**Avinashilingam Institute for Home Science and Higher Education for women**
**(Deemed to be University) Coimbatore – 641 043.**
**Bachelor's Degree Examination – November 2018**
**V Semester**

**Class : III UG** **Time : 3 HRS**
**Major : Special Education & Reh, and Mathematics** **Max.Marks: 100**

**15BSMC 12 – Complex Analysis-I**

1. In a bounded ____ region a continuous function is uniformly continuous.
   a) closed b) open c) continuous d) differentiable

2. The function f(z) = 3z – 2 is ____ in $|z| \leq 10$.
   a) continuous b) uniformly continuous c) constant
   d) nowhere differentiable

3. The function 1/(1+z) is analytic at ____.
   a) $\infty$ b) 0 c) 1 d) –1

4. ____ is not an entire function.
   a) $e^z$ b) sinz c) cosz d) |x|

5. A function is said to be harmonic if it satisfies ______ equation.
   a) harmonic b) Laplace c) differential d) C-R

6. If u and v are conjugate harmonic functions then ____ are also conjugate harmonic function.
   a) u & v b) v & u c) –u & v d) v & –u

7. The fixed points of the transformation w = z+b is z = __.
   a) $\infty$ b) 0 c) b d) –b

8. Points which are invariant under a transformation are called ____ points.
   a) variable b) variant c) constant d) translation

9. $\int_C \frac{dz}{z-a} =$ ____ where C is the positively oriented circle whose radius is r and centre is z = a.
   a) $2\pi$ b) $2\pi i$ c) 0 d) $\infty$

10. Integrals along scr curves are called ____ integrals.
    a) definite b) line c) surface d) contour

**PART – B** **5x6=30**

**Answer the following**

11. a) If f(z) is differentiable at $z_0$, then show f(z) is continuous at $z_0$.

-OR-

11. b) Show that the function $f(z) = \bar{z}$ is nowhere differentiable.

12. a) Prove that for an analytic function f(z) the C-R equation can be put in the condensed form $\partial f/\partial \bar{z} = 0$.

-OR-

12. b) If f(z) and $\overline{f(z)}$ are analytic in a region, show that f(z) is constant in that region.

13. a) Find the analytic function f(z) if $u = x^2 - y^2$.

-OR-

13. b) Check whether xy(x+y) is a harmonic function. Can it be the real part of an analytic function?

14. a) Prove that the cross ratio is preserved by a bilinear transformation.

-OR-

14. b) Find the bilinear transformation which map $z = \infty, i, 0$ into $w = 0, i, \infty$.

15. a) If l(L) is the length of the simple arc C and if f(z) is continuous on C, max|f(z)≤ M, then

prove that $\left|\int_C f(z)dz\right| \leq ML$.

-OR-

15. b) If C is positively oriented circle $|z-1| = 1$, show that. $\int_C \left(\frac{3z^2+z}{z^2-1}\right) dz = 4\pi i$

**PART – C** **5x12=60**

**Answer the following**

16. a) State and prove the sufficient conditions for differentiability.

-OR-

16.b) Derive C-R equations in Cartesian coordinates.

17. a) Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log|f(z)| = 0$.

-OR-

17.b) Derive the C-R equations in polar coordinates.

18. a) Find the analytic function f(z) if $u - v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$

-OR-

18. b) Show that the function $u(x,y) = e^x(x\cos y - y\sin y)$ is harmonic. Find its harmonic conjugate v(x,y) and the analytic function f(z) = u+iv.

19. a) Prove that the bilinear transformations which map $\text{Im}z \geq 0$ onto $|w| \leq 1$ are of the form $w = e^{i\lambda} \frac{z - z_1}{z - \overline{z_1}}$ where $\lambda$ is a real number.

-OR-

19. b) Prove that the set of all bilinear transformations is a group under composition.

20. a) State and prove the Cauchy's fundamental theorem.

-OR-

20. b) State and prove the Cauchy's formula for first derivative.

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