Review of Literature

REVIEW OF LITERATURE

Matrices over semirings play a significant role while dealing with different questions in matrix theory and its applications. Due to numerous applications in combinatorics, graph theory, and information theory, there is constantly increasing interest in linear algebra over semirings.

During the past century a lot of literature has been devoted to investigations of semirings. Briefly, a semiring is essentially a ring where only the zero element is required to have an additive inverse. Therefore, all rings are also semirings. Moreover, among semirings there are such combinatorially interesting systems as the Boolean algebra of subsets of a finite set (with addition being union and multiplication being intersection), nonnegative integers and reals (with the usual arithmetic), fuzzy scalars (with fuzzy arithmetic), etc. Matrix theory over semirings is an object of much study in the last decades.

A number of works on theory of matrices over semirings were carried out by many famous researchers- Gbosh. S, Golan, J.S, Gregory, D.A, Pullman, N.J, Reutenauer, C, Straubing, H, Guterman, A.E.

Regular matrices play a central role in the theory of matrices, and they have many applications in network and switching theory and information theory. Recently regular matrices over semirings are studied by Seok-Zun Song, Kyung-Tae Kang, LeRoy B.Beasley, Nung-Sing Sze, Bapat, R.B, Denes, J, Kim, K.H, Plemmons, R.J, Rao, P.S.S.N.V.P, Rao, K.P.S.B and many others.

Prime and semiprime matrices over semirings are studied by Han Hyuk Cho and Suh-Ryung Kim.

Invertible matrices are an important type of matrices. Since the beginning of the 1950s, many aurthors have studied this type of matrices for some special cases of antirings. In 1952, Luce [21] showed that a matrix over a Boolean algebra of atleast two elements is invertible if and only if it is an orthogonal matrix. Zhao [36] proved that a fuzzy square matrix is invertible if and only if it is a permutation matrix. Given'on [10] developed the theory of invertible lattice matrices, thus generalizing the result of Luce [21], Zhao [37, 38] discussed the conditions for invertibility of matrices over a kind of Brouwerian lattice and arbitrary distributive lattice, respectively. Skornyakov [32] gave an extensive description of the invertible lattice matrices. Cao et. al. [4] were the first to study the condition for an incline matrix to be invertible and showed that the statement of Luce [21] holds for the incline matrices as well. In addition, Zarko [35] established Cramer's rule over general Boolean algebras. Tian et al [34] presented Cramer's rule over complete and completely distributive lattices. Recently, Han et al [12] gave the complete description of the invertible incline matrices, they studied some necessary and sufficient conditions for an incline matrices to be invertible and presented Cramer's rule over inclines. In 2007, Yijia Tan consider the invertible matrices over general commutative antirings.

The first characterization of idempotent matrices over semirings dates back to 1963 [25], where idempotent Boolean relation matrices of finite order were characterized from the graph theoretical point of view. In parallel, in 1969 structural characterization of idempotent matrices over the semirings of the nonnegative reals was obtained in [8] in terms of a special rectangle block structure of matrices. Later, in [26] the characterization of Boolean idempotent matrices was given in terms of quasi-orders and in [11] it was provided in terms of limit-dominating matrices.

The investigations of matrix transformations which leave fixed different matrix properties and invariants is an actively developing part of matrix theory. This research was started in the works by Frobenius, [9, 20] and Dieudonne [7, 20], where bijective linear transformations on matrices over fields which preserve the determinant and the set of singular matrices, correspondingly, were characterized. During the last three decades many authors investigated linear transformations on more general algebraic structures, such as matrices over rings and semirings. There is much literature on the study of linear preserver problem for matrices over semirings.

In this Review of Literature, a brief survey of some of the articles published on matrices over semirings and linear preserver problems for matrices over semirings are given.

Matrices over Semirings

Shamik Ghosh, (1976) [31]

Introducing the concept of semi-invertibility of square matrices over semirings, some sufficient conditions for semi-inveritibility of square matrices over various kinds of semirings are obtained. Also a necessary and sufficient conditions for semi-invertibility of square matrices over semirings is obtained.

Linear preservers of perimeters of nonnegative real matrices Seok-Zun song and Kyung-Tae Kang, (2008) [27]

For the nonnegative real matrix A of rank 1, A can be factored as ab^t for some vectors a and b. The perimeter of A is the number of nonzero entries in both a and b. If B is a matrix of rank k, then B is the sum of k matrices of rank 1. The perimeter of B is the minimum of the sums of perimeters of k matrices of rank 1, where the minimum is taken over all possible rank-1

decompositions of B. In this paper characterizations of the linear operators which preserve perimeters 2 and k for some $k \geq 4$ is obtained.

Transformations of non-negative integer-valued matrices that preserve the determinant

Guterman, A.E., (2003) [12]

In this paper, bijective additive transformations that preserve the determinant is characterized.

On nilpotent incline matrices

Song-Chol Han, Hong-Xing Li and Jia-Yin Wang, (2005) [33]

This paper studies the nilpotent incline matrices in detail. It is proved that an incline matrix is nilpotent if and only if it has index and the zero vector is its unique standard eigenvector. The nilpotent matrices over an incline without nilpotent elements are characterized in terms of principal minors, main diagonals, nilpotent indices and adjoint matrices. Also some properties of the reduction of nilpotent matrices over an additively residuated incline without nilpotent elements are established.

Rank inequalities over semiring

LeRoy B.Beasley and Alexander E.Guterman, (2005) [16]

In this paper inequalities on the rank of the sum and the product of two matrices over semirings are surveyed. Preferences are given to the factor rank, row and column ranks, term rank and zero-term rank of matrices over antinegative semirings.

Invertible and nilpotent matrices over antirings

David dolzan and Polona oblak, (2009) [5]

In this paper invertible matrices over an arbitrary commutative antirings S with 1 are characterized. The number of nilpotent matrices over an entire commutative finite antirings are found. If it proved that every nilpotent $n \times n$ matrix over an entire antiring can be written as a sum of $\lceil \log_2 n \rceil$ square-zero matrices.

Linear preservers of Boolean nilpotent matrices

Seok-Zun Song, Kyung-Tae Kang and Young-Bae-Jun, (2006) [13]

For an $n \times n$ Boolean matrix A, A is called nilpotent if $A^m = 0$ for some positive integer m. Linear operatiors that strongly preserve nilpotent matrices over Boolean algebras are characterized.

Linear operators that strongly preserve the index of imprimitivity

LeRoy B. Beasley and Norman J. Pullman, (1992) [18]

In this paper, the linear operators on the square matrices (even those with non-zero diagonal) over the 2-element Boolean algebra that strongly preserve index of imprimitivity k is characterized.

Linear operators strongly preserving idempotent matrices over semirings

LeRoy Beasley and N. J. Pullman, (1992) [19]

In this paper, the problem of characterizing those linear operators L on the matrices over a semiring such that L(X) is idempotent if and only if X is considered. Complete characterizations are obtained for many semirings, including the nonnegative reals, the nonnegative integers, the two element Boolean algebra, and the fuzzy scalars.

Factor and term ranks for matrix union over semirings

Pshenitsyna, O.A., (2006) [23]

In the present paper, classifications of the linear preservers of the extremes of inequalities on the factor rank and on the term rank of matrix union are obtained.