CHAPTER – V

CHAPTER V

FUZZY δ – SEMI IRRESOLUTE FUNCTIONS

Definition: 5.1

A fuzzy set λ in a fts (X,τ) is said to be **fuzzy δ-semi open** if there exists fuzzy δ-open set η such that $\eta \leq \lambda \leq \mathrm{cl}\,\eta$.

The complement of a fuzzy δ-semi open set is said to be a **fuzzy δ-semi closed** set.

Notation: 5.2

We denote **F_X (δ-SO)** is the set of all fuzzy δ-semi open sets in a fuzzy topological space (X,τ).

Definition: 5.3

A fuzzy set α is called a **fuzzy δ-semi neighbourhood** of a fuzzy point x_p in X if there exists a $\beta \in F_X(\delta\text{-SO})$ such that $x_p \in \beta \leq \alpha$.

Definition: 5.4

A fuzzy set α is called a **fuzzy δ-semi q-neighbourhood** of a fuzzy point x_p in X if there exists a $\beta \in F_X(\delta\text{-SO})$ such that $x_p \; q \; \beta \leq \alpha$.

Note: 5.5

We denote the set of all fuzzy δ-semi neighbourhood (resp. fuzzy δ-semi q-neighbourhood) of x_p by $N(x_p)$(resp. $N_q(x_p)$).

Definition: 5.6

A function f : $(X,\tau) \rightarrow (Y,\delta)$ is called a **fuzzy δ-semi irresolute function** if $f^{-1}(\beta) \in F_X(\delta\text{-SO})$ for each $\beta \in F_Y(\delta\text{-SO})$.

Example: 5.7

Let X= {a, b} and $\tau = \{\phi, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9\}$

Where $A_1(a) = \frac{1}{10}$, $A_2(a) = \frac{1}{10}$, $A_3(a) = \frac{1}{4}$,

$A_4(a) = \frac{1}{4}$, $A_5(a) = \frac{1}{4}$;

$A_1(b) = \frac{1}{10}$, $A_2(b) = \frac{1}{5}$, $A_3(b) = \frac{1}{10}$,

$A_4(b) = \frac{1}{5}$, $A_5(b) = \frac{9}{40}$,

$A_6(a) = \frac{3}{4}$, $A_7(a) = \frac{3}{4}$, $A_8(a) = \frac{9}{10}$, $A_9(a) = \frac{9}{10}$;

$A_6(b) = \frac{3}{4}$, $A_7(b) = \frac{8}{9}$, $A_8(b) = \frac{3}{4}$, $A_9(b) = \frac{8}{9}$.

Then (X, τ) is a fuzzy topology.

Here A_5 is the only fuzzy δ-semi open set in (X, τ).

$\therefore$ The identity function f : $(X, \tau) \rightarrow (X, \tau)$ is a fuzzy δ-semi irresolute function.

Theorem: 5.8

For a function f : X→ Y we have the following equivalent relations:

(i) f is fuzzy δ-semi irresolute.

(ii) For each fuzzy point x_p in X and each $\beta \in F_Y(\delta\text{-SO})$ with $f(x_p) \in \beta$ $\exists$ a $\alpha \in F_X(\delta\text{-SO})$ such that $x_p \in \alpha$ and $f(\alpha) \leq \beta$.

(iii) For each fuzzy point x_p in X and each $\beta \in F_Y(\delta\text{-SO})$ satisfying $f(x_p)\, q\, \beta$ $\exists$

$\alpha \in F_X(\delta\text{-SO})$ such that $x_p \, q \, \alpha$ and $f(\alpha) \leq \beta$.

Then (i) ⇒ (ii) ⇒ (iii) ⇒ (i).

Proof:

(i) ⇒ (ii) :

Let x_p be a fuzzy point in X and $\beta \in F_Y(\delta\text{-SO})$ such that $f(x_p) \in \beta$. Then $x_p \in f^{-1}(\beta)$. Since f is fuzzy δ-semi irresolute, $f^{-1}(\beta) \in F_X(\delta\text{-SO})$ Let $f^{-1}(\beta) = \alpha$ then we have $f(\alpha) = f\ f^{-1}(\beta) \leq \beta$.

(ii) ⇒ (iii) :

Let x_p be a fuzzy point in X and $\beta \in F_Y(\delta\text{-SO})$ satisfying $f(x_p) \, q \, \beta$.

Now take f(x) = y then $\beta(y) + p > 1$ and there is a possible real number t (say) such that $\beta(y) > t > 1 - p$ so that $\beta \in N(y_t)$, where $N(y_t)$ is the set of all fuzzy δ-semi neighbourhood of y_t.

By (ii), there exists a $\alpha \in F_X(\delta\text{-SO})$ such that $x_t \in \alpha$ and $f(\alpha) \leq \beta$.

Now $\alpha(x) \geq t \Longrightarrow \alpha(x) > 1 - p \Longrightarrow \alpha(x) + p > 1$. Thus $\alpha \in N_q(x_p)$ Where $N_q(x_p)$ is the set of all fuzzy δ-semi q-neighbourhood of x_p. i.e. $x_p \, q \, \alpha$.

(iii) ⇒ (i) :

Let x_p be a fuzzy point in X and λ be a fuzzy δ-semi open set in Y such that $x_p \, q \, f^{-1}(\lambda)$ i.e. $f(x_p) \, q \, \lambda$. Then ∃ a fuzzy δ-semi open set μ in X containing x_p such that $f(\mu) \leq \lambda$.

∴ We obtain $x_p \in \mu \leq cl(\delta - int\ \mu) \leq cl(\delta - int(f^{-1}(\lambda)))$

and hence, $f^{-1}(\lambda) \leq cl(\delta - int\ f^{-1}(\lambda))$.

Thus $f^{-1}(\lambda)$ is a δ-semi open set in X.

i.e. f is a fuzzy δ-semi irresolute function.

Theorem: 5.9

Let $f : X \rightarrow Y$ is a fuzzy δ-semi irresolute function then for each fuzzy δ-semi closed set λ *in* Y , $f^{-1}(\lambda)$ is a fuzzy δ-semi closed set in X.

Theorem: 5.10

For any fuzzy δ-semi irresolute function $f : X \rightarrow Y$,

(i) $f(F_\delta - scl(\gamma)) \leq F_\delta - scl(f(\gamma))$, for any fuzzy set γ in X.

(ii) $f^{-1}(F_\delta - sint(\lambda)) \leq F_\delta - sint(f^{-1}(\lambda))$, for any fuzzy set λ *in* Y.

Proof:

(i):

Let x_p be a fuzzy point of X such that $x_p \in F_\delta - scl(\gamma)$.

Then by (ii) of theorem 5.8,

$$\gamma \, q \, \alpha \Rightarrow f(\gamma) \, q \, f(\alpha) \Rightarrow \beta \, q \, f(\gamma)$$

$$\Rightarrow f(x_p) \in F_\delta - scl(f(\gamma))$$

$$\Rightarrow x_p \in f^{-1}(F_\delta - scl(f(\gamma)))$$

$$\Rightarrow F_\delta - scl(\gamma) \leq f^{-1}(F_\delta - scl(f(\gamma)))$$

$$\Rightarrow f(F_\delta - scl(\gamma)) \leq f(f^{-1}(F_\delta - scl(f(\gamma))))$$

$$\leq F_\delta - scl(f(\gamma)).$$

(ii) :

For any fuzzy set λ in Y, $F_\delta - sint(\lambda)$ is a fuzzy δ-semi open set in Y and since f is fuzzy δ- semi irresolute $f^{-1}(F_\delta - \text{sint}(\lambda))$ is a fuzzy δ-semi open set in X.

Hence, $f^{-1}(F_\delta - \text{sint}(\lambda)) = F_\delta - \text{sint}(f^{-1}(F_\delta - \text{sint}(\lambda)))$

$$\leq F_\delta - \text{sint}(f^{-1}(\lambda)) \ .$$

Theorem: 5.11

The function $f : X \longrightarrow Y$ is fuzzy δ-semi irresolute iff $F_\delta - \text{scl}\ (f^{-1}(\mu)) \leq f^{-1}(F_\delta - \text{scl}\ (\mu))$, for any fuzzy set μ in Y.

Proof:

Let $f : X \longrightarrow Y$ is a fuzzy δ-semi irresolute function then for any fuzzy set μ in Y, $F_\delta - \text{scl}(f^{-1}(\mu)) \leq f^{-1}(F_\delta - \text{scl}(\mu))$, follows from the theorem 5.10.

Conversely, let λ be a fuzzy δ-semi closed set in Y, then

$F_\delta - \text{scl}\ (f^{-1}(\lambda)) \leq f^{-1}\ (F_\delta - \text{scl}\ (\lambda)) = f^{-1}(\lambda)$

$\therefore f^{-1}(\lambda)$ is a fuzzy δ-semi closed set in X.

Thus, f is a fuzzy δ-semi irresolute function.

Definition: 5.12

A fuzzy topological space X is said to be **fuzzy semi compact** if every fuzzy semi open cover of X has a finite sub cover.

Definition: 5.13

A fuzzy topological space X is said to be **fuzzy δ-semi compact** if every fuzzy δ-semi open cover X has a finite sub cover.

Theorem: 5.14

A fts X is fuzzy δ-semi compact iff for every collection $\{A_j : j \in J\}$ of fuzzy

δ-semi closed sets of X having the finite intersection property, $\wedge_{j \in J} A_j \neq 0_X$.

Proof:

Let { A_j : j ∈ J } be a collection of fuzzy δ-semi closed sets with the finite intersection property.

If possible let $\wedge_{j \in J} A_j = 0_X$.

Then $\wedge_{j \in J} A_j' = 1_X$.

Since { A_j': j ∈ J} is a collection of fuzzy δ-semi open sets cover of X, then from the fuzzy δ-semi compactness of X it follows that there exists a finite subset F ⊆ J such that $\vee_{j \in J} A_j' = 1_X$. Then $\vee_{j \in F} A_j = 0_X$ which gives a contradiction and therefore $\vee_{j \in F} A_j \neq 0_X$.

Conversely, let {A_j : j ∈ J} be a collection of δ-semi open sets cover of X. Suppose that for every finite subset F ⊆ J , we have $\vee_{j \in F} A_j \neq 1_X$.

Then $\vee_{j \in J} A_j' \neq 0_X$.

Hence {A_j : j ∈ J} satisfies the finite intersection property. Then from the hypothesis we have $\vee_{j \in J} A_j' \neq 0_X$. which implies $\vee_{j \in F} A_j \neq 1_X$ and this contradicting that is a {A_j: j ∈ J} is a δ-semi open cover of X.

Thus X is fuzzy δ-semi compact.

Theorem: 5.15

If f : X ⟶ Y be a fuzzy δ-semi irresolute function and X be a fuzzy δ-semi compact space then f (X) is also a fuzzy δ-semi compact space.

Proof:

Let f: X ⟶ Y be a fuzzy δ-semi irresolute function and U = {A_j: j∈ Λ} be a collection of fuzzy δ-semi open sets cover of f (X) $\leq Y$. Then M = $\{f^{-1}(A_j): j\in \Lambda\}$ is a fuzzy δ-semi open cover of X. As X is a fuzzy δ-semi compact $\exists$ a finite subcover $M' = \{ f^{-1}(A_j) : j\in \Lambda'\}$, where Λ' is a finite subset of Λ of X.

Then $f(M') = \{f\ f^{-1}(A_j) : j\in\Lambda'\}$ is a finite sub cover of fuzzy δ-semi open sets of f (X). Hence f (X) is a fuzzy δ-semi compact space.