



### $G\pi$ - Closed Sets in Bi $\check{C}$ ech Closure Spaces

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#### ABSTRACT

In this paper, we introduce the concepts of  $g\pi$ - closed and  $g\pi$ - open sets in Bi  $\check{C}$ ech closure space and study some of their properties.

**Key words:** Bi  $\check{C}$ ech closure operator, Bi  $\check{C}$ ech closure spaces, Bi  $\check{C}$ ech  $g\pi$  closed sets.

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#### 1. INTRODUCTION:

$\check{C}$ ech closure spaces were introduced by E.  $\check{C}$ ech [1]. In  $\check{C}$ ech's approach the operator satisfies idempotent condition among kuratowski axioms. This condition need not hold for every set  $A$  of  $X$ . When this condition is also true, the operator becomes topological closure operator. Thus the concept of closure space is the generalization of a topological space. In this paper, we introduce the concept of  $(k_1, k_2)$ -  $g\pi$  closed sets,  $g\pi C_0$  bi-  $\check{C}$ ech space and study their basic properties.

#### 2. PRELIMINARIES:

**Definition: 2.1** [2] Two functions  $k_1$  and  $k_2$  from power set  $X$  to itself are called bi-  $\check{C}$ ech closure operators (simply biclosure operator) for  $X$  if they satisfy the following properties.

- (i)  $k_1(\emptyset) = \emptyset$  and  $k_2(\emptyset) = \emptyset$
- (ii)  $A \subseteq k_1(A)$  and  $A \subseteq k_2(A)$  for any set  $A \subseteq X$
- (iii)  $k_1(A \cup B) = k_1(A) \cup k_1(B)$  and  $k_2(A \cup B) = k_2(A) \cup k_2(B)$  for any  $A, B \subseteq X$  ( $X, k_1, k_2$ ) is called bi-  $\check{C}$ ech closure space.

**Example: 2.2** Let  $X = \{a, b, c\}$  and let  $k_1$  and  $k_2$  be defined as  $k_1(\{a\}) = \{a\}$ ,  $k_1(\{b\}) = k_1(\{c\}) = k_1(\{b, c\}) = \{b, c\}$ ,  $k_1(\{a, b\}) = k_1(\{a, c\}) = k_1(\{X\}) = X$ , and  $k_1(\emptyset) = \emptyset$ .  $k_2(\{a\}) = \{a\}$ ,  $k_2(\{b\}) = \{b, c\}$ ,  $k_2(\{c\}) = k_2(\{a, c\}) = \{a, c\}$ ,  $k_2(\{a, b\}) = k_2(\{b, c\}) = k_2(\{X\}) = X$ ,  $k_2(\emptyset) = \emptyset$ .

Now,  $(X, k_1, k_2)$  is a bi-  $\check{C}$ ech closure space.

**Definition: 2.3** [3] A subset  $A$  in a bi-  $\check{C}$ ech closure space  $(X, k_1, k_2)$  is said to be

- 1.  $k_i$ -regular open if  $A = \text{int}_{k_i}(k_i(A))$ ,  $i = 1, 2$
- 2.  $k_i$ -regular closed if  $A = k_i(\text{int}_{k_i}(A))$ ,  $i = 1, 2$
- 3.  $k_i$ -semi open if  $A \subseteq k_i(\text{int}_{k_i}(A))$ ,  $i = 1, 2$
- 4.  $k_i$ -semi closed if  $\text{int}_{k_i}(k_i(A)) \subseteq A$ ,  $i = 1, 2$

The finite union of  $k_i$  regular open sets is said to be  $k_i \pi$ -open.

The complement of a  $k_i \pi$ -open set is said to be  $k_i \pi$ -closed. The smallest  $k_i \pi$  closed set containing  $A$  is called  $k_i \pi$ -closure of  $A$  and it is denoted by  $k_{\pi \text{cl}_i}(A)$ .

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### 3. $(k_1, k_2)$ - $g\pi$ closed sets:

**Definition: 3.1** A subset  $A$  is a bi- $\sim$  Cech closure space  $(X, k_1, k_2)$  is said to be  $(k_1, k_2)$ -  $g\pi$  closed if  $k_{\pi cl_2}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $k_1$ - open set in  $X$ .

**Example: 3.2** In example 2.2, the  $(k_1, k_2)$  -  $g\pi$  closed sets are  $X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}$ .

**Theorem: 3.3** If  $A$  and  $B$  are  $(k_1, k_2)$  -  $g\pi$  closed sets and so is  $A \cup B$ .

**Proof:** Let  $A$  and  $B$  be two  $(k_1, k_2)$ -  $g\pi$  closed sets. Let  $U$  be  $k_1$ -open set in  $X$ . Let  $(A \cup B) \subseteq U$ , we have  $A \subseteq U$  and  $B \subseteq U$ . Then  $k_{\pi cl_2}(A) \subseteq U$  and  $k_{\pi cl_2}(B) \subseteq U$  implies  $(k_{\pi cl_2}(A) \cup k_{\pi cl_2}(B)) \subseteq U$ . Hence  $k_{\pi cl_2}(A \cup B) \subseteq U$ .

Thus

$A \cup B$  is  $(k_1, k_2)$ -  $g\pi$  closed set.

**Theorem: 3.4** If  $A$  is  $(k_1, k_2)$ -  $g\pi$  closed set, then  $k_{\pi cl_2}(A)$ - $A$  contains no nonempty  $k_1$  - closed sets.

**Proof:** Let  $A$  be  $(k_1, k_2)$ -  $g\pi$  closed. Let  $U$  be  $k_1$  - closed contained in  $k_{\pi cl_2}(A)$ - $A$ .

Now,  $U \subseteq k_{\pi cl_2}(A)$  and  $U \subseteq A^c$ . Now,  $U \subseteq A^c$  then  $A \subseteq U^c$ . Since  $U$  is  $k_1$ -closed.  $k_{\pi cl_2}(A) \subseteq U^c$ . Consequently  $U \subseteq [k_{\pi cl_2}(A)]^c$ . As  $U \subseteq k_{\pi cl_2}(A) \cap [k_{\pi cl_2}(A)]^c = \emptyset$ ,  $U = \emptyset$ . Hence  $k_{\pi cl_2}(A)$ - $A$  contains no non-empty  $k_1$ -closed sets.

**Theorem: 3.5** If  $A$  is  $(k_1, k_2)$ -  $g\pi$  closed set, then  $k_{\pi cl_1}(x) \cap A \neq \emptyset$  holds for each  $x \in k_{\pi cl_2}(A)$ .

**Proof:** Let  $A$  be  $(k_1, k_2)$ -  $g\pi$  closed set. Suppose  $k_{\pi cl_1}(x) \cap A = \emptyset$ , for some  $x \in k_{\pi cl_2}(A)$ , we have  $A \subseteq [k_{\pi cl_1}(x)]^c$ .

Now,  $k_{\pi cl_1}(x)$  is  $k_1$ -  $\pi$ -closed. Therefore  $[k_{\pi cl_1}(x)]^c$  is  $k_1$ -  $\pi$  open. Thus  $[k_{\pi cl_1}(x)]^c$  is  $k_1$ - open. Since  $A$  is  $(k_1, k_2)$ -  $g\pi$  closed set, we have  $k_{\pi cl_2}(A) \subseteq [k_{\pi cl_1}(x)]^c$  implies  $k_{\pi cl_2}(A) \cap k_{\pi cl_1}(x) = \emptyset$ . Then  $x \notin k_{\pi cl_2}(A)$  is a contradiction.

Hence  $k_{\pi cl_1}(x) \cap A \neq \emptyset$  holds for each  $x \in k_{\pi cl_2}(A)$ .

**Theorem: 3.6** Let  $(X, k_1, k_2)$  be bi- $\sim$  Cech closure space. For each  $x$  in  $X$ ,  $\{x\}$  is  $k_1$  - closed or  $\{x\}^c$  is  $(k_1, k_2)$ -  $g\pi$  closed set.

**Proof:** Let  $(X, k_1, k_2)$  be bi- $\sim$  Cech closure space. Suppose that  $\{x\}$  is not  $k_1$ - closed,  $\{x\}^c$  is not  $k_1$ - open. Therefore, the only  $k_1$ - open set containing  $\{x\}^c$  is  $X$ . Thus,  $\{x\}^c \subseteq X$ . Now,  $k_{\pi cl_2}[\{x\}^c] \subseteq k_{\pi cl_2}(X) = X$ .

Hence  $\{x\}^c$  is  $(k_1, k_2)$  -  $g\pi$  closed set.

**Theorem: 3.7** Let  $A$  be  $(k_1, k_2)$  -  $g\pi$  closed set and if  $A$  is  $k_1$  - open then  $A = k_{\pi cl_2}(A)$ .

**Proof:** Let  $A$  be  $(k_1, k_2)$  -  $g\pi$  closed subset of a bi- $\sim$  Cech closure spaces  $(X, k_1, k_2)$  and let  $A$  be  $k_1$  - open set. Then  $k_{\pi cl_2}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $k_1$  - open set in  $X$ . Since  $A$  is  $k_1$  - open and  $A \subseteq A$ , we have  $k_{\pi cl_2}(A) \subseteq A$ . But always,  $A \subseteq k_{\pi cl_2}(A)$ . Thus,  $A = k_{\pi cl_2}(A)$ .

**Theorem: 3.8** Let  $A \subseteq Y \subseteq X$  and suppose that  $A$  is  $(k_1, k_2)$ -  $g\pi$  closed in  $(X, k_1, k_2)$ . Then  $A$  is  $(k_1, k_2)$  -  $g\pi$  closed relative to  $Y$ .

**Proof:** Let  $S$  be any  $k_1$  - open set in  $Y$  such that  $A \subseteq S$ . Then  $S = U \cap Y$  for some  $U$  is  $k_1$  - open in  $X$ . Therefore  $A \subseteq U \cap Y$  implies  $A \subseteq U$ . Since  $A$  is  $(k_1, k_2)$  -  $g\pi$  closed set in  $X$ , we have  $k_{\pi cl_2}(A) \subseteq U$ . Hence  $Y \cap k_{\pi cl_2}(A) \subseteq Y \cap U = S$ . Thus  $A$  is  $(k_1, k_2)$  -  $g\pi$  closed set relative to  $Y$ .

### 4. ${}_{g\pi}C_0$ bi- $\sim$ Cech spaces:

**Definition: 4.1** A bi- $\sim$  Cech closure space  $(X, k_1, k_2)$  is said to be a  ${}_{g\pi}C_0$  bi- $\sim$  Cech space if for every  $g\pi$ -open subset  $U$  of  $(X, k_1)$ ,  $x \in U$  implies  $k_2(\{x\}) \subseteq U$ .

**Theorem: 4.2** A bi- $\sim$  Cech closure space  $(X, k_1, k_2)$  is a  ${}_{g\pi}C_0$  bi- $\sim$  Cech space if and only if for every  $g\pi$ -closed subset  $F$  of  $(X, k_1)$  such that  $x \notin F$ ,  $k_2(\{x\}) \cap F = \emptyset$ .

**Proof:** Let  $F$  be a  $g\pi$ -closed subset of  $(X, k_1)$  and let  $x \notin F$ . Since  $x \in X - F$  and  $X - F$  is a  $g\pi$ -open subset of  $(X, k_1)$ ,  $k_2(\{x\}) \subseteq X - F$ . Consequently  $k_2(\{x\}) \cap F = \emptyset$ .

Conversely, let  $U$  be a  $g\pi$ -open subset of  $(X, k_1)$  and let  $x \in U$ . Since  $X - U$  is a  $g\pi$ -closed subset of  $(X, k_1)$  and  $x \notin X - U$ ,  $k_2(\{x\}) \cap (X - U) = \emptyset$ . Consequently  $k_2(\{x\}) \subseteq U$ .

Hence,  $(X, k_1, k_2)$  is a  $g\pi C_0$  bi- $\sim$  Cech space.

**Theorem: 4.3** Let  $\{(X_i, k_i^1, k_i^2) : i \in I\}$  be a family of bi- $\sim$  Cech closure spaces. If  $\prod_{i \in I} (X_i, k_i^1, k_i^2)$  is an  $g\pi C_0$  bi- $\sim$  Cech space, then  $(X_i, k_i^1, k_i^2)$  is an  $g\pi C_0$  bi- $\sim$  Cech space for each  $i \in I$ .

**Proof:** Suppose that  $\prod_{i \in I} (X_i, k_i^1, k_i^2)$  is an  $g\pi C_0$  bi- $\sim$  Cech space. Let  $j \in I$  and let  $G$  be an  $g\pi$ -open subset of  $(X_j, k_j^1)$  such that  $x_j \in G$ . Then  $G \times \prod_{i \neq j, i \in I} X_i$  is an  $g\pi$ -open subset of  $\prod_{i \in I} (X_i, k_i^1)$  such that  $(x_i)_{i \in I} \in G \times \prod_{i \neq j, i \in I} X_i$ . Since  $\prod_{i \in I} (X_i, k_i^1, k_i^2)$  is an  $g\pi C_0$  bi- $\sim$  Cech space,  $\prod_{i \in I} k_i^2 \pi_i(\{(x_i)_{i \in I}\}) \subseteq G \times \prod_{i \neq j, i \in I} X_i$ .

Consequently,  $k_j^2 \{x_j\} \subseteq G$ .

Hence  $(X_i, k_i^1, k_i^2)$  is an  $g\pi C_0$  bi- $\sim$  Cech space.

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