Applied Mathematical Sciences, Vol. 6, 2012, no. 64, 3199 - 3207

Double Acceptance Sampling Based on Truncated Life Tests in Generalized Exponential Distribution

A. R. Sudamani Ramaswamy

Department of Mathematics
Avinashilingam University
Coimbatore – 641043, Tamil Nadu, India
arsudamani@hotmail.com

Priyah Anburajan

Department of Mathematics
Avinashilingam University
Coimbatore – 641043, Tamil Nadu, India
priyahanburajan@gmail.com

Abstract

In this paper, double acceptance sampling plans for truncated life tests are developed when the lifetimes of test items follows generalized exponential distribution. Probability of acceptance is calculated for different consumer's confidence levels fixing the producer's risk. Probability of acceptance and producer's risk are discussed with the help of tables and examples.

Keywords: Truncated Life test, consumer's risk, producer's risk, Generalized exponential distribution, Double acceptance sampling plan.

1. Introduction

An acceptance sampling plan involves quality contracting on product orders between the producers and the consumers. In a time – truncated sampling plan, a random sample is selected from a lot of products and put on the test where the number of failures is recorded until the pre – specified time. If the number of failures observed is not greater than the specified acceptance number, then the lot will be accepted. Two risks are always attached to an acceptance sampling. The probability of rejecting a good lot is known as the producer's risk and the probability of accepting a bad lot is called the consumer's risk. An acceptance sampling plan should be designed so that both risks are smaller than the required values. These life tests are discussed by many authors, Goode and Kao (1961) have studied sampling plans based on the distributions, Gupta and Groll (1961) have studied application of Gamma distribution in acceptance sampling based on life tests, Aslam M. and Shabaz M.Q. (2007) have studied Economic reliability test plans using the generalized exponential distribution, Baklizi A. and El Masri A.E.K. (2004) have studied Acceptance sampling plans based on truncated life tests in the Birnbaum Saunders model, Muhammad Aslam, Debasis Kunda, Munir Ahmad have studied Time truncated acceptance sampling plans for generalized exponential distribution and also Muhammad Aslam have studied Double acceptance sampling based on truncated life tests in Rayleigh distribution, Srinivasa Rao S.(2011) have studied Double acceptance sampling plans based on truncated life tests for the Marshall – Olkin extended exponential distribution. All these authors developed the sampling plans for life tests using single acceptance sampling.

We in this paper propose a plan to find the probability of acceptance for the double acceptance sampling assuming the experiment is truncated at pre – assigned time and lifetime follows Generalized exponential distribution.

2. Generalized Exponential Distribution

The cumulative distribution function (cdf) of the Generalized exponential distribution is given by

$$F(t,\sigma) = \left(1 - e^{-\frac{t}{\sigma}}\right)^{\alpha} \tag{1}$$

where σ is a scale parameter. If some other parameters are involved, then they are assumed to be known, for an example, if shape parameter of a distribution is unknown it is very difficult to design the acceptance sampling plan. In quality control analysis, the scale parameter is often called the quality parameter or characteristics parameter. Therefore it is assumed that the distribution function depends on time only through the ratio of t/σ.

3. Design of the proposed plan

It is known that the double acceptance sampling plan (DASP) is more efficient than the single sampling plan in terms of the sample size required.

Further, a DASP is expected to reduce the producer's risk when specifying the consumer's risk. We propose a following DASP based on a truncated life test:

1. Draw the first sample of size n_l and put them on test during time t_0.
2. Accept the lot if there are no more than c_l failures. Reject the lot and terminate the test if there are more than c_2 failures.
3. If the number of failures is between c_l and c_2, then draw the second sample of size n_2 and put them on test during time t_0.
4. Accept the lot if the total number of failures from the first and second samples is no greater than c_2. Otherwise, reject the lot and terminate the test.

The DASP is composed of four parameters of (n_l, n_2, c_l, c_2) if t_0 is specified. Here n_l and n_2 are sample sizes of the first and the second sample, respectively, whereas c_l and c_2 are the acceptance numbers associated with the first and the second sample, respectively. The single sampling plan is a special case of DASP when $c_l = c_2$.

We assume that the lot size is large enough to use the binomial distribution to find the probability of lot acceptance. The probability of acceptance $L(p_l)$ and $L(p_2)$ for the sampling plan (n_l, c_l, a) and (n_2, c_2, a) is calculated using the following equations,

$$L(p_1) = \sum_{i=0}^{c_1=0} \binom{n_1}{i} p^i (1-p)^{n_1-i} \quad (2)$$

$$L(p_2) = \sum_{i=0}^{c_2=2} \binom{n_2}{i} p^i (1-p)^{n_2-i} \quad (3)$$

Where,

$$p = \left(1 - e^{-\frac{t}{\sigma}}\right)^{\alpha} \quad (4)$$

Then the probability of acceptance for DASP is given by

P(A) = P(no failure occur in sample 1) + P(1 failure occur in sample 1 and 0,1 failure occur in sample 2) + P(2 failures occur in sample 1 and 0 failure occurs in sample 2).

$$L(p) = \binom{n_1}{0} p^0 q^{n_1} + \binom{n_1}{1} p^1 q^{n_1-1} \left[\sum_{i=0}^{1} \binom{n_2}{i} p^i q^{n_2-i} \right]$$
$$+ \binom{n_1}{2} p^2 q^{n_1-2} \left[\binom{n_2}{0} p^0 q^{n_2} \right] \quad (5)$$

4. Notation

n_1	-	First sample size
n_2	-	Second sample size
c_1	-	Acceptance number of sample first
c_2	-	Acceptance number of sample second
d	-	Number of defectives
t	-	Termination time
α	-	Shape parameter
σ	-	Scale parameter
β	-	Consumer's risk
p	-	Failure probability
$L(p)$	-	Probability of acceptance
p^*	-	Minimum probability
σ_0	-	Specified life

5. Description of tables and examples

In this study we fixed the consumer's risk which did not exceed 1-p*, where p* = 0.75, 0.90, 0.95, 0.99 and satisfy the following inequality.

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - p^* \quad (6)$$

If in this experiment no failure occurs when sample first is put on test we accept the lot. Probability of acceptance for sample first using the Generalized exponential distribution for α =2 is placed in Table 1 and the probability of acceptance for DASP is calculated and placed in Table 2 using (2), (3) and (4).

Assume that an experimenter wants to establish that true unknown mean life is at least 1000 hours with confidence 0.95. Let the acceptance numbers for this experiment be $c_1 = 0$ and $c_2 = 2$ with sample sizes $n_1 = 9$ and $n_2 = 20$. The lot is accepted if during 628 hours no failure is observed in a sample of 9. Probability of acceptance for this single sampling from Table 1 is 0.507364.

Probability of acceptance for the same measurements using DASP for Table 2 is 0.735299. In DASP scheme as σ/σ_0 increases probability of acceptance for DASP decreases. For the above consideration probability of acceptance is 0.999951 when the ratio of unknown average life to specified life is 12. As the time of experiment increases, the probability of acceptance for DASP decreases. From Table 2, it is clear that when the time of experiment is 4712 hours, probability of acceptance for ratio $\sigma/\sigma_0 = 2$ is 0.062053. For this same experiment time, as increases, probability of acceptance also increases. We here note that double acceptance sampling scheme minimizes the producer's risk, and also exerts pressure on producer to improve the quality of the product. At 4712 hours, $\sigma/\sigma_0 = 12$, and $P^* = 0.95$, the probability of acceptance is 0.978865. Producer's risk for the sample first and double sampling are placed for $P^* = 0.95$ in Table 3.

Table 1:
Operating characteristic values for sampling plan (n_1, c_1, t/σ) when c_1= 0

p*	t/σ	n_1	σ/σ_0					
			2	4	6	8	10	12
0.75	0.628	5	0.685944	0.898809	0.951588	0.971823	0.981612	0.986399
	0.912	3	0.633636	0.873651	0.937995	0.963437	0.975950	0.981683
	1.257	2	0.611956	0.859851	0.929826	0.958163	0.975950	0.978294
	1.571	2	0.495540	0.800136	0.896684	0.937432	0.958175	0.966302
	2.356	2	0.271430	0.642995	0.800202	0.874087	0.913822	0.925803
	3.141	2	0.138863	0.495715	0.695385	0.800234	0.859961	0.871947
	3.927	2	0.068137	0.370812	0.591844	0.721695	0.800175	0.807199
	4.712	2	0.032620	0.271430	0.495657	0.642995	0.737561	0.734640
0.90	0.628	7	0.589935	0.861260	0.932886	0.960776	0.974351	0.981011
	0.912	3	0.633636	0.873651	0.937995	0.963437	0.975950	0.981683
	1.257	2	0.611956	0.859851	0.929826	0.958163	0.975950	0.978294
	1.571	2	0.495540	0.800136	0.896684	0.937432	0.958175	0.966302
	2.356	2	0.271430	0.642995	0.800202	0.874087	0.913822	0.925803
	3.141	2	0.138863	0.495715	0.695385	0.800234	0.859961	0.871947
	3.927	2	0.068137	0.370812	0.591844	0.721695	0.800175	0.807199
	4.712	2	0.032620	0.271430	0.495657	0.642995	0.737561	0.734640
0.95	0.628	9	0.507364	0.825280	0.914552	0.949855	0.967145	0.975652
	0.912	5	0.467447	0.798417	0.898809	0.939808	0.960239	0.969659
	1.257	4	0.374490	0.739344	0.864577	0.918076	0.945347	0.957059
	1.571	3	0.271430	0.715725	0.849100	0.907631	0.937923	0.949882
	2.356	2	0.271430	0.642995	0.800202	0.874087	0.913822	0.925803
	3.141	2	0.138863	0.495715	0.695385	0.800234	0.859961	0.871947
	3.927	2	0.068137	0.370812	0.591844	0.721695	0.800175	0.807199
	4.712	2	0.032620	0.271430	0.495657	0.642995	0.737561	0.734640
0.99	0.628	14	0.348023	0.741769	0.870277	0.923091	0.949361	0.962383
	0.912	8	0.296191	0.697538	0.843078	0.905446	0.937145	0.951897
	1.257	5	0.292955	0.685580	0.833690	0.898666	0.932157	0.946615
	1.571	4	0.245560	0.640218	0.804042	0.878778	0.918100	0.933741
	2.356	3	0.141412	0.515599	0.715812	0.817206	0.873560	0.890796
	3.141	2	0.138863	0.495715	0.695385	0.800234	0.859961	0.871947
	3.927	2	0.068137	0.370812	0.591844	0.721695	0.800175	0.807199
	4.712	2	0.032620	0.271430	0.495657	0.642995	0.737561	0.734640

Table 2: Operating characteristic values for the double sampling plan (n_2, c_2, t/σ) when c_1=0, c_2=2

P*	t/σ	n_1	n_2	σ/σ_0					
				2	4	6	8	10	12
0.75	0.628	5	13	0.905117	0.996009	0.999540	0.999908	0.999974	0.999989
	0.912	3	8	0.862862	0.992745	0.999089	0.999809	0.999945	0.999976
	1.257	2	6	0.831248	0.989288	0.998551	0.999684	0.999907	0.999955
	1.571	2	5	0.734251	0.978330	0.996787	0.999266	0.999778	0.999883
	2.356	2	4	0.460343	0.920436	0.985110	0.996171	0.998755	0.999202
	3.141	2	3	0.305620	0.855110	0.968019	0.990991	0.996900	0.997630
	3.927	2	3	0.142985	0.720071	0.923063	0.975615	0.990983	0.991902
	4.712	2	3	0.062053	0.569141	0.855060	0.948507	0.979556	0.978865
0.90	0.628	7	17	0.820271	0.990743	0.998882	0.999771	0.999935	0.999974
	0.912	3	10	0.825158	0.989577	0.998652	0.999714	0.999917	0.999963
	1.257	2	7	0.800499	0.986110	0.998075	0.999576	0.999874	0.999939
	1.571	2	6	0.684910	0.970934	0.995535	0.998965	0.999684	0.999834
	2.356	2	4	0.460343	0.920436	0.985110	0.996171	0.998755	0.999202
	3.141	2	4	0.216182	0.791794	0.949416	0.985117	0.994758	0.995975
	3.927	2	3	0.142985	0.720071	0.923063	0.975615	0.990983	0.991902
	4.712	2	3	0.062053	0.569141	0.855060	0.948507	0.979556	0.978865
0.95	0.628	9	20	0.735299	0.983931	0.997981	0.999581	0.999881	0.999951
	0.912	5	12	0.671669	0.974581	0.996488	0.999235	0.999776	0.999900
	1.257	4	8	0.587555	0.960566	0.994058	0.998647	0.999593	0.999801
	1.571	3	6	0.560055	0.952542	0.992392	0.998206	0.999448	0.999708
	2.356	2	5	0.387466	0.891495	0.978350	0.994276	0.998113	0.998785
	3.141	2	4	0.216182	0.791794	0.949416	0.985117	0.994758	0.995975
	3.927	2	3	0.142985	0.720071	0.923063	0.975615	0.990983	0.991902
	4.712	2	3	0.062053	0.569141	0.855060	0.948507	0.979556	0.978865
0.99	0.628	14	26	0.538502	0.960505	0.994593	0.998840	0.999664	0.999861
	0.912	8	15	0.456770	0.939782	0.990834	0.997927	0.999381	0.999721
	1.257	5	10	0.444797	0.939782	0.988749	0.997358	0.999192	0.999603
	1.571	4	8	0.369992	0.902714	0.982592	0.995706	0.998648	0.999278
	2.356	3	6	0.199413	0.792931	0.952584	0.986688	0.995472	0.997059
	3.141	2	4	0.216182	0.791794	0.949416	0.985117	0.994758	0.995975
	3.927	2	4	0.092745	0.625546	0.883936	0.960993	0.985104	0.986584
	4.712	2	4	0.039365	0.460343	0.791729	0.920436	0.967077	0.966006

Table 3:
Producer's risk with respect to time of experiment for double sampling ($p^* = 0.95$)

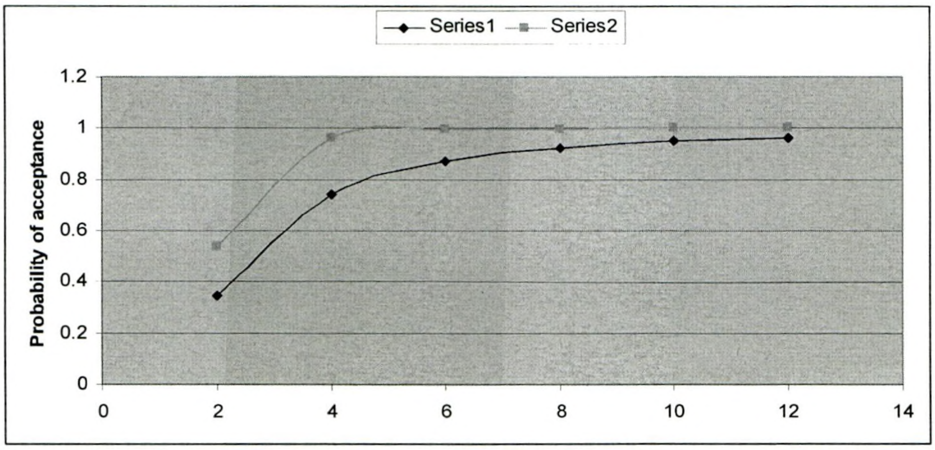
σ/σ_0	$c_1=0, c_2=2, n_1=9, n_2=20, t/\sigma_0=0.628$	$c_1=0, c_2=2, n_1=2, n_2=3, t/\sigma_0=4.712$
2	0.264701	0.937947
4	0.016069	0.430859
6	0.002019	0.144940
8	0.000419	0.051493
10	0.000119	0.020444
12	0.000049	0.021135

For $\sigma/\sigma_0 = 2$ (if unknown average life is twice of specified average life) producer's risk when time of experiment is 628 hrs and 4712 hrs for $\alpha = 2$ are 0.264701 and 0.937947 respectively.

Producer's risk decreases as the quality level of the product increases with $p^*=0.95$. Table 3. Fig 1 and Fig 2 illustrate our idea.

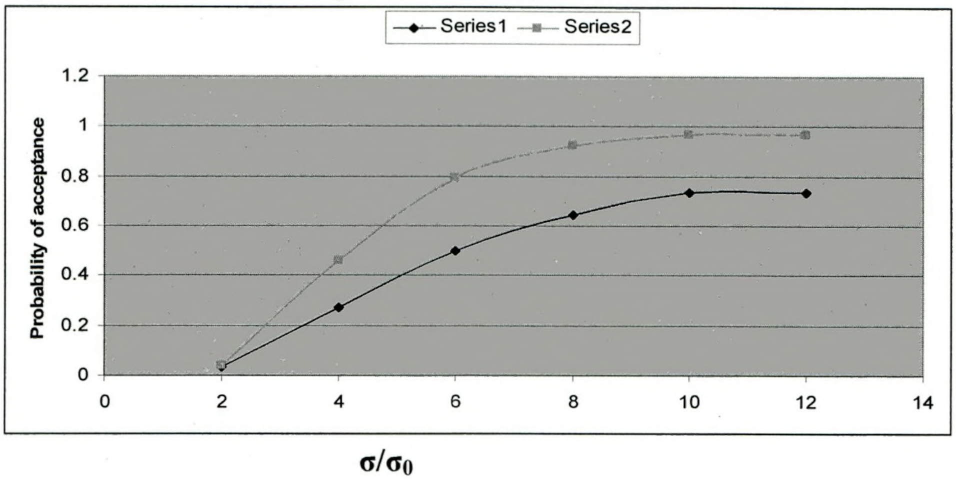
Figure 1 :

OC Curve with $p^* = 0.99$ and $t_0 = 628$ hrs



σ/σ_0

Figure 2: OC Curve with $p^* = 0.99$ and $t_0 = 4712$ hrs



Conclusion

We find the acceptance sampling plans for various values of σ/σ_0 and different experiment times assuming that the life test follows the Gamma distribution. This distribution provides the high probability for $\sigma/\sigma_0 > 6$.

References

[1] M. Aslam and M. Q. Shabaz, Economic reliability test plans using the generalized exponential distribution, Journal of Statistics, vol. 14, 52 – 59, 2007.

[2] A. Baklizi and A. E. K. El Masri, Acceptance sampling plans based on truncated life tests in the Birnbaum Saunders model, Risk Analysis, vol. 24, 1453 – 1457, 2004.

[3] S. S. Gupta and P. A. Groll, Gamma distribution in acceptance sampling based on life tests, Journal of the American Statistical Association, vol. 56, 942 – 970, 1961.

[4] Muhammad Aslam, Debasis Kunda, Munir Ahmad, Time truncated acceptance sampling plans for generalized exponential distribution, Journal of Applied Statistics, Vol. 37, No. 4, April 2010, 555-566

[5] Muhammad Aslam, Double acceptance sampling based on truncated life tests in Rayleigh distribution, European Journal of Scientific Reasearch, Vol. 17 No. 4 (2007), pp. 605 – 610.

[6] S. Srinivasa Rao, Double acceptance sampling plans based on truncated life tests for the Marshall – Olkin extended exponential distribution, Austrian Journal of Statistics Volume 40 (2011), Number 3, 169 – 176.

Received: January, 2012

3.	3	10	0.15	1.6100	0.2260	3.030	0.026	38.40
4.	4	10	0.2	1.2660	0.400	3.150	0.056	17.86
5.	5	10	0.25	1.1600	0.500	3.471	0.090	11.11
6.	6	10	0.30	0.6524	0.600	3.339	0.182	5.49
7	7	10	0.35	0.5597	0.700	3.518	0.285	3.51
8	8	10	0.40	0.3650	0.800	3.542	0.4536	2.20
9	9	10	0.45	0.1838	0.900	3.528	0.6896	1.45

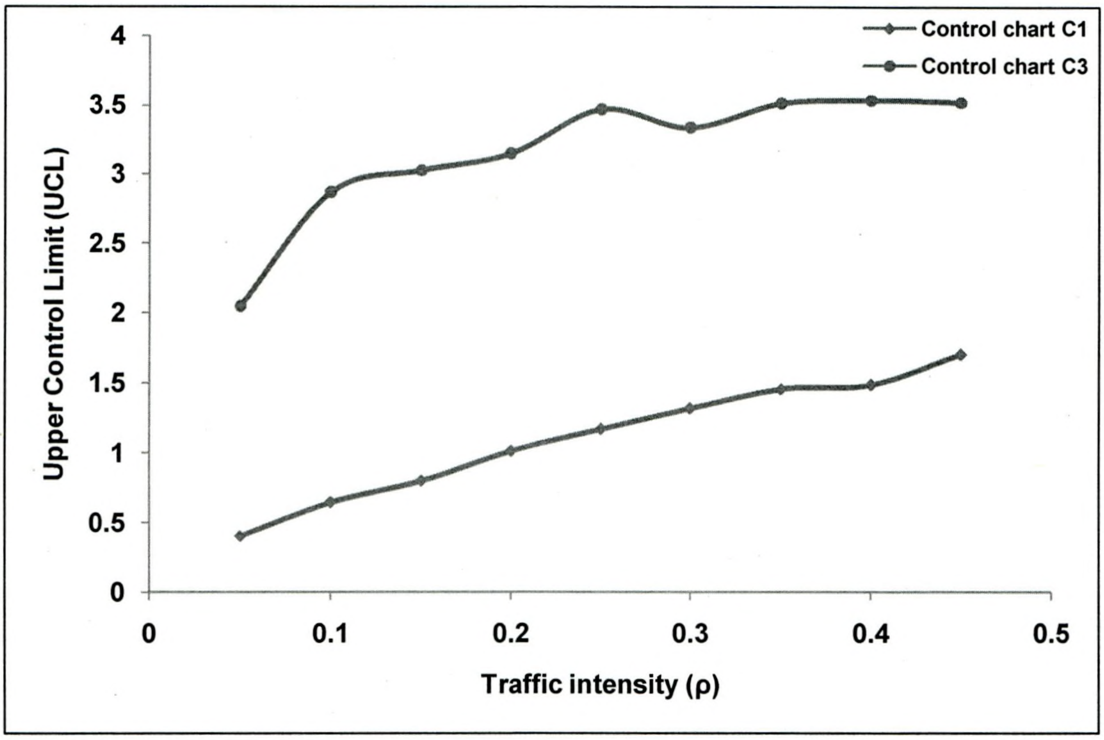


FIGURE II :COMPARISON OF UPPER CONTROL LIMIT (UCL) OF CONTROL CHARTS C_1 AND C_3 WITH L = 3 AND C = 2 FOR DIFFERENT VALUES OF ρ

It is noticed that the upper control limit of control chart C_3 outperforms the control chart C_1 for all values of ρ. Upper control limit values in control chart C_3 are very very large than that of chart C_1. Therefore for skewed population the use of control chart C_3 is advisable. This can be observed from Figure II

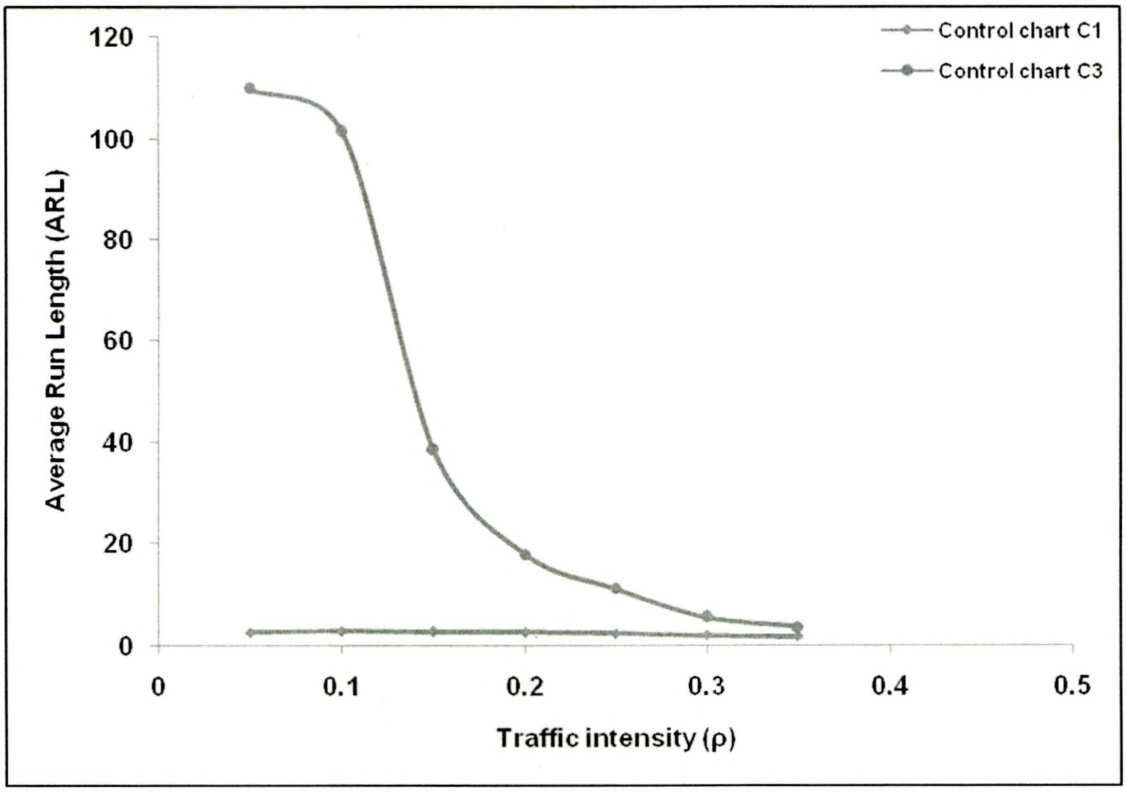


FIGURE III: COMPARISON OF AVERAGE RUN LENGTH OF CONTROL CHARTS C_1 AND C_3 WITH L = 3 AND C = 2 FOR DIFFERENT VALUES OF ρ

CONCLUSON

The curves of ARL values ρ are displayed in Figure III for charts C_1 and C_3. The horizontal axis of ρ spans 0.05 to 0.45, Figure III demonstrates that the ARL produced by chart C_3 is superior to the chart C_1.

REFERENCES

1. Arnold, A.O. (1978). Probability, Statistics and Queueing Theory with Computer Science Applications, Second Edition, Academic Press Limited, New York.

2. Barrer, D.Y. (1957). Queueing with Impatient Customers and Indifferent Clerks, Operations Research, 5, 644-649.

3. Khaparde, M.V. and Dhabe, S.D. (2010). Control Charts for Random Queue Length N for (M / M / 1) : (∞ / FCFS) Model, International Journal of Agricultural and Statistical Sciences, 1, 319-334.

4. Khaparde, M.V. and Dhabe, S.D. (2011). Control Charts for Random Queue Length N for (M / M / 1) : (∞ / FCFS) Queueing Model Using Skewness and Power Transformation, Bulletin of Pure and Applied Sciences, 30 E, 71-83.

5. Montgomery, D.C. (2005). Introduction to Statistical Quality Control, Fifth Edition, John Wiley and Sons Inc.

6. Shore, H. (2000). General Control Charts for Attributes, IIE Transactions, 32, 1149-1160.

7. Sudamani Ramaswamy, A.R. (2010). Statistical Analysis of M / M / c Model, Research Highlights, 20, 27-32.