Avinashilingam Institute for Home Science and Higher Education for Women
(Deemed to be University, Estd. u/s 3 of UGC Act 1956 Category 'A' by MHRD)
Re-accredited with 'A++' Grade by NAAC. Recognised by UGC Under Section 12 B
Coimbatore - 641 043, Tamil Nadu, India

Master's Degree Examination – November 2024
III Semester

Class : II PG | **Time: 3 Hours**
Major : Mathematics | **Max. Marks: 100**

23MMAC15 Differential Geometry

Course Outcomes:

CO1: Calculate the curvature and torsion of a curve.
CO2: Find the osculating surface and osculating curve at any point of a given curve.
CO3: Calculate the first and the second fundamental forms of surface.
CO4: Solve the problems related to Gaussian curvature, the mean curvature, the curvature lines, the asymptotic lines.
CO5: Identify the appropriate approach to solve the problems on geodesics of a surface.

Part A — 10 x 1 = 10
Choose the Correct Answer

1. When a function is infinitely differentiable then f is said to be ________ CO1K3
 a. $C^{\infty\infty}$ b. A^{∞} c. C^{∞} d. class of m

2. The point P on the curve for which ______ is called a point of inflexion CO1K2
 a. $r'=0$ b. $r''=0$ c. $r''\neq 0$ d. $r'''=0$

3. The osculating plane at any point P has three point contacts with the..... at P CO2K3
 a. curve b. line c. tangent d. path

4. If a curve on a sphere is a helix, then the curve is called a ______ CO2K1
 a. cylinder b. spherical helix c. cone d. curvature

5. Tangent to any curve drawn on a surface is called a ________ CO3K4
 a. normal b. line c. tangent line d. circle

6. The differential quadratic form is called ________ CO3K1
 a. parameter b. orthogonal c. helicoids d. metric on the surface

7. The curves of the family $\Phi(u, v)$= constant, are the solutions of the differential equation ____ CO4K2
 a. $\Phi_1 du+\Phi_2 dv=0$ b. $\Phi_1 dv+\Phi_2 dw=0$ c. $\Phi_1 du$ d. $\Phi_1 du+\Phi_2 dw=0$

8. When the parametric curves are orthogonal, then v _______ CO4K2
 a. $E_1=0$ b. $F=0$ c. $G_1=0$ d. $E_2=0$

9. A curve on a sphere is a geodesic iff it is a _______ CO5K4
 a. cylinder b. cone c. circle d. normal

10. A curve on a plane is a geodesic iff it is a _______ CO5K1
 a. tangent line b. straight line c. orthogonal d. constant

Part B **5 x 6 = 30**

Answer ALL questions

Each answer should not exceed 400 words or two pages

11.a. Prove that the arc length of a curve is invariant under parametric transformation. CO1K3

(or)

11.b. A necessary and sufficient condition that a given curve to be a plane curves is that $\tau=0$ at all points of the curve prove it. CO1K4

12.a. Prove that the locus of the centres of spherical curvature is an evolute iff the curve is a plane curve. CO2K4

(or)

12.b. Prove that a spherical helix projects on a plane perpendicular to its axis in an arc of an epicyloid. CO2K4

13.a. Prove that the equation of a tangent plane at P on a surface with position vector r=r(u, v) is $R= r+ar_1+br_2$, where a and b are parameters. CO3K3

(or)

13.b. Prove that the first fundamental form of a surface is a positive definite quadratic form in du, dv. CO3K4

14.a. The parameters on a surface can always be chosen so that the curves of the given family and the orthogonal trajectories become parametric curves. Explain. CO4K5

(or)

14.b. Prove the following: (i) If the curves on a surface are not parametric curves, then the sufficient condition for a curve to be a geodesic is either U=0 or V=0.
(ii) For a parametric curve u= constant to be a geodesic a sufficient condition is U=0 and V= constant to be a geodesic , the sufficient condition is V=0. CO4K5

15.a. Prove that a curve on a surface is a geodesic iff the rectifying plane is tangent to the surface. CO5K4

(or)

15.b.prove that the surface generated by the tangents to any space curve is a surface of constant zero curvature. CO5K5

Part C **5 x 12 = 60**

Answer ALL questions

Each answer should not exceed 800 words or four pages

16.a. State and Prove Serret- Frenet Formulae. CO1K4

(or)

16.b. Find the curvature and torsion of the circular helix r= (a cosu, a sinu, bu). CO1K5

17.a.Derive the conditions of a surface having n point contact with the curve γ are invariant over a change of parameter. CO2K4

(or)

17.b. Find the intrinsic equation of the curve r= (ae^ucosu, ae^usinu, be^u). CO2K4

18.a.Prove that the notion of r-equivalence of representations of a surface is an equivalence relation. CO3K4

(or)

18.b. Prove that the metric is invariant under a parametric transformation. CO3 K4

19.a. Find the orthogonal trajections of the circle r=a cosθ CO4K4

(or)

19.b. Prove that three types of geodesics on a surface of revolution $r = (g(u)\cos v, g(u)\sin v, f(u))$ are (i) $v = \alpha\phi(u,a) + \beta$ where α and β are constants. (ii) Every meridian v=constant.
(iii) A parallel u= constant is a geodesic iff its radius is stationary. CO4K5

20.a.Prove that any curve u=u(t), v=v(t) on a surface r=r(u, v) is a geodesic iff the principle normal at every point on the curve is normal to the surface. CO5K4

(or)

20.b. A geodesic can be found to pass through any given point and have any given direction on a surface. The geodesic is uniquely determined by the initial conditions. Explain. CO5K5

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