	1.5	0.398997	0.916764	0.985646	0.996392	0.99886	0.999579
		1	2.0	0.144557	0.773385	0.94887	0.985375	0.995015	0.999806
0.05	2	11	1.0	0.607447	0.947590	0.987355	0.995500	0.998041	0.999031
		5	1.5	0.494978	0.910667	0.976153	0.991080	0.996003	0.997982
		3	2.0	0.421908	0.872885	0.962780	0.995426	0.993271	0.996535
	4	8	1.0	0.774646	0.989703	0.998775	0.999738	0.999925	0.999974
		3	1.5	0.639484	0.973043	0.996204	0.999121	0.999734	0.999904
		2	2.0	0.472697	0.938158	0.999708	0.997416	0.999178	0.999640
	6	3	1.0	0.699783	0.982676	0.997818	0.999524	0.999861	0.999951
		2	1.5	0.387210	0.928051	0.988639	0.997254	0.999150	0.999690
		1	2.0	0.343787	0.887812	0.978321	0.994192	0.998087	0.999272
	8	2	1.0	0.598124	0.970964	0.996132	0.999139	0.999746	0.999911
		1	1.5	0.398997	0.916764	0.985646	0.996392	0.998860	0.999579
		1	2.0	0.144557	0.773385	0.948870	0.985375	0.995015	0.998064
0.01	2	17	1.0	0.462831	0.920171	0.980525	0.993054	0.996974	0.998503
		7	1.5	0.373611	0.877210	0.966775	0.987547	0.994408	0.997176
		4	2.0	0.316439	0.874211	0.950683	0.980615	0.991038	0.995383
	4	13	1.0	0.660790	0.983322	0.998010	0.999575	0.999877	0.999957
		5	1.5	0.474661	0.955577	0.998536	0.998536	0.999557	0.999840
		3	2.0	0.324993	0.908686	0.996126	0.996126	0.998767	0.999540
	6	5	1.0	0.551576	0.971923	0.996366	0.999208	0.999769	0.999919
		2	1.5	0.387210	0.928051	0.988639	0.997254	0.999150	0.999690
		1	2.0	0.343787	0.887812	0.978321	0.994192	0.990887	0.992720
	8	3	1.0	0.462580	0.956764	0.994204	0.998709	0.999620	0.999856
		1	1.5	0.398997	0.916764	0.985646	0.996392	0.998860	0.999579
		1	2.0	0.144557	0.773385	0.948870	0.985375	0.995015	0.998064

Table.1.6. Operating characteristics value of group sampling plan with c =1 under Poisson model

β	R	g	a	μ / μ_0					
				2	4	6	8	10	12
0.25	2	5	1.0	0.955326	0.997953	0.999751	0.999946	0.999996	1.000000
		3	1.5	0.892547	0.991778	0.998796	0.999716	0.999993	1.000000
		3	2.0	0.779410	0.972951	0.995242	0.998771	0.999603	0.999999
	4	4	1.0	0.802082	0.988233	0.998486	0.999667	0.999992	1.000000
		2	1.5	0.674965	0.964320	0.994211	0.998578	0.999956	0.999999
		1	2.0	0.676698	0.946356	0.989227	0.997045	9990130	0.999837
	6	2	1.0	0.749196	0.982087	0.997570	0.999454	0.999839	0.999943
		1	1.5	0.629872	0.950102	0.991167	0.997746	0.999281	0.999733
		1	2.0	0.423216	0.865561	0.969115	0.991003	0.996895	0.998784
	8	1	1.0	0.761463	0.980744	0.997260	0.999373	0.999812	0.999933
		1	1.5	0.446677	0.902554	0.981083	0.994981	0.998367	0.999386
		1	2.0	0.238126	0.761463	0.937683	0.980744	0.993137	0.997260
0.10	2	8	1.0	0.929486	0.996727	0.999602	0.999914	0.999995	1.000000
		5	1.5	0.827406	0.986334	0.997993	0.999527	0.999989	1.000000
		4	2.0	0.715477	0.964099	0.993661	0.998362	0.999899	0.999991
	4	6	1.0	0.718337	0.982402	0.997729	0.999500	0.999854	0.999949
		3	1.5	0.554526	0.946960	0.991329	0.997868	0.999334	0.999755
		3	2.0	0.309873	0.847548	0.968027	0.991162	0.997042	0.998864
	6	3	1.0	0.648475	0.973251	0.996358	0.999182	0.999758	0.999910
		2	1.5	0.396738	0.902693	0.982413	0.995497	0.998563	0.999466
		2	2.0	0.179112	0.749196	0.939183	0.982087	0.993800	0.997570
	8	2	1.0	0.579826	0.961859	0.994528	0.998745	0.999625	0.999866
		1	1.5	0.446477	0.902554	0.981083	0.994981	0.998367	0.999386
		1	2.0	0.238126	0.761463	0.937683	0.980744	0.993137	0.997260
0.05	2	10	1.0	0.912648	0.995910	0.999502	0.999893	0.999969	0.999989
		6	1.5	0.796641	0.983623	0.997592	0.999432	0.999826	0.999937
		5	2.0	0.658028	0.955326	0.992082	0.997953	0.999338	0.999751
	4	8	1.0	0.643360	0.976605	0.996973	0.999334	0.999805	0.999931
		4	1.5	0.455578	0.929913	0.988455	0.997159	0.999112	0.999674
		3	2.0	0.309873	0.847543	0.968027	0.991162	0.997042	0.998864
	6	4	1.0	0.561295	0.964494	0.995147	0.998909	0.999677	0.999886
		2	1.5	0.396738	0.902693	0.982413	0.995497	0.998563	0.999466
		2	2.0	0.17912	0.749196	0.939183	0.982087	0.993800	0.997570
	8	3	1.0	0.441516	0.943338	0.991804	0.998119	0.999438	0.999800
		2	1.5	0.199341	0.814604	0.962525	0.989987	0.996736	0.998772
		1	2.0	0.238126	0.761463	0.937683	0.980744	0.993137	0.997260
0.01	2	16	1.0	0.863944	0.993465	0.999203	0.999828	0.999950	0.999983
		9	1.5	0.711039	0.975536	0.996391	0.999148	0.997390	0.999905
		7	2.0	0.556599	0.938021	0.988932	0.997136	0.999074	0.999651
	4	12	1.0	0.516008	0.965113	0.995463	0.999001	0.999707	0.999897
		6	1.5	0.307499	0.896733	0.982733	0.995741	0.998668	0.999511
		5	2.0	0.141897	0.759055	0.947281	0.985313	0.995075	0.998107
	6	6	1.0	0.420520	0.947217	0.992729	0.998364	0.999516	0.999829
		3	1.5	0.249894	0.857650	0.973735	0.993253	0.997846	0.999199
		3	2.0	0.075803	0.648475	0.910176	0.973251	0.990714	0.996358
	8	4	1.0	0.336199	0.925173	0.989086	0.997492	0.999250	0.999733
		2	1.5	0.199349	0.814604	0.962525	0.989987	0.996736	0.998772
		2	2.0	0.056704	0.579826	0.879250	0.961859	0.986321	0.994528

Minimum Median ratios

The producer may be interested in enhancing the quality level of the product so that the acceptance probability should be larger than a specified level. For a given producer's risk, α the minimum ratio can be obtained by satisfying the following inequality,

$$L(p) \geq 1 - \alpha$$

(1.9)

where L(p), the probability of acceptance is given by equation (1.4), p is given by equation (1.5) and g is chosen according to the consumer's risk β when $\mu/\mu_0 = 1$. the minimum median ratios to the specified life of generalized exponential distribution with $\delta = 2$ and the producers risk $\alpha = 0.05$ are given in Tables 1.7 and 1.8.

From the numerical values of Tables 1.7and 1.8, it is seen that

i. increase in the consumer's risk decrease the minimum ratio value for given producer's risk, acceptance number and termination ratio.

ii. increase in number of items in group increases the minimum ratio value for given consumer's risk, acceptance number and producer's risk.

iii. increase in acceptance number decreases the minimum median ratio value for given producer's risk and consumer's risk.

Table 1.7. Minimum Median ratios to the specified life at producer's risk of 0.05 under binomial model

β	r	c	a 0.6	0.8	1.0	1.5	2	2.5
0.25	2	0	7.62	8.20	10.26	11.00	14.17	17.72
		1	2.85	3.16	3.00	3.62	3.84	4.80
	4	0	8.85	11.79	10.26	15.45	20.60	25.75
		1	3.00	3.26	3.28	4.92	6.56	8.21
		2	2.16	2.31	2.48	3.17	3.56	4.45
	6	0	7.62	10.15	12.69	19.06	25.59	31.75
		1	3.15	3.41	4.26	6.39	8.51	10.64
		2	2.29	2.55	2.91	3.71	4.95	6.19
		3	1.82	1.98	2.19	2.51	3.34	4.18
	8	0	8.85	11.80	14.75	22.11	29.49	36.88
		1	3.74	4.05	5.07	7.60	10.15	12.67
		2	2.33	2.90	3.03	4.55	6.06	7.58
		3	1.90	2.10	2.43	3.19	4.25	5.31
		4	1.66	1.79	2.01	2.39	3.19	3.98
		5	1.59	1.70	1.83	2.19	2.75	3.07

β	r	c	a 0.6	0.8	1.0	1.5	2	2.5
0.10	2	0	9.93	11.79	12.69	15.39	14.18	17.73
		1	3.52	4.20	4.06	4.82	5.49	6.04
	4	0	8.90	11.80	10.30	15.39	20.51	25.65
		1	3.52	4.00	4.07	4.92	6.60	8.31
		2	2.35	2.56	2.80	3.48	4.22	4.45
	6	0	10.99	10.15	12.69	19.06	25.38	31.72
		1	3.85	4.20	5.25	6.39	8.51	10.65
		2	2.52	2.72	3.19	3.71	4.95	6.19
		3	1.98	2.15	2.35	2.88	3.34	4.19
	8	0	12.66	11.80	14.89	22.11	29.50	36.85
		1	4.20	4.98	5.07	7.60	10.13	12.67
		2	2.78	3.10	3.55	4.55	6.06	7.58
		3	2.15	2.43	2.62	3.65	4.25	5.31
		4	1.83	2.01	2.19	2.70	3.19	3.98
		5	1.64	1.76	1.90	2.29	2.75	3.07

Continuation of Table 1.7

β	r	c	a					
			0.6	0.8	1.0	1.5	2	2.5
0.05	2	0	7.62	8.20	10.26	11.00	14.17	17.72
		1	2.85	3.16	3.00	3.62	3.84	4.80
	4	0	8.85	11.79	10.26	15.45	20.60	25.75
		1	3.00	3.26	3.28	4.92	6.56	8.21
		2	2.16	2.31	2.48	3.17	3.56	4.45
	6	0	10.99	10.15	12.69	19.06	25.38	31.72
		1	3.85	4.20	5.25	6.39	8.51	10.65
		2	2.52	2.72	3.19	3.71	4.95	6.19
		3	1.98	2.15	2.35	2.88	3.34	4.19
	8	0	8.85	11.80	14.75	22.11	29.49	36.88
		1	3.74	4.05	5.07	7.60	10.15	12.67
		2	2.33	2.90	3.03	4.55	6.06	7.58
		3	1.90	2.10	2.43	3.19	4.25	5.31
		4	1.66	1.79	2.01	2.39	3.19	3.98
		5	1.59	1.70	1.83	2.19	2.75	3.07

β	r	c	a					
			0.6	0.8	1.0	1.5	2	2.5
0.01	2	0	9.93	11.79	12.69	15.39	14.18	17.73
		1	3.52	4.20	4.06	4.82	5.49	6.04
	4	0	10.91	11.79	14.74	15.38	20.51	25.63
		1	3.78	4.26	4.59	6.10	6.56	8.20
		2	2.48	2.70	2.88	3.49	4.22	4.45
	6	0	7.62	10.15	12.69	19.06	25.59	31.75
		1	3.15	3.41	4.26	6.39	8.51	10.64
		2	2.29	2.55	2.91	3.71	4.95	6.19
		3	1.82	1.98	2.19	2.51	3.34	4.18
	8	0	12.66	11.80	14.89	22.11	29.50	36.85
		1	4.20	4.98	5.07	7.60	10.13	12.67
		2	2.78	3.10	3.55	4.55	6.06	7.58
		3	2.15	2.43	2.62	3.65	4.25	5.31
		4	1.83	2.01	2.19	2.70	3.19	3.98
		5	1.64	1.76	1.90	2.29	2.75	3.07

Table 1.8. Minimum Median ratios to the specified life at producer's risk of 0.05 under Poisson model

β	r	c	a = 0.6	0.8	1.0	1.5	2	2.5
0.25	2	0	7.62	8.20	10.26	11.00	14.17	17.72
		1	2.85	3.16	3.00	3.62	3.84	4.80
	4	0	8.85	11.79	10.26	15.45	20.60	25.75
		1	3.00	3.26	3.28	4.92	6.56	8.21
		2	2.16	2.31	2.48	3.17	3.56	4.45
	6	0	7.62	10.15	12.69	19.06	25.59	31.75
		1	3.15	3.41	4.26	6.39	8.51	10.64
		2	2.29	2.55	2.91	3.71	4.95	6.19
		3	1.82	1.98	2.19	2.51	3.34	4.18
	8	0	8.85	11.80	14.75	22.11	29.49	36.88
		1	3.74	4.05	5.07	7.60	10.15	12.67
		2	2.33	2.90	3.03	4.55	6.06	7.58
		3	1.90	2.10	2.43	3.19	4.25	5.31
		4	1.66	1.79	2.01	2.39	3.19	3.98
		5	1.59	1.70	1.83	2.19	2.75	3.07

β	r	c	a = 0.6	0.8	1.0	1.5	2	2.5
0.10	2	0	9.93	11.79	12.69	15.39	14.18	17.73
		1	3.52	4.20	4.06	4.82	5.49	6.04
	4	0	8.90	11.80	10.30	15.39	20.51	25.65
		1	3.52	4.00	4.07	4.92	6.60	8.31
		2	2.35	2.56	2.80	3.48	4.22	4.45
	6	0	10.99	10.15	12.69	19.06	25.38	31.72
		1	3.85	4.20	5.25	6.39	8.51	10.65
		2	2.52	2.72	3.19	3.71	4.95	6.19
		3	1.98	2.15	2.35	2.88	3.34	4.19
	8	0	12.66	11.80	14.89	22.11	29.50	36.85
		1	4.20	4.98	5.07	7.60	10.13	12.67
		2	2.78	3.10	3.55	4.55	6.06	7.58
		3	2.15	2.43	2.62	3.65	4.25	5.31
		4	1.83	2.01	2.19	2.70	3.19	3.98
		5	1.64	1.76	1.90	2.29	2.75	3.07

Continuation of Table 1.8

β	r	c	a: 0.6	0.8	1.0	1.5	2	2.5
0.05	2	0	7.62	8.20	10.26	11.00	14.17	17.72
		1	2.85	3.16	3.00	3.62	3.84	4.80
	4	0	8.85	11.79	10.26	15.45	20.60	25.75
		1	3.00	3.26	3.28	4.92	6.56	8.21
		2	2.16	2.31	2.48	3.17	3.56	4.45
	6	0	10.99	10.15	12.69	19.06	25.38	31.72
		1	3.85	4.20	5.25	6.39	8.51	10.65
		2	2.52	2.72	3.19	3.71	4.95	6.19
		3	1.98	2.15	2.35	2.88	3.34	4.19
	8	0	8.85	11.80	14.75	22.11	29.49	36.88
		1	3.74	4.05	5.07	7.60	10.15	12.67
		2	2.33	2.90	3.03	4.55	6.06	7.58
		3	1.90	2.10	2.43	3.19	4.25	5.31
		4	1.66	1.79	2.01	2.39	3.19	3.98
		5	1.59	1.70	1.83	2.19	2.75	3.07

β	r	c	a: 0.6	0.8	1.0	1.5	2	2.5
0.01	2	0	9.93	11.79	12.69	15.39	14.18	17.73
		1	3.52	4.20	4.06	4.82	5.49	6.04
	4	0	10.91	11.79	14.74	15.38	20.51	25.63
		1	3.78	4.26	4.59	6.10	6.56	8.20
		2	2.48	2.70	2.88	3.49	4.22	4.45
	6	0	7.62	10.15	12.69	19.06	25.59	31.75
		1	3.15	3.41	4.26	6.39	8.51	10.64
		2	2.29	2.55	2.91	3.71	4.95	6.19
		3	1.82	1.98	2.19	2.51	3.34	4.18
	8	0	12.66	11.80	14.89	22.11	29.50	36.85
		1	4.20	4.98	5.07	7.60	10.13	12.67
		2	2.78	3.10	3.55	4.55	6.06	7.58
		3	2.15	2.43	2.62	3.65	4.25	5.31
		4	1.83	2.01	2.19	2.70	3.19	3.98
		5	1.64	1.76	1.90	2.29	2.75	3.07

Numerical illustration

An illustration is provided to explain the application of the given sampling plan for life testing. Assume that the lifetime of the product under consideration follows generalized exponential distribution. It is desired to design a GASP to test if the median is greater than 1,000 hours based on a testing time of 1500 hours. This gives the termination multiplier a = 1.5. When the tester is equipped with 2 items each is used. It is assumed that c=1 and $\beta = 0.10$.

From Table.1.3 the minimum number of groups required is obtained as g=4. Thus, a random sample of size n= 8 items is to be drawn and allocate 8 items to each of the 4 groups to put on the test for 1500 hours. This indicates that a total of 8 products are needed and that 2 items are allocated to each of the 4 testers. The lot is accepted if no more than 1 failure occurs before 1500 hours in each of the 4 groups. The experiment is truncated as soon as the 2nd failure occurs before 1500th hour.

For this proposed sampling plan the probability of acceptance is L (p) =0.980877 when the true value of the median is $\mu = 6{,}000$ hours. This shows that, if the true value of the median is 6 times of the required value $\mu_0 = 1000$ hours the producer's risk is $\alpha = 0.019123$.

Given the ratio to assure a producer's risk of $\alpha = 0.05$, when $\beta = 0.10$, r=2, g = 4 and a= 1.5, the required ratio is $\mu/\mu_0 = 4.92$.

Chapter - II

Chapter II

Life test group acceptance sampling plan based on

Kumaraswamy-Log-logistic Distribution

In this chapter, a truncated life test group acceptance sampling plan is developed using median for when the lifetime of item follows Kumaraswamy-log-logistic distribution. The beauty and importance of this distribution lies in accommodating several important distributions as sub-models. For a given group size, the minimum number of groups and the acceptance number required are determined for specified consumer's risk and the test termination time. The values of operating characteristic function for various quality levels are analysed and the minimum ratios of the true median life to the specified median life at given producer's risk are obtained under binomial and Poisson models. The results are illustrated by numerical examples.

Kumaraswamy-log-logistic distribution is the result of the work done by Kumaraswamy (1980) and Cordeiro and de Castro (2011). This new distribution contains several important distributions as sub-models. This motivated the researcher to use Kumaraswamy-log-logistic distribution as the distribution of the life time of the product under study in order to have wider applicability.

The group acceptance sampling plan for the truncated life test assuming that the lifetime of a product follows the Kumaraswamy-log-logistic distribution is proposed in this chapter which gives description of Kumaraswamy-log-logistic distribution, procedure of group sampling plan, analysis of the minimum number of groups at the specified consumer's risk, analysis of the operating characteristics as a function of the ratio of the true median life to the specified median life and the evaluation of minimum ratios of the median life to the specified median life so as to lower the producer's risk at the specified level. Numerical examples are provided to explain the application of the proposed sampling plan in real life situations.

Kumaraswamy-Log-Logistic Distribution

The lifetime of a product is assumed to have Kumaraswamy-log-logistic distribution (KLLD), whose probability density function and cumulative distribution function are given respectively as

$$f(t;a,b,c,\gamma)=\frac{ab\gamma}{c^{a\gamma}}t^{a\gamma-1}\left[1+(\frac{t}{c})^{\gamma}\right]^{-(a+1)}\left\{1-\left[1-\frac{1}{1+(\frac{t}{c})^{\gamma}}\right]^{a}\right\}^{b-1}\quad t>0,a,b,c,\gamma>0 \quad (2.1)$$

and

$$F(t;a,b,c,\gamma)=1-\left\{1-\left[1-\frac{1}{1+\left(\frac{t}{c}\right)^{\gamma}}\right]^{a}\right\}^{b},t\geq 0,a,b,c,\gamma>0 \quad (2.2)$$

where c is a scale parameter and a, b and γ are the shape parameters. If T is a random variable relating to lifetime of the product with density function (2.1) is denoted as T~KLLD (a,b,c,γ) The median of the Kumaraswamy-log-logistic distribution is derived by

$$median,\mu=c\left[\frac{\left\{1-(0.5)^{\frac{1}{b}}\right\}^{\frac{1}{a}}}{1-\left\{1-(0.5)^{\frac{1}{b}}\right\}^{\frac{1}{a}}}\right]^{\frac{1}{\gamma}} \quad (2.3)$$

Expression (2.3) shows that the median is proportional to the scale parameter, c when the two shape parameters a and b are fixed. It is also seen that for Kumaraswamy-log-logistic distribution the median reduces to c regardless of γ when a=b=1.

Kumaraswamy-log- logistic distribution with scale parameter c and shape parameter a, b and γ reduces to

Burr distribution when a=1,

Exponentiated log- logistic distribution when b=1 and

Log-logistic distribution when a=b=1.

Design of the Group Acceptance Sampling Plan

The operating procedure of group acceptance sampling plan for the truncated life test has the following steps,

(1) Select a random sample of size n from a lot.
(2) Allocate r items to each of g groups (or testers) so that n= r.g where r is pre-defined.
(3) Put r items on test before the termination time t_0.
(4) Accept the lot if the total number of failures from g groups is less than or equal to c.
(5) Reject the lot if the total number of failures from g groups is greater than c before t_0 and truncate the test.

Assume that the quality of a product can be represented by its median lifetime, μ. The lot will be accepted if the submitted lot has a good quality when the experimental data supports the null hypothesis, $H_0 : \mu \geq \mu_0$ against the alternative hypothesis, $H_1 : \mu < \mu_0$, where μ_0 is a specified median life time. The significance level for the test is considered as $\beta = 1 - P^*$, where P^* is the consumer's confidence level.

The group sampling plan for the truncated life test consists of deriving (1) the minimum number of groups, (2) acceptance numbers and (3) the ratio of true median life to the specified median life μ/μ_0.

The determination of the number of group g required for in the case of the Kumaraswamy-log-logistic distribution and various values of acceptance number c, whereas the group size r and the termination time t_0 are assumed to be given. Since, it is convenient to set the termination time as a multiple of the specified value μ_0 of the median, we will consider $t_0 = \eta\mu_0$ for a given constant η (termination ratio).

To develop the Group Acceptance Sampling Plan, we can use the binomial distribution if the lot size, n is large and the probability of success, p is same in each trial and Poisson distribution if the lot size, n is very large and the probability of the success, p is very small such that np is finite.

The probability of acceptance of the lot for the proposed group acceptance sampling plan under binomial and Poisson models are

$$\left.\begin{aligned} L(p) &= \left(\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i} \right)^g \\ L(p) &= \left(\sum_{i=0}^{c} e^{-rp} \frac{(rp)^i}{i!} \right)^g \end{aligned}\right\} \qquad (2.4)$$

where p is the probability that an item in a group fails before the termination time t_0, where $t_0 = \eta\mu_0$ and η is the time multiplier.

The probability p for the Kumaraswamy-log-logistic distribution is given by

$$p = 1 - \left\{ 1 - \left[1 - \frac{1}{1 + \left(\frac{t_0}{c} \right)^{\gamma}} \right]^a \right\}^b$$

$$= 1 - \left\{ 1 - \left[\frac{(\eta\delta)^{\gamma}}{\left(\frac{\mu}{\mu_0} \right)^{\gamma} + (\eta\delta)^{\gamma}} \right]^a \right\}^b \qquad (2.5)$$

where $$\delta = \left[\frac{\left\{ 1 - (0.5)^{\frac{1}{b}} \right\}^{\frac{1}{a}}}{1 - \left\{ 1 - (0.5)^{\frac{1}{b}} \right\}^{\frac{1}{a}}} \right]^{\frac{1}{\gamma}}$$

The minimum number of groups can be determined by considering the consumer's risk when the true median life equals the specified median life by the following inequality

$$L(p_0) \leq \beta \tag{2.6}$$

where p_0 is the failure probability at $\mu = \mu_0$ and it is given by

$$p_0 = 1 - \left\{ 1 - \left(\frac{(\eta\delta)^{\gamma}}{1 + (\eta\delta)^{\gamma}} \right)^{a} \right\}^{b} \tag{2.7}$$

Particularly for c=0, the number of groups g can be determined by the minimum integer satisfying the following inequalities,

$$\left.\begin{array}{ll} g \geq \dfrac{\ln \beta}{r \ln(1 - p_0)} & \text{under binomial model} \\[2ex] g \leq \dfrac{-\ln \beta}{rp} & \text{under Poisson model} \end{array}\right\} \tag{2.8}$$

The minimum number of groups required for the proposed sampling plan in case of Kumaraswamy-log-logistic distribution are calculated and displayed in Tables 2.1.to 2.4. for the specified values of group size, acceptance number time multiplier and consumer's risk using equations (2.6), (2.7) and (2.8).

Numerical values in Tables 2.1. to 2.4 reveal the following facts:

i. increase in consumer's risk (β), decreases the number of groups for fixed time multiplier η and r.

ii. increase in time multiplier (η), the number of groups is reduced for fixed β, the shape parameter (γ) and r.

iii. increase in r, increases the number of groups for fixed c and η.

iv. increase in c, increases the number of groups for fixed β, η and r.

Table2.1.Minimum number of groups, g in Zero failure test under binomial model

β	r	η					
		0.6	0.8	1.0	1.5	2.0	2.5
0.25	2	3	2	1	1	1	1
	4	2	1	1	1	1	1
	6	1	1	1	1	1	1
	8	1	1	1	1	1	1
0.10	2	4	3	2	1	1	1
	4	2	2	1	1	1	1
	6	2	1	1	1	1	1
	8	1	1	1	1	1	1
0.05	2	5	4	3	2	1	1
	4	3	2	2	1	1	1
	6	2	2	1	1	1	1
	8	2	1	1	1	1	1
0.01	2	8	5	4	2	2	2
	4	4	3	2	1	1	1
	6	3	2	2	1	1	1
	8	2	2	1	1	1	1

Table2.2. Minimum number of groups, g in Zero failure test under Poisson model

β	r	η					
		0.6	0.8	1.0	1.5	2.0	2.5
0.25	2	3	2	2	2	1	1
	4	2	1	1	1	1	1
	6	1	1	1	1	1	1
	8	1	1	1	1	1	1
0.10	2	5	3	3	2	2	2
	4	3	2	2	1	1	1
	6	2	1	1	1	1	1
	8	2	1	1	1	1	1
0.05	2	6	4	3	2	2	2
	4	3	2	1	1	1	1
	6	3	2	1	1	1	1
	8	1	1	1	1	1	1
0.01	2	3	2	2	1	1	1
	4	2	2	1	1	1	1
	6	2	2	1	1	1	1
	8	2	1	1	1	1	1

Table 2.3. Minimum number of groups (g) for the proposed plan with a=b=1 under binomial model

γ	β	r	c	η = 0.6	0.8	1.0	1.5	2	2.5
2	0.25	2	1	20	9	5	2	2	2
		4	1	5	2	2	1	1	1
			2	23	8	4	2	1	1
		6	1	2	1	1	1	1	1
			2	7	3	2	1	1	1
			3	30	8	4	2	1	1
		8	1	2	1	1	1	1	1
			2	4	2	1	1	1	1
			3	10	3	2	2	1	1
			4	40	9	4	2	1	1
			5	240	31	9	2	1	1

γ	β	r	c	η = 0.6	0.8	1.0	1.5	2	2.5
2	0.10	2	1	32	14	9	4	3	2
		4	1	7	4	2	1	1	1
			2	38	13	7	3	2	1
		6	1	4	2	2	1	1	1
			2	11	5	3	1	1	1
			3	50	13	6	2	1	1
		8	1	3	2	1	1	1	1
			2	6	3	2	1	1	1
			3	16	5	3	1	1	1
			4	66	14	6	2	1	1
			5	399	52	15	4	2	1

Continuation of Table 2.3

γ	β	r	c	η					
				0.6	0.8	1.0	1.5	2	2.5
2	0.05	2	1	42	19	11	5	3	3
		4	1	9	5	3	2	1	1
			2	49	17	8	3	2	2
		6	1	5	3	2	1	1	1
			2	14	6	3	2	1	1
			3	64	17	8	3	2	1
		8	1	3	2	1	1	1	1
			2	7	3	2	1	1	1
			3	21	7	3	2	1	1
			4	85	18	7	2	2	1
			5	518	67	20	4	2	2

γ	β	r	c	η					
				0.6	0.8	1.0	1.5	2.0	2.5
2	0.01	2	1	64	28	17	8	5	4
		4	1	14	7	4	2	2	1
			2	76	26	13	5	3	2
		6	1	7	4	3	2	1	1
			2	22	9	5	2	2	1
			3	99	26	11	4	2	2
		8	1	5	3	2	1	1	1
			2	11	5	3	2	1	1
			3	32	10	5	2	2	1
			4	131	27	11	3	2	2
			5	797	103	30	7	3	2

Table 2.4. Minimum number of groups (g) for the proposed plan with a=b=1 under Poisson model

γ	β	r	c	η					
				0.6	0.8	1.0	1.5	2	2.5
2	0.25	2	1	14	7	5	3	3	2
		4	1	5	3	2	1	1	1
			2	15	6	4	2	2	2
		6	1	3	2	1	1	1	1
			2	6	3	2	1	1	1
			3	18	6	4	2	2	1
		8	1	2	1	1	1	1	1
			2	4	2	1	1	1	1
			3	8	3	2	1	1	1
			4	21	7	3	2	1	1
			5	65	14	6	3	2	2

γ	β	r	c	η					
				0.6	0.8	1.0	1.5	2	2.5
2	0.10	2	1	23	12	8	5	4	4
		4	1	7	4	3	2	2	2
			2	25	10	6	4	3	3
		6	0	2	1	1	1	1	1
			1	4	3	2	1	1	1
			2	10	5	3	2	2	2
			3	29	10	6	3	2	2
		8	1	3	2	1	1	1	1
			2	6	3	2	1	1	1
			3	13	5	3	2	2	1
			4	35	11	5	3	2	2
			5	108	23	10	4	3	2

Continuation of Table 2.4

γ	β	r	c	η					
				0.6	0.8	1.0	1.5	2	2.5
2	0.05	2	1	29	23	10	8	7	6
		4	1	9	5	4	3	2	2
			2	32	13	8	6	4	3
		6	1	5	3	2	2	1	1
			2	13	6	4	2	2	2
			3	38	13	7	4	3	3
		8	1	4	2	2	1	1	1
			2	7	4	3	2	1	1
			3	17	7	4	2	2	2
			4	46	14	7	3	3	2
			5	40	30	13	5	4	3

γ	β	r	c	η					
				0.6	0.8	1.0	1.5	2.0	2.5
2	0.01	2	1	45	23	16	9	8	7
		4	1	14	8	6	4	3	3
			2	49	20	12	7	5	5
		6	1	8	5	3	2	2	2
			2	20	9	6	4	3	3
			3	58	20	11	6	4	4
		8	1	5	4	2	2	2	1
			2	11	5	4	2	2	2
			3	26	10	6	3	3	2
			4	70	21	10	5	4	3
			5	215	46	20	8	5	4

Operating characteristic value

The probability of acceptance can be regarded as a function of the deviation of the specified value μ_0 of the median. This function is called operating characteristic (OC) function of the sampling plan. Once the minimum number of groups g is obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is sufficiently good. The product is considered to be good if $\mu \geq \mu_0$ (or) $(\mu / \mu_0) \geq 1$.

For $\gamma = 2$, the probabilities of acceptance under binomial and Poisson models are given in Tables 2.5 and 2.6 based on various values of the median ratios μ / μ_0, consumer's risk β and time multiplier η.

It is observed from the operating characteristic values given in the Tables 2.5 and 2.6

i. increase in the consumer's risk increases the operating characteristic value for a given ratio μ / μ_0.

ii. increase in r decreases the operating characteristic value for a given ratio μ / μ_0 and fixed shape parameter (a, b, γ).

iii. increase in the median ratio increase the operating characteristic values for any given η and any specified consumer's risk.

Table.2.5. Operating characteristics value of group sampling plan with c=1, (γ ,a,b=2) under binomial model

β	r	g	η	μ/μ_0					
				2	4	6	8	10	12
0.25	2	5	1.0	0.949739	0.999555	0.999980	0.999998	1.000000	1.000000
		3	1.5	0.763320	0.995162	0.999733	0.999970	0.999995	0.999999
		2	2.0	0.561539	0.979584	0.998569	0.999822	0.999967	0.999992
	4	2	1.0	0.895740	0.998945	0.999951	0.999995	0.999999	1.000000
		1	1.5	0.663296	0.990820	0.999472	0.999940	0.999989	0.999997
		1	2.0	0.311540	0.946436	0.995857	0.999472	0.999901	0.999976
	6	1	1.0	0.883178	0.998698	0.999939	0.999994	0.999999	1.000000
		1	1.5	0.434514	0.978552	0.998698	0.999850	0.999973	0.999994
		1	2.0	0.108776	0.883178	0.990007	0.998698	0.999754	0.999939
	8	1	1.0	0.809245	0.997599	0.999887	0.999988	0.999998	1.000000
		1	1.5	0.268538	0.961518	0.997599	0.999721	0.999950	0.999988
		1	2.0	0.034877	0.809245	0.981999	0.997599	0.999544	0.999887
0.10	2	9	1.0	0.911356	0.999199	0.999963	0.999996	0.999999	1.000000
		4	1.5	0.697604	0.993554	0.999644	0.999960	0.999993	0.999998
		3	2.0	0.420795	0.969533	0.997854	0.999733	0.999950	0.895740
	4	2	1.0	0.895740	0.998945	0.999951	0.999995	0.999999	1.000000
		1	1.5	0.663296	0.990820	0.999472	0.999940	0.999989	0.999997
		1	2.0	0.311540	0.946436	0.995857	0.999472	0.999901	0.999976
	6	2	1.0	0.780004	0.997397	0.999878	0.999987	0.999998	0.999999
		1	1.5	0.434514	0.978952	0.998698	0.999850	0.999973	0.999994
		1	2.0	0.108776	0.883178	0.990007	0.998698	0.999754	0.999939
	8	1	1.0	0.809245	0.997599	0.999887	0.999988	0.999998	1.000000
		1	1.5	0.268536	0.961518	0.997599	0.999721	0.999950	0.999988
		1	2.0	0.034877	0.809245	0.981999	0.997599	0.999544	0.999887
0.05	2	11	1.0	0.892750	0.999021	0.999955	0.999995	0.999999	1.000000
		5	1.5	0.637546	0.991950	0.999555	0.999949	0.999991	0.999998
		3	2.0	0.420795	0.969533	0.997854	0.999733	0.999950	0.999988
	4	3	1.0	0.847761	0.998418	0.999927	0.999992	0.999999	1.000000
		2	1.5	0.439962	0.981724	0.998945	0.999879	0.999978	0.999995
		1	2.0	0.311540	0.946436	0.995857	0.999472	0.999901	0.999976
	6	3	1.0	0.780004	0.997397	0.999878	0.999987	0.999998	0.999999
		2	1.5	0.434514	0.978252	0.998698	0.999850	0.999973	0.999994
		1	2.0	0.108776	0.883178	0.990007	0.998698	0.999754	0.999939
	8	1	1.0	0.809245	0.997599	0.999887	0.999988	0.999998	1.00000
		1	1.5	0.268538	0.961518	0.997599	0.999721	0.999950	0.999988
		1	2.0	0.034877	0.809245	0.981999	0.997599	0.999544	0.999887
0.01	2	17	1.0	0.839180	0.998487	0.999931	0.999993	0.999999	1.000000
		8	1.5	0.486651	0.987150	0.999288	0.999919	0.999986	0.999997
		5	2.0	0.236293	0.949739	0.996426	0.999555	0.999917	0.999980
	4	4	1.0	0.802351	0.997891	0.999902	0.999990	0.999998	1.000000
		2	1.5	0.439962	0.981724	0.998945	0.999879	0.999978	0.999995
		2	2.0	0.097057	0.895740	0.991731	0.998945	0.999802	0.999951
	6	3	1.0	0.688883	0.996098	0.999818	0.999981	0.999997	0.999999
		2	1.5	0.188803	0.956978	0.997397	0.999699	0.999946	0.999946
		1	2.0	0.108776	0.883178	0.990007	0.998698	0.999754	0.999754
	8	2	1.0	0.654877	0.995204	0.999774	0.999976	0.999996	0.999999
		1	1.5	0.268538	0.961518	0.997599	0.999721	0.999950	0.999988
		1	2.0	0.034877	0.809245	0.981999	0.997599	0.999544	0.999887

Table.2.6. Operating characteristics value of group sampling plan with c=1, (γ ,a,b=2) under Poisson model

β	r	g	η	μ/μ_0					
				2	4	6	8	10	12
0.25	2	5	1.0	0.913419	0.999121	0.999959	0.999996	0.999999	1.000000
		3	1.5	0.687100	0.990840	0.999473	0.999940	0.999989	0.999997
		3	2.0	0.397532	0.947113	0.995861	0.999473	0.999901	0.999976
	4	2	1.0	0.878071	0.998611	0.999935	0.999993	0.999999	1.000000
		1	1.5	0.672167	0.988382	0.999305	0.999920	0.999986	0.999997
		1	2.0	0.405313	0.937055	0.994666	0.999305	0.999869	0.999968
	6	1	1.0	0.875535	0.998456	0.999927	0.999992	0.999999	1.000000
		1	1.5	0.474695	0.975202	0.998456	0.999820	0.999968	0.999992
		1	2.0	0.198575	0.875535	0.975202	0.998456	0.999707	0.999927
	8	1	1.0	0.805066	0.997290	0.999871	0.999986	0.999998	0.999999
		1	1.5	0.320087	0.958161	0.997290	0.999682	0.999943	0.999986
		1	2.0	0.091204	0.805066	0.980116	0.997290	0.999481	0.999871
0.10	2	8	1.0	0.865111	0.998594	0.999935	0.999993	0.999999	1.000000
		5	1.5	0.535016	0.984781	0.999121	0.999989	0.999982	0.999996
		4	2.0	0.292300	0.930113	0.994486	0.999297	0.999868	0.999967
	4	3	1.0	0.822801	0.997917	0.999903	0.999990	0.999998	1.000000
		2	1.5	0.451809	0.976900	0.998611	0.999840	0.999971	0.999993
		2	2.0	0.164279	0.878071	0.989361	0.998611	0.999738	0.999935
	6	1	1.0	0.875535	0.998456	0.999927	0.999992	0.999999	1.000000
		1	1.5	0.474695	0.975202	0.998456	0.999820	0.999968	0.999992
		1	2.0	0.198575	0.875535	0.988415	0.998456	0.999707	0.999927
	8	1	1.0	0.805066	0.997290	0.999871	0.999986	0.999998	0.999999
		1	1.5	0.320087	0.958161	0.997290	0.999682	0.999943	0.999986
		1	2.0	0.091204	0.805066	0.980116	0.997290	0.999481	0.999871
0.05	2	8	1.0	0.865111	0.998594	0.999935	0.999993	0.999999	1.000000
		5	1.5	0.535016	0.984781	0.999121	0.999899	0.999982	0.999996
		4	2.0	0.292300	0.930113	0.994486	0.999297	0.999868	0.999967
	4	3	1.0	0.822801	0.997917	0.999903	0.999990	0.999998	1.000000
		2	1.5	0.451809	0.976900	0.998611	0.999840	0.999971	0.999993
		2	2.0	0.164279	0.878071	0.989361	0.998611	0.999738	0.999935
	6	2	1.0	0.766562	0.996915	0.999854	0.999984	0.999997	0.999999
		1	1.5	0.474695	0.975202	0.998456	0.999820	0.999968	0.999984
		1	2.0	0.198575	0.875535	0.988415	0.998456	0.999707	0.999927
	8	1	1.0	0.648131	0.994588	0.999742	0.999972	0.999995	0.999999
		1	1.5	0.320087	0.958161	0.997290	0.999682	0.999943	0.999986
		1	2.0	0.091204	0.805066	0.980116	0.997290	0.999481	0.999871
0.01	2	16	1.0	0.748417	0.997190	0.999870	0.999986	0.999998	0.999999
		9	1.5	0.324384	0.972772	0.998418	0.999819	0.999968	0.999992
		8	2.0	0.085440	0.865111	0.989002	0.998594	0.999737	0.999935
	4	6	1.0	0.677001	0.995839	0.999805	0.999979	0.999996	0.999999
		4	1.5	0.204131	0.954334	0.997224	0.999679	0.999943	0.999986
		3	2.0	0.066584	0.822801	0.984085	0.997917	0.999607	0.999903
	6	3	1.0	0.671151	0.995377	0.999782	0.999977	0.999996	0.999999
		2	1.5	0.225335	0.951019	0.996915	0.999641	0.999936	0.999984
		2	2.0	0.039432	0.766562	0.976965	0.996915	0.999414	0.999854
	8	2	1.0	0.648131	0.994588	0.999742	0.999972	0.999995	0.999999
		2	1.5	0.102456	0.918073	0.994588	0.999364	0.999886	0.999972
		1	2.0	0.091204	0.805066	0.980116	0.997290	0.999481	0.999871

Minimum Median Ratios

The producer may be interested in improving the quality level of the product so that the acceptance probability should be larger than a specified level. For a given producer's risk, α the minimum ratio μ/μ_0 is obtained from the following inequality,

$$L(p) \geq 1 - \alpha \tag{2.9}$$

where L (p) is given in Equation (2.4) and p is given in Equation (2.5). and g is chosen according to the consumer's risk β when (μ/μ_0) = 1.

The minimum ratios to the specified life at the given consumer's risk and test times corresponding to the specified producer's risk α under binomial and Poisson models are given in Tables 2.7 and 2.8.

From the Tables 2.7 and 2.8, it is seen that

i. increase in consumer's risk decreases the minimum ratio value for given shape parameter (a,b,γ) and η.
ii. increase in r increase the minimum ratio value for given shape parameter (a,b,γ) and η.
iii. increase in (a,b,γ) decreases the value for given r, β and η.
iv. increase in η increases the minimum median ratios for any given combination of (a,b,γ) and β for a given producer's risk.

Table 2.7. Minimum Median ratios to the specified life at producer's risk of 0.05 and c=1 under binomial model

β	r	(a,b,γ)	η = 0.6	0.8	1.0	1.5	2	2.5
0.25	2	(1,1,2)	2.60	2.81	2.99	3.88	4.60	5.75
		(2,2,2)	1.50	1.77	2.01	2.76	3.42	4.28
	4	(1,1,2)	2.85	2.97	3.70	4.57	6.09	7.61
		(2,2,2)	1.58	1.82	2.28	3.04	4.05	5.07
	6	(1,1,2)	2.82	3.09	3.87	5.80	7.73	9.66
		(2,2,2)	1.57	1.87	2.34	3.50	4.67	5.83
	8	(1,1,2)	3.30	3.63	4.54	6.81	9.07	11.34
		(2,2,2)	1.71	2.05	2.56	3.84	5.12	6.39
0.10	2	(1,1,2)	2.94	3.16	3.51	4.21	5.17	5.75
		(2,2,2)	1.61	1.89	2.21	2.90	3.68	4.28
	4	(1,1,2)	3.12	3.58	3.71	4.57	6.09	7.61
		(2,2,2)	1.66	2.03	2.28	3.04	4.06	5.07
	6	(1,1,2)	3.40	3.75	4.69	5.80	7.73	9.66
		(2,2,2)	1.74	2.09	2.61	3.50	4.67	5.83
	8	(1,1,2)	4.00	4.78	4.93	7.40	9.86	12.33
		(2,2,2)	1.82	2.28	2.56	3.84	5.12	6.39
0.05	2	(1,1,2)	3.16	3.42	3.70	4.48	5.17	6.47
		(2,2,2)	1.67	1.98	2.25	3.01	3.68	4.59
	4	(1,1,2)	3.34	3.80	4.14	5.56	6.09	7.61
		(2,2,2)	1.72	2.10	2.43	3.42	4.05	5.07
	6	(1,1,2)	3.6	4.19	4.69	5.80	7.73	9.66
		(2,2,2)	1.8	2.22	2.61	3.5	4.67	5.83
	8	(1,1,2)	3.68	4.40	4.54	6.81	9.07	11.34
		(2,2,2)	1.82	2.28	2.56	3.84	5.12	6.39
0.01	2	(1,1,2)	3.52	3.79	4.15	5.09	5.97	7.01
		(2,2,2)	1.78	2.10	2.43	3.25	4.01	4.83
	4	(1,1,2)	3.74	4.16	4.48	5.56	7.61	1.84
		(2,2,2)	1.89	2.21	2.54	3.42	4.55	5.07
	6	(1,1,2)	3.94	4.53	5.24	7.03	7.73	9.66
		(2,2,2)	1.89	2.32	2.78	3.91	4.67	5.83
	8	(1,1,2)	4.22	4.91	5.51	6.81	9.10	11.34
		(2,2,2)	1.96	2.43	2.85	3.84	5.12	6.39

Table 2.8. Minimum Median ratios to the specified life at producer's risk of 0.05 and c=1 under Poisson model

β	r	(a,b,γ)	η = 0.6	0.8	1.0	1.5	2	2.5
0.25	2	(1,1,2)	2.50	3.09	3.51	4.55	6.06	6.71
		(2,2,2)	1.56	1.87	2.21	3.04	4.05	4.70
	4	(1,1,2)	3.04	3.52	3.93	4.81	6.41	8.01
		(2,2,2)	1.64	2.01	2.36	3.14	4.18	5.23
	6	(1,1,2)	3.27	3.89	3.99	5.98	7.98	9.97
		(2,2,2)	1.70	2.13	2.38	3.57	4.75	5.94
	8	(1,1,2)	3.39	3.72	4.64	6.96	9.28	11.60
		(2,2,2)	1.74	2.08	2.59	3.89	5.18	6.48
0.10	2	(1,1,2)	3.20	3.58	4.01	5.27	6.51	8.24
		(2,2,2)	1.68	2.03	2.38	3.31	4.25	5.31
	4	(1,1,2)	3.33	3.82	4.41	5.89	7.85	9.82
		(2,2,2)	1.73	2.11	2.52	3.53	4.71	5.89
	6	(1,1,2)	3.53	4.35	4.86	5.98	7.98	9.97
		(2,2,2)	1.78	2.27	2.66	3.57	4.75	5.94
	8	(1,1,2)	3.79	4.52	4.64	6.96	9.28	11.60
		(2,2,2)	1.85	2.32	2.59	3.89	5.18	6.48
0.05	2	(1,1,2)	3.40	4.26	4.26	6.01	7.72	9.24
		(2,2,2)	1.74	2.24	2.47	3.57	4.66	5.69
	4	(1,1,2)	3.56	4.06	4.77	6.60	7.85	9.82
		(2,2,2)	1.79	2.18	2.63	3.77	4.71	5.89
	6	(1,1,2)	3.75	4.37	4.86	7.29	7.98	9.97
		(2,2,2)	1.84	2.27	2.66	3.99	4.75	5.94
	8	(1,1,2)	4.09	4.52	5.64	6.96	9.28	11.60
		(2,2,2)	1.93	2.32	2.89	3.89	5.18	6.48
0.01	2	(1,1,2)	3.81	4.26	4.83	6.20	8.01	9.05
		(2,2,2)	1.86	2.24	2.65	3.64	4.76	5.83
	4	(1,1,2)	4.00	4.60	5.32	7.15	8.80	11.0
		(2,2,2)	1.91	2.34	2.80	3.95	5.03	6.29
	6	(1,1,2)	4.25	5.00	5.44	7.99	9.72	12.15
		(2,2,2)	1.97	2.45	2.84	3.99	5.32	6.65
	8	(1,1,2)	4.34	5.45	5.64	8.47	11.28	11.60
		(2,2,2)	1.99	2.51	2.89	4.34	5.78	6.48

Numerical Example

Assume that the lifetime of the product under consideration follows Kumaraswamy-log-logistic distribution with shape parameters a=1, b=1 and $\gamma = 1$.

It is desired to design a GASP to test if the median is greater than 1,000 hours based on a testing time of 1500 hours and using tester equipped with 2 items each. It is assumed that c=1 and $\beta = 0.01$. This gives the termination multiplier $\eta = 1.5$.

From Table.2.3 the minimum number of groups required is obtained as g=8. Thus, a random sample of size n=16 items is to be drawn and allocate 2 items to each of the 8 groups to put on the test for 1500 hours. This indicates that a total of 16 products are needed and that 2 items are allocated to each of the 8 testers. The lot is accepted if no more than 1 failure occurs before 1500 hours in each of the 8 groups. The experiment is truncated as soon as the 2nd failure occurs before 1500th hour.

For this proposed sampling plan the probability of acceptance is L (p) =0.998487 when the true value of the median is $\mu = 4{,}000$ hours. This shows that, if the true value of the median is 4 times of the required value $\mu_0 = 1000$ hours the producer's risk is $\alpha = 0.00113$.

Given the ratio to assure a producer's risk of $\alpha = 0.05$, when $\beta = 0.01$, r= 2, g= 8 and $\eta = 1.5$, the required ratio is $\mu/\mu_0 = 5.09$.

Summary and Conclusion

SUMMARY AND CONCLUSION

This dissertation discusses on the development of group acceptance sampling plan for truncated life test based on product quality following generalized exponential distribution and Kumaraswamy-log-logistic distribution.

Basic terms, definitions and mathematical concepts relevant for the preparation of this dissertation are included in the Basic concepts.

In Chapter I, group acceptance sampling plan for the truncated life test based on median with the product quality following generalized exponential distribution is presented. Tables are constructed for minimum number of groups, operating characteristic values and minimum median ratios for certain fixed parameters. Illustration is provided to establish their applicability.

The group acceptance sampling plan for truncated life test based on the median of Kumaraswamy-log-logistic distribution is proposed in Chapter II. The procedure is provided to construct the proposed sampling plans based on the median of the Kumaraswamy-log-logistic distribution. Tables are constructed on the minimum number of groups, operating characteristic values and minimum median ratios. Illustration is provided to establish their applicability.

Bibliography is given at the end.

Recommendation for further Studies

Group acceptance sampling for life testing proposed in this dissertation provide a good foundation for further studies on designing life test plans.

i. Economic designing of group sampling plans may be carried out with different life time distributions.
ii. Group acceptance sampling plan for life test developed in terms of median may be modified with percentiles.

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