Chapter III

CHAPTER-3

SEMI STAR GENERALIZED W-CONTINUITY IN TOPOLOGICAL

In this chapter semi star generalized open sets due to Chandra sekhara Rao and Jaseph [8] , Semi star generalized w-open sets and s^* gw-$T_{1/2}$ spaces due to Chandra sekhara Rao and Narasimhan [13] and semi star generalized w-closed map due to Chandra sekhara Rao[12] are studied. Properties and characterization of these concepts are analyzed.

SECTION: 3.1

PRELIMINARIES

Definition: 3.1.1

Let (X, τ) be a topological space .A point $x \in X$ is called a **Condensation point** of A if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable.

Definition: 3.1.2

A subset A of a topological space (X, τ) is called **w-closed** if it contains all its condensation points.

The complement of a w-closed set is called **w-open.**

Definition: 3.1.3

A set A of a topological space (X, τ) is called **semi open** if there exists an open set U such that $U \subseteq A \subseteq cl(U)$.

Definition: 3.1.4

A set A of a topological space (X, τ) is called **semi closed** if X– A is semi open, equivalently, a set A of a topological space (X, τ) is called **semi closed** if there exists a closed set F such that int(F) $\subseteq$ A $\subseteq$ F.

Definition: 3.1.7

A set A of a topological space (X, τ) is called **generalized w-closed (gw-closed)** if $cl_w(A) \subseteq U$ whenever A $\subseteq$ U and U is open in X.

Definition: 3.1.8

A set A of a topological space (X, τ) is called **generalized w-open (gw-open)** if X – A is gw-closed.

Definition: 3.1.9

A set A of a topological space (X, τ) is called **regular generalized w-closed (rgw-closed)** if $cl_w(A) \subseteq U$ whenever A $\subseteq$ U and U is regular open in X.

Definition: 3.1.10

A set A of a topological space (X, τ) is called **regular generalized w-open** (rgw-open) if X – A is rgw-closed.

Definition: 3.1.11

A set A of a topological space (X, τ) is called **semi star generalized w-closed (s*gw-closed)** if $cl_w(A) \subseteq U$ whenever A $\subseteq$ U and U is semi open in X.

Definition: 3.1.12

A set A of a topological space (X, τ) is called **semi star generalized w-open** (s*gw-open) if X – A is s*gw-closed.

Definition: 3.1.13

A map f: X→ Y is called **w-irresolute** if the inverse image of every w-closed subset of Y is w-closed in X.

Definition: 3.1.14

A map f: X→ Y is called **R map** if the inverse image of every regular closed subset of Y is regular closed in X.

Definition: 3.1.15

A map f: X→ Y is called **gw-closed** if the image of every closed subset of X is gw-closed in Y.

Definition: 3.1.16

A map f: X→ Y is called rgw-**closed** if the image of every closed subset of X is rgw-closed in Y.

Definition: 3.1.17

A map f: X→ Y is called **pre w-closed** if the image of every w-closed subset of X is w-closed in Y.

Definition: 3.1.18

A map f: X→ Y is called **rgw-continuous** if the inverse image of every w-closed subset of Y is rgw-closed in X.

Definition: 3.1.19

A map f: X→ Y is called **rgw-irresolute** if the inverse image of every rgw-closed subset of Y isrgw-closed in X.

Definition: 3.1.20

A space (X, τ) is said to be **semi star generalized w-$T_{1/2}$** (simply, s*gw-$T_{1/2}$) if every s*gw-closed set in (X, τ) is w-closed.

SECTION 3.2

SEMI STAR GENERALIZED W-CONTINUITY

Definition: 3.2.1

A map f: X → Y is called s***gw-continuous** if the inverse image of every w -closed subset of Y is s*gw-closed in X.

Definition: 3.2.2

A map f: X → Y is called **s*gw-irresolute** if the inverse image of every s*gw-closed subset of Y iss*gw-closed in X.

Definition: 3.2.3

A map f: X → Y is called s***gw-closed** if the image of every closed subset of X is s*gw-closed in Y.

Proposition: 3.2.4

If f: X → Y, g: Y→ Z are s*gw-irresolute, then gof is also s*gw-irresolute.

Proposition: 3.2.5

If f: X → Y is s*gw-irresolute and g: Y → Z is s*gw-continuous, then gof is also s*gw-continuous.

Proposition: 3.2.6

The composition of two s*gw-continuous functions is not s*gw-continuous.

Proposition: 3.2.8

Since every s*gw-closed set is gw-closed and rgw-closed, every s*gw-continuous function is gw-continuous and rgw-continuous.

Theorem: 3.2.9

Let f: X $\rightarrow$ Y be onto s*gw-irresolute and pre w -closed map. If X is s*gw- $T_{1/2}$then Y is also s*gw-$T_{1/2}$

Proof:

Let A be a s*gw-closed subset of Y. Since f is a s*gw-irresolute map,
$f^{-1}(A)$ is a s*gw-closed subset of X. Since X is a s*gw-$T_{1/2}$ space, f^{-1} (A) is w-closed in X. Since f is pre w-closed, f [f^{-1} (A)] = A is w-closed in Y. Therefore, Y is a s*gw-$T_{1/2}$ space.

Remark: 3.2.10

Since every s*gw-closed set is gw-closed, every s*gw-closed map is a gw-closed map and hence it is a rgw-closed map [since every gw-closed map is a rgw-closed map].

Since every w-closed set is s*gw-closed, every s*gw-irresolute map is a s*gw-continuous map.

Definition: 3.2.11

A subset A $\subseteq$ X is said to be **w-c-closed** provided that there is a proper subset B for which A = $Cl_w(B)$.

Definition: 3.2.12

A map f: X $\rightarrow$ Y is said to be **gw-c-closed** if f(A) is gw-closed in Y for every w-c-closed subset A $\subseteq$ X.

Definition: 3.2.13

A map f: X $\rightarrow$ Y is said to be **rgw-c -closed** if f (A) is rgw-closed in Y for every w-c-closed subset A $\subseteq$ X.

Definition: 3.2.14.

A map f: X $\rightarrow$ Y is called s***gw-c-closed** if f (A) is s*gw-closed in Y for every w-c-closed subset A $\subseteq$ X.

Definition: 3.2.15

A map f: X $\rightarrow$ Y is said to be **irresolute map** if the inverse image of every semi closed subset of Y is semi closed in X.

Theorem: 3.2.16

Let f: X $\rightarrow$ Y be irresolute map and s*gw-c-closed. Then f (A) is s*gw-closed in Y for every s*gw-closed subset A of X.

Proof:

Let A be any s*gw-closed subset of X. Let U be any semi open subset of Y such that f (A) $\subseteq$ U. Then A $\subseteq$ f^{-1} (U) and f^{-1} (U) is semi open in X since f is an irresolute map. Since A is a s*gw-closed subset of X, $cl_w(A) \subseteq f^{-1}$ (U).Hence f [$cl_w(A)$] $\subseteq$ U. Since cl_w (A) is a w-c-closed subset of X and f is s*gw-c-closed, f[$cl_w(A)$] is s*gw -closed. Therefore, cl_w [f(A)] $\subseteq$ cl_w{f[$cl_w(A)$]} $\subseteq$ U. Hence f (A) is s*gw-closed in Y.

Theorem: 3.2.17

Let f: X $\rightarrow$ Y be a semi open preserving and w-irresolute map. If B is s*gw-closed in Y, then f^{-1}(B) is s*gw-closed in X.

Proof:

Let B be any s*gw-closed subset of Y . Let U be any semi open subset of X such that f^{-1} (B) $\subseteq$ U. Since f is semi open preserving g , B $\subseteq$ f(U)and

f (U) is semi open. Since B is s*gw-closed, cl_w(B) $\subseteq$ f (U) and f^{-1} [cl_w(B)] $\subseteq$ U. Since f is w-irresolute, f^{-1} [cl_w(B)] is w-closed and cl_w{f^{-1} [cl_w(B)]} = f^{-1} [cl_w(B)]. Hence cl_w {f^{-1} (B)} $\subseteq$ cl_w {f^{-1} [clw(B)]} $\subseteq$ U. Hence f^{-1} (B) is s*gw-closed.

Theorem: 3.2.18

Let f: X $\rightarrow$ Y be an irresolute map and s*gw-closed and A is s*g-closed sets in X. Then f(A) is s*gw-closed.

Proof:

Let A be any s*gw-closed subset of X. Let U be any semi open subset of Y such that f (A) $\subseteq$U. Since f is an irresolute map, A $\subseteq$ f^{-1} (U) and f^{-1} (U) is semi open in X. Since A is a s*g-closed subset of X, cl(A) $\subseteq$ f^{-1} (U). Hence f [cl (A)] $\subseteq$ U. Since f is s*gw-closed, f [cl (A)] is s*gw-closed. Therefore, cl_w {f[cl(A)]} $\subseteq$ U which implies that cl_wf(A) $\subseteq$ U. Hence f(A) is s*gw-closed in Y .

Theorem: 3.2.19

Let f : X $\rightarrow$ Y be an irresolute map and pre w-closed and A is s*gw-closed in X. Then f (A) is s*gw-closed.

Proof:

Let A be any s*gw-closed subset of X. Let U be any semi open subset of Y such that f (A) $\subseteq$ U. since f is an irresolute map A $\subseteq$ f^{-1} (U) and f^{-1} (U) is semi open in X. $cl_w(A) \subseteq f^{-1}$ (U),Since A is a s*gw-closed subset of X. $cl_w(A) \subseteq f^{-1}$ (U). Hence f [$cl_w(A)$] $\rightarrow$ U. Since f is pre w-closed, $cl_w[f(A)] \subseteq cl_w\{f[clw(A)]\} = f[cl_w(A)] \subseteq$ U. Hence f (A) is s*gw-closed in Y.

Definition: 3.2.20

A map f: X $\rightarrow$ Y is said to be **w-contra-R-map** if for every regular open subset V of Y, f^{-1} (V) is w-closed.

Definition: 3.2.21

A map f: X $\rightarrow$ Y is said to be **w-contra-S-map** if for every semi open subset V of Y , f^{-1} (V) is w-closed.

Theorem 3.2.22

Let f: X $\rightarrow$ Y be a w-contra-S-map and s*gw-c -closed. Then f (A) is s*gw-closed in Y for every subset A of X.

Proof:

Let A be any subset of X. Let U be any semi open subset of Y such that f (A) $\subseteq$ U. Then A $\subseteq$ f^{-1} (U). Since f is a w-contra-S-map, f^{-1} (U) is w-closed and so $cl_w(A) \subseteq cl_w[f^{-1}(U)] \subseteq f^{-1}$ (U). Hence f [$cl_w(A)$] $\subseteq$ U. Since $cl_w(A)$ is a w-c-closed subset of X and f is s*gw-c-closed, f[$cl_w(A)$] is s*gw-closed.

Therefore, $cl_w[f(A)] \subseteq clw\{f[clw(A)]\} \subseteq$ U.

Hence f(A) is s*gw-closed in Y .

Theorem: 3.2.23

If f: X $\rightarrow$ Y is s*gw-continuous and X is s*gw-$T_{1/2}$ then f is w-continuous.

Proof:

Let A be any closed subset of Y. Since f is s*gw-continuous, f^{-1} (A) is a s*gw-closed subset of X. Since X is a s*gw-$T_{1/2}$ space, f^{-1} (A) is w-closed in X. Therefore, f is w-continuous.

Theorem: 3.2.24

If f: X $\rightarrow$ Y is s*gw-irresolute and X is s*gw-$T_{1/2}$ then f is w- irresolute.

Proof:

Let A be any w-closed subset of Y. Since f is s*gw- irresolute, f^{-1} (A) is a s*gw-closed subset of X. Since X is a s*gw-$T_{1/2}$ space, f^{-1} (A) is w-closed in X. Therefore, f is w- irresolute.

Theorem: 3.2.25

Let f: X $\rightarrow$ Y be a bijective, semi open preserving and s*gw-continuous map. Then f is a s*gw-irresolute map.

Proof:

Let V be any s*gw-closed subset of Y. Let U be any semi open subset of X such that f^{-1} (V) $\subseteq$ U. Then V $\subseteq$ f(U)and f(U) is semi open, Since f is semi open preserving. Since V is s*gw-closed, $cl_w(V) \subseteq f(U)$ and f^{-1} $[cl_w(V)] \subseteq$ U. Since f is s*gw-continuous and $cl_w(V)$ is w-closed, f^{-1} $[cl_w(V)]$ is s*gw-closed. Therefore $cl_w[f^{-1}[cl_w(V)] \subseteq$ U.

Hence $cl_w f^{-1}(V)] \subseteq cl_w[f^{-1}[cl_w(V)]] \subseteq$ U. Hence f is s*gw-irresolute

Theorem: 3.2.26

Let $f: X \rightarrow Y$ is s*gw-closed if and only if for each subset B of Y and for each w-open set U containing $f^{-1}(B)$, there is a s*gw-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof:

Let f: $X \rightarrow Y$ is s*gw-closed. Let $B \subseteq Y$ and U is a w-open set such that $f^{-1}(B) \subseteq U$. Since $X - U$ is w-closed and f: $X \rightarrow Y$ is s*gw-closed, $f(X - U)$ is s*gw-closed, $V = Y - f(X - U)$ is s*gw-open. Then $f^{-1}(V) \subseteq X - (X - U) = U$.

Conversely, for each subset B of Y and for each w-open set U containing $f^{-1}(B)$, there is a s*gw-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$ Let F be w-closed in X. Then $f^{-1}[Y - f(F)] \subseteq X - F$ and $X - F$ is w-open in X. Then by assumption there is a s*gw open set V of Y such that $Y - f(F) \subseteq V$ and $f^{-1}(V) \subseteq X - F$. Therefore, $F \subseteq X - f^{-1}(V)$ and $Y - V \subseteq f(F)$. Hence, $Y - V \subseteq f(F) \subseteq Y - V$. Hence, $Y - V = f(F)$ is s*gw-closed. Hence, f: $X \rightarrow Y$ is s*gw-closed.