Chapter III

CHAPTER III

Supra Semi* Generalized Closed (Open) Soft Sets in Supra Soft Topological Spaces

Section 3.1

Supra semi* g-closed soft sets

Definition: 3.1.1

A soft set (F,E) is called a **supra semi* generalized closed soft set** (supra semi*g-closed soft) in a supra soft topological space (X, μ, E) if $cl^s(F, E) \tilde{\subseteq} (G, E)$ whenever $(F, E) \tilde{\subseteq} (G, E)$ and (G,E) is a supra semi open soft in X.

Example: 3.1.2

Suppose that are three cars in the universe X given by, $X = \{h_1, h_2, h_3\}$. Let $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "expensive" and "beautiful" respectively.

Let $(G_1, E), (G_2, E)$ be two soft sets over the common universe X, which describe the composition of the cars, where

$G_1(e_1) = \{h_2, h_3\}$, $G_1(e_2) = \{h_1, h_2\}$,

$G_2(e_1) = \{h_1, h_2\}$, $G_2(e_2) = \{h_1, h_3\}$.

Then, $\mu = \{X, \emptyset, (G_1, E), (G_2, E)\}$ is a supra soft topology over X. Hence, the soft sets $(F_1, E), (F_2, E)$, where

$F_1(e_1) = \{h_2, h_3\}$, $F_1(e_2) = \{h_1, h_3\}$,

$F_2(e_1) = \{h_1, h_2\}$, $F_2(e_2) = \{h_1, h_2\}$.

are supra semi*g-closed soft sets in (X, μ, E), but the soft sets $(G_1, E), (G_2, E)$ are not supra semi*g-closed soft sets in (X, μ, E).

Remark: 3.1.3

The soft intersection (resp. soft union) of any two supra semi*g-closed soft sets is not supra semi*g-closed soft in general as shown in the following examples.

Examples: 3.1.4

(1) In Example 3.1.2, $(F_1, E), (F_2, E)$ are supra semi*g-closed soft in (X, μ, E), but their soft intersection $(F_1, E) \tilde{\cap} (F_2, E) = (M, E)$ where

$M(e_1) = \{h_2\}$, $M(e_2) = \{h_1\}$ is not supra semi*g-closed soft.

(2) Suppose that there are four alternatives in the universe of houses $X = \{h_1, h_2, h_3, h_4\}$ and consider $E = \{e_1, e_2\}$ be the set of decision parameters which are stands for "quality of houses" and "green surroundings" respectively.

Let $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E)$,

$(F_{10}, E), (F_{11}, E), (F_{12}, E)$

Be twelve soft sets over the common universe X which describe the goodness of the houses, where

$F_1(e_1) = \{h_1\}$, $F_1(e_2) = \{h_4\}$,

$F_2(e_1) = \{h_1, h_4\}$, $F_2(e_2) = \{h_1, h_4\}$,

$F_3(e_1) = \{h_4\}$, $F_3(e_2) = \{h_1\}$,

$F_4(e_1) = \{h_1, h_2\}$, $F_4(e_2) = \{h_2, h_4\}$,

$F_5(e_1) = \{h_2, h_4\}$, $F_5(e_2) = \{h_1, h_2\}$,

$F_6(e_1) = \{h_1, h_2, h_3\}$, $F_6(e_2) = \{h_1, h_2, h_3\}$,

$F_7(e_1) = \{h_2, h_3, h_4\}$, $F_7(e_2) = \{h_2, h_3, h_4\}$,

$F_8(e_1) = \{h_1, h_2, h_4\}$, $F_8(e_2) = \{h_1, h_2, h_4\}$,

$F_9(e_1) = X$, $F_9(e_2) = \{h_1, h_2, h_3\}$

$F_{10}(e_1) = \{h_2, h_3, h_4\}$, $F_{10}(e_2) = X$

$F_{11}(e_1) = \{h_1, h_2, h_3\}$, $F_{11}(e_2) = X$

$F_{12}(e_1) = X$, $F_{12}(e_2) = \{h_2, h_3, h_4\}$.

Hence,

$\mu = \{\widetilde{X}, \widetilde{\emptyset}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E),$

$(F_{10}, E), (F_{11}, E), (F_{12}, E)\}$is a supra soft topology over X. therefore, the soft sets (G,E),(H,E), where

$G(e_1) = \{h_4\}$, $G(e_2) = \{h_4\}$,

$H(e_1) = \{h_1\}$, $H(e_2) = \{h_1\}$

are supra semi*g-closed soft sets in (X, μ, E), but their soft union $(G, E) \widetilde{\cup} (H, E) = (A, E)$ where $A(e_1) = \{h_1, h_4\}, A(e_2) = \{h_1, h_4\}$ is not supra semi*g-closed soft.

Remark: 3.1.5

Every supra closed soft set is supra semi*g-closed soft. But, the converse is not true in general as shown in the following example.

Example: 3.1.6

In Example 3.1.2, $(F_1, E), (F_2, E)$ are supra semi*g-closed soft in (X, μ, E), but are not supra closed soft over X.

Theorem: 3.1.7

If (F,E) is supra semi*g-closed soft and supra semi open soft, then it is supra closed soft.

Proof:

Let (F,E) be a supra semi*g-closed soft and supra semi open soft. Since $(F, E) \widetilde{\subseteq} (F, E)$, then $cl^s(F, E) \widetilde{\subseteq} (F, E)$. This implies that,$cl^s(F, E) = (F, E)$. Therefore, (F,E) is supra closed soft.

Theorem: 3.1.8

Every supra semi*g-closed soft set is supra g-closed soft.

Proof:

It is clear from the fact that, every supra open soft set is supra semi open soft.

Remark: 3.1.9

The converse of Theorem 3.1.8 is not true in general as shown in the following example.

Example: 3.1.10

In Example 3.1.2, the soft set (G,E) where

$G(e_1) = \{h_2, h_3\}$, $G(e_2) = \{h_1, h_3\}$,

Is supra g-closed soft set, but it is not supra semi*g-closed soft. Because of, the soft set (H,E), which defined by:

$H(e_1) = \{h_2, h_3\}$, $H(e_2) = X$,

is supra semi open soft set such that $(G,E) \tilde{\subset} (H,E)$, but we have $cl^s(G,E) \tilde{\not\subseteq} (H,E)$.

Theorem: 3.1.11

In a supra soft topological space $(X, \mu, E), \mu = \text{supra} - SOS(X) = \mu'$ if and only if every soft subset of X is supra semi*g-closed soft.

Proof:

Necessity: Let (A,E) be any soft subset of X such that$(A,E) \tilde{\subset} (G,E)$, where (G,E) is supra semi open soft set in X. By hypothesis, $(G,E) \in \mu'$. Hence, $cl^s(A,E) \tilde{\subseteq} cl^s(G,E) = (G,E)$. Therefore, (A,E) is supra semi*g-closed soft.

Sufficiency: We first prove that $\mu = \mu'$. Let $(O,E) \in \mu$. By hypothesis, (O,E) is supra semi*g-closed soft. Therefo,re, $cl^s(O,E) \subseteq (O,E)$. This implies that, $(O,E) \in \mu'$. Thus,$\mu \subseteq \mu'$. Next, $(B,E) \in \mu'$. By a similar argument, we can get $\mu' \subseteq \mu$. This means that, $\mu = \mu'$.

Now, we prove that $\mu = \text{supra} - SOS(X)$. Let $(Z,E) \in \text{supra} - SOS(X)$. Since $(Z,E) \tilde{\subseteq} (Z,E)$. Then, $cl^s(Z,E) \tilde{\subseteq} (Z,E)$. This implies that, $(Z,E) \in \mu'$. Thus, supra −

$SOS(X) \subseteq \mu' = \mu$. But, we have $\mu' = \mu \tilde{\subseteq}$ supra $-$ SOS(X). This means that $\mu =$ supra $-$ SOS(X) $= \mu'$.

Theorem: 3.1.12

Let (X, μ, E) be a supra soft topological space and (F,E) be a supra semi*g-closed soft in X. If $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} cl^s(F, E)$, then (H,E) is supra semi*g-closed soft.

Proof:

Let $(H, E) \tilde{\subseteq} (G, E)$and $(G, E) \in$ supra $-$ SOS(X). Since $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} (G, E)$ and (F,E) is supra semi*g-closed soft in X, then $cl^s(F, E) \tilde{\subseteq} (G, E)$. Hence, $cl^s(H, E) \tilde{\subseteq} cl^s(F, E) \tilde{\subseteq} (G, E)$. Thus, $cl^s(H, E) \tilde{\subseteq} (G, E)$. Therefore, (H,E) is supra semi*g-closed soft.

Theorem: 3.1.13

Let (X, μ, E) be a supra soft topological space. Then, (H,E) is supra semi*g-closed soft in X if and only if $cl^s(H, E) \backslash (H, E)$ contains only null supra semi closed soft set.

Proof:

Necessity: Let (H,E) be a supra semi*g-closed soft set, (F,E) be a non null supra semi closed soft set in X and $(F, E) \tilde{\subseteq} cl^s(H, E) \backslash (H, E)$. Then, $(F, E)'$ is supra semi open soft, $(F, E) \tilde{\subseteq} cl^s(H, E)$ and $(F, E) \tilde{\subseteq} (H, E)'$. Hence, $(H, E) \tilde{\subseteq} (F, E)'$. Since (H,E) is supra semi*g-closed soft. Then, $cl^s(H, E) \tilde{\subseteq} (F, E)'$. Hence,$(F, E) \tilde{\subseteq} [cl^s(H, E)]'$. This means that, $(F, E) \tilde{\subseteq} cl^s(H, E) \tilde{\cap} [cl^s(H, E)]' = \tilde{\emptyset}$. Thus, $(F, E) = \tilde{\emptyset}$, which is a contradiction. Therefore, $cl^s(H, E) \backslash (H, E)$ contains only null supra semi closed soft set.

Sufficiency: Assume that $cl^s(H, E) \backslash (H, E)$ contains only null supra semi closed soft set, $(H, E) \tilde{\subseteq} (G, E), (G, E)$ is supra semi open soft and suppose that$cl^s(H, E) \tilde{\nsubseteq} (G, E)$. Then, $cl^s(H, E) \tilde{\cap} (G, E)'$ is non null supra semi closed soft subset of $cl^s(H, E) \backslash (H, E)$, which is a contradiction. Thus, (H,E) is supra semi*g-closed soft in X. This completes the proof.

Corollary: 3.1.14

Let (F,E) be supra semi*g-closed soft set. Then, (F,E) is supra closed soft if and only if $cl^s(F, E)\backslash(F, E)$ is supra semi closed soft.

Proof:

Necessity: Let (F,E) be a supra semi*g-closed soft which is also supra closed soft. Then, $cl^s(F, E)\backslash(F, E) = \widetilde{\emptyset}$, which is supra semi closed soft.

Sufficiency: Suppose that $cl^s(F, E)\backslash(F, E) = \widetilde{\emptyset}$, is supra semi closed soft and (F,E) is supra semi*g-closed soft. Then $cl^s(F, E)\backslash(F, E) = \widetilde{\emptyset}$, from Theorem 3.4. Hence, $cl^s(F, E) = (F, E)$. Thus, (F,E) is supra closed soft.

Section 3.2

Supra semi* g-open soft sets

Definition: 3.2.1

A soft set (F,E) is called **supra semi*generalized open soft set** (supra semi*g-open soft) in a supra soft topological space (X, μ, E) if its relative complement $(F, E)'$ is supra semi*g-closed soft in X.

Example: 3.2.2

In Example 3.1.1, $(F_1, E)', (F_2, E)'$ are supra semi*g-open soft in (X, μ, E).

Remark: 3.2.3

Every supra open soft set is supra semi*g-open soft. But, the converse is not true in general as shown in the following example.

Example: 3.2.4

In Example 3.1.2 $(F_1, E)', (F_2, E)'$ are supra semi*g-open soft (X, μ, E), but it is not supra open soft over X.

Theorem: 3.2.5

Let (X, μ, E) be supra soft topological space. Then the supra soft set (F,E) is supra semi*g-open soft set if and only if $(F, E) \widetilde{\subseteq} int^s(G, E)$ whenever $(F, E) \widetilde{\subseteq} (G, E)$ and (G,E) is supra semi closed soft in X.

Proof:

Necessity: Let (F,E) be a supra semi*g-open soft in X, $(F, E) \widetilde{\subseteq} (G, E)$ and (F,E) is supra semi closed soft in X. Then, $(F, E)'$ is is supra semi*g-closed soft from Definition 3.2.1 and $(G, E)' \widetilde{\subseteq} (F, E)'$. Since (F,E) is supra semi*g-open soft in X. Then, $cl^s(G, E)' \widetilde{\subseteq} (F, E)'$. Hence, $(F, E) \widetilde{\subseteq} [cl^s(G, E)']' = int^s(G, E)$.

Sufficiency: Let $(F, E)' \widetilde{\subseteq} (H, E)$ and (H,E) is supra semi open soft in X. Then, $(H, E)' \widetilde{\subseteq} (F, E)$ and $(H, E)'$ is supra semi closed soft in X. Hence, $(H, E)' \widetilde{\subseteq} int^s(F, E)$ from the necessary condition. Thus, $[int^s(F, E)]' = cl^s[(F, E)'] \widetilde{\subseteq} (H, E)$ and (H,E) is supra semi open soft in X. This means that, $(F, E)'$ is supra semi*g-closed soft in X. Therefore, (F,E) is supra semi*g-open soft set from Definition 3.2.1. This completes the prrof.

Remark: 3.2.6

The following example deduce that, each supra semi*g-open soft set and supra semi open soft set are independent concepts.

Example: 3.2.7

In Example 3.1.1, the soft set (G_1, E) is a supra semi open soft set in (X, μ, E), but it is not supra semi*g-open soft. Also, the soft set (F_1, E) is supra semi*g-open soft set in (X, μ, E), but it is not supra semi open soft.

Remark: 3.2.8

The soft intersection (resp. soft union) of any two supra semi*g-open soft sets is not supra semi*g-open soft in general as shown in the following examples.

Examples: 3.2.9

In Example 3.1.1, the soft sets (A,E), (B,E), where

$A(e_1) = \{h_1, h_2\}$, $A(e_2) = \{h_1, h_3\}$,

$B(e_1) = \{h_2, h_3\}$, $B(e_2) = \emptyset$,

are supra semi*g-open soft set in (X, μ, E), but their soft union $(A, E) \tilde{\cup} (B, E) = (M, E)$ where

$M(e_1) = X$, $M(e_2) = \{h_1, h_3\}$is not supra semi*g-open soft.

(2): In Example 3.1.1(2), the soft sets (P,E), (O,E), where

$P(e_1) = \{h_1, h_2, h_3\}$, $P(e_2) = \{h_1, h_2, h_3\}$,

$O(e_1) = \{h_2, h_3, h_4\}$, $O(e_2) = \{h_2, h_3, h_4\}$,

are supra semi*g-open soft set in (X, μ, E), but their soft intersection$(P, E) \tilde{\cap} (O, E) = (N, E)$ where

$N(e_1) = \{h_2, h_3\}$, $N(e_2) = \{h_2, h_3\}$is not supra semi*g-open soft.

Theorem: 3.2.10

Let (X, μ, E) be asupra soft topological space and (F,E) be a supra semi*g-open soft in X. If $int^s(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} (F, E)$, then (H,E) is supra semi*g-open soft.

Proof:

Let $(G, E) \tilde{\subseteq} (H, E)$ and $(G, E) \in supra - SCS(X)$. Since $(G, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} (F, E)$ and (F,E) is supra semi*g-open soft in X, then $(G, E) \tilde{\subseteq} int^s(F, E) \tilde{\subseteq} int^s(H, E)$. Thus, $(G, E) \tilde{\subseteq} int^s(H, E)$. Therefore, (H,E) is supra semi*g-open soft.