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$M^X/M/1$ QUEUE WITH WORKING VACATION

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In this paper a batch arrival $M^X/M/1$ queue with exponentially distributed multiple and single **working vacations** is analyzed.The queuing system is modeled as a two-dimensional Markov-chain to obtain the Chapman-Kolmogrov equations. The probability generating function (pgf) of the steady state system size probabilities is derived for the model and the expected system size probabilities are presented in closed form. Further various performance measures including the expected queue length and their graphical representations are presented. Special cases are also discussed.

INTRODUCTION

Over the last two decades, the queuing system with vacations have been widely studied because of their applications in modeling the computer networks, communications and manufacturing / service system. Various vacations policies provide more flexibility for optimal design. The explanations are seen in monographs of Takagi [10], Tian and Zhang [11], the survey of Doshi [3] and Teghem [12].

In various studies of vacation models, it is assumed that the server stops the main service completely during vacation. In 2002, Servi and Finn [9] analyzed an $M/M/1$ queue with working vacation, which was denoted by $M/M/1/WV$ where the same server works at a lower service rate rather than completely stopping the service during vacation period. Subsequently Kim, Choir and Chae [6], Wu and Takagi [13] generalized this model as an $M/G/1$ queue with grand working vacation. Baba [1] provided a study of GI/M/1 queue with multiple working vacation by using matrix geometric method. Banik *et. al.* [2] discussed about $GI/M/1/N$ queue with working vacations. Liu *et. al.* [7] proved the stochastic decomposition of the $M/M/1$ queue with working vacations and Li and Tian [8] considered two types of discrete-time GI/Geo/1 queues with working vacations. Various authors discussed about working under different conditions, but so far the batch arrival $M^X/M/1$ queue with multiple working vacation is not analyzed.

In this paper we consider a batch arrival queuing model $M^X/M/1$ with exponentially distributed working vacation. A set of Chapman –Kolmogrov equations satisfied by the system size probabilities is derived for the model. We have used the generating function approach to solve the equations. Various performance measures including the expected system length, are calculated. Particular cases and numerical examples are also presented.

Section 1 : $M^X/M/1$ multiple working vacation

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MODEL DESCRIPTION

In the previous papers [5, 9] all the authors have discussed working vacation concept for single arrival queuing system and obtained various steady state results using matrix geometric method. In this paper, we examine a batch arrival Markovian queueing system arising out of some non-birth and death processes. By relaxing simple assumption under lying $M/M/1$ queue, let us assume that the arrival **stream** forms a Poisson process, and the actual number of customers in a batch is denoted by the random variable x. The random variable x may take any positive integral values $n < \infty$, with probability g_n. This queuing model is Monrkovian in the sense that the future behavior depends only on the present but not on the past.

If λ_n denotes the arrival rate of the Poisson process of batch size $x = n$, then $g_n = \dfrac{\lambda_n}{\lambda}$, where λ is the composite arrival rate of all batches and is equal to $\sum_{i=1}^{\infty} \lambda_i$. Also, the total arrival process which arises from the overlap of the set of Poisson process with the rates $\{\lambda_i\}=1, 2, \ldots$ forms a multiple Compound Poisson process Gross and Haris [4].

There is only one server in the system and he works, also during vacation time (V) whose duration follows as exponential distribution with parameter η. During a working vacation period arriving customers are served at a rate $\mu_v < \mu_0$ where μ_0 denotes the regular arrival rate of the server during the busy period. When a vacation ends, if there are customers in the system, the server changes his service rate from μ_v to μ_0 and the regular busy period starts. Otherwise if there are no customers in the system the server takes another working vacation until a new batch arrives. Such a vacation policy is called repeated or **multiple working vacation** policy and the queuing model is denoted by $M^X/M/1/MWV$ **queue**. It is assumed that the inter arrival time, service time and working vacations times follow exponential distributions and are mutually independent. In addition the service discipline follows FIFO.

Let $N_s(t)$ denote the number of customers in the system at time t,

and $J(t) = 0$ when the system is in working vacation state at time t

1 when the system is in a regular busy state at time t.

Hence the queue is studied as a Markovian chain and the state space is denoted by

$(N_s(t), J(t)) = \{(n, j); n \geq 0; j = 0, 1\}$ where n denotes the number of customers present in the system and $j = 0$ or 1 according as the server is in working vacation or in regular busy period.

The state (0,0) denote the server is idle in vacation and the system is empty.

By defining $p_{nj}(t)$ as probability that the system is in state (n, j) at time t, for $n \geq 0; j = 0$, 1, and assuming that the steady state probabilities $P_{nj} = \lim_{t \to \infty} P_{nj}(t)$ exist for $n \geq 0$ and $j = 0, 1$, the steady state equations satisfied by P_{nj}'s are obtained as:

Steady state equations:

$$\lambda p_{00} = \mu_v p_{10} + \mu_0 p_{11} \qquad \ldots (1.1)$$

$$(\lambda + \eta + \mu_v) p_{n0} = \lambda \sum_{k=1}^{n} p_{n-k\,0}\, g_k + \mu_v p_{n+1\,0}, n \geq 1 \qquad \ldots (1.2)$$

$$(\lambda + \mu_0)\, p_{11} = \mu_0 p_{21} + \eta\, p_{10} \qquad \ldots (1.3)$$

$$(\lambda + \mu_0)\, p_{n1} = \lambda \sum_{k=1}^{n-1} p_{n-k\,1}\, g_k + \mu_0 p_{n+11} + \eta\, p_{n0},\ n \geq 2 \qquad \ldots (1.4)$$

We define the following partial probability generating function of the steady state system size probabilities to derive the total *pgf* of the model.

$$p_0(z) = \sum_{n=0}^{\infty} p_{n\,0}\, z^n$$

$$p_1(z) = \sum_{n=1}^{\infty} p_{n\,1}\, z^n$$

and the total *pgf* is given by $p(z) = p_0(z) + p_1(z)$.

Multiplying equation (1.2) by z^n, summing over n and then adding with (1.1) we get

$$(\lambda + \eta + \mu_v) \sum_{n=0}^{\infty} P_{n0}\, z^n - (\eta + \mu_v)\, p_{00}$$

$$= \lambda \sum_{n=1}^{\infty} z^n \sum_{k=1}^{n} z^n p_{n-k0}\, g_k + \frac{\mu_p}{z} \sum_{n=1}^{\infty} p_{n0} z^n + \mu_0\, p_{11}$$

$$\Rightarrow (\lambda + \eta + \mu_v)\, p_0(z) = (\eta + \mu_v)\, p_{00} + \frac{\mu_v}{z}(p_0(z) - p_{00}) + \mu_0\, p_{11} + \lambda p_0(z)\, x(z)$$

$$\left(\text{Since } \sum_{n=1}^{\infty} z^n \sum_{k=1}^{n} p_{n-k0}\, g_k = \left(\sum_{n=0}^{\infty} = p_{n0} z^n\right)\left(\sum_{k=1}^{\infty} g_k z^k\right) = p_0(z) = p_0(z)\, x(z)\right)$$

Hence we get,

$$(\lambda z\,(1 - x(z)) + \eta z + \mu_v\,(z - 1))\, p_0(z) = z\,(\mu_0\, p_{11} + \eta\, p_{00}) + \mu_v\,(z - 1)\, p_{00} \qquad \ldots (1.5)$$

Similarly multiplying equation (1.4) by z^n and (1.3) by z and summing over $n = 1$ to ∞ we get

$$(\lambda + \mu_0)\, p_1(z) = \frac{\mu_0}{z}[p_1(z) - p_{11} z] + \eta[p_0(z) = p_{00}] + \lambda \sum_{n=2}^{\infty} z^n \sum_{k=1}^{n-1} p_{n-k\,1} g_k \qquad \ldots (1.6)$$

since $$\sum_{n=2}^{\infty} z^n \sum_{k=1}^{n-1} p_{n-k\,1}\, g_k = \left(\sum_{n=1}^{\infty} p_{n\,1}\, z^n\right)\left(\sum_{n=1}^{\infty} g_k\, z^k\right) = p_1(z)\, x(z))$$

Equation (1.6) can be written as

$$(\lambda + \mu_0)\, p_1(z) = \frac{\mu_0}{z} p_1(z) - \mu_0 p_{11} + \eta(p_0(z) - p_{00}) + \lambda p_1(z)\, x(z)$$

$$\lambda z\,(1 - x(z)) + \mu_0 z)\; p_1(z) = \eta z(p_0(z)) - (\eta p_{00} + \mu_0 p_{11})\, z + \mu_0 p_1(z) \qquad \ldots (1.7)$$

Let $p = \eta\, p_{00} + \mu_0\, p_{11}$, then the equations (1.5) and (1.7) can be written respectively as

$$(\lambda\, z\,(1 - x(z)) + \eta\, z + \mu_v\,(z - 1))\, p_0(z) = pz + \mu_v\,(z - 1)\, p_{00} \qquad \ldots (1.8)$$

$$(\lambda z\,(1-x(z))+\mu_0(z-1))\;p_1(z)=\eta z\,(p_0(z))-p_z \qquad \dots (1.9)$$

By defining $f(z)=\lambda z\,(1-x(z))+\eta z+\mu_v\,(z-1)$ and $g(z)=\lambda\,z\,(1-x(z))+\mu_0\,(z-1)$, we find $f\,(0)=-\,\mu_v<0$ and $f(1)=\eta>0$. This implies that there exists a real root $z_1\in(0,\,1)$ for the equation $f(z)=\lambda z\,(1-x(z))+\eta z+\mu_v\,(z-1)=0$ ... (1.10)

Thus at $z=z_1$, equation (1.8) becomes

$$pz_1=\mu_v\,(1-z_1)\,p_{00}$$

i.e.,

$$p=\frac{\mu_v\,(1-z_1)\,p_{00}}{z_1} \qquad \dots (1.11)$$

Substituting for $p_0(z)$ and p from equations (1.8) and (1.11) in equation (1.9) $p_1(z)$ becomes

$$p_1(z)=\frac{p_{00}z\mu_v}{(\lambda z\,(1-x(z))+\mu_v(z-1)+\eta z)}\left[\frac{(z-1)}{z_1}(\eta+\mu_v)\,z_1-\mu_v)+\frac{\lambda z\,(x\,(z)-1)\,(1-z_1)}{z_2}\right]$$

... (1.12)

Since z_1 is a root of $f(z)=0$ from equation (1.10) we have

$$(\eta+\mu_v)\,z_1-\mu_v=\lambda z_1(x\,(z_1)-1)$$

using this in equation (1.12) we get

$$p_1(z)=\frac{p_{00}\lambda z\mu_{v[(z-1)\,z_2\,(X(z_2)-2)-2)-(z_1-1)\,z\,(X\,(z)-1)]}}{z_2(\lambda z\,(1-X(z))+\mu_0(z-1))\,(\lambda z\,(1-X(z))+\mu_v(z-1)+\eta z)} \qquad \dots (1.13)$$

where $z_1<1$.

Similarly by substituting for p in equation (1.8) we get $p_0\,(z)$ as

$$p_0(z)=\frac{\mu_v(z-z_1)\,p_{00}}{z_2(\lambda z\,(1-X(z))+\mu_v(z-1)+\eta z)} \qquad \dots (1.14)$$

Hence the total *pgf* is given by

$$p(z)=p_0(z)+p_1(z)$$

$$p(z)=\frac{p_{00}\,\mu_v(x-1)\,[\lambda_{zz_{\lambda}}(X(z_1)-z)+\mu_0(z-z_1)]}{z_2(\lambda z\,(1-X(z))+\mu_v(z-1)+\eta z)\,(\lambda z(1-X(z))+\mu_0(z-1))} \qquad \dots (1.15)$$

By using the normalizing condition $p(1)=1$ the value of p_{00} can be calculated as

$$p_{00}=\frac{\eta z_1\mu_0(1-\rho_0)}{\mu_v\,(\lambda x_1(X(x_1)-1)+\mu_0(1-x_1))} \qquad \left(\text{where } \rho_0=\frac{\lambda_E(X)}{\mu_0}\right)$$

By substituting for p_{00} in equation (1.14), $p(z)$ becomes

$$p(z)=\left[\frac{(1-\rho_0)\,(1-z)\,\mu_0}{\mu_0(1-z)-\lambda z\,(1-X(z))}\right]\left[\frac{\eta\,(\lambda_{zz_1}(X\,(z_2)-X(z))-\mu_0(z_1-z))}{(\lambda z\,(1-X(z))+\mu_0(z-1)+\eta z)\,(\lambda z_1(X(z_1)-1)-\mu_0\,(z_1-1))}\right]$$

Moreover p(z) can also be written as

$$p(z) = \left[\frac{(1-\rho_0)(1-z)\mu_0}{\mu_0(1-z) - \lambda z(1-X(z))}\right]\left[\frac{\eta(z-z_1)}{(1-z_1)(\lambda z(1-X(z)) + \mu_v(z-1) + \eta z)}\right] \left[\frac{\left(\mu_0 + \lambda_{zz_2}\left(\frac{X(z_1)-X(z)}{z-z_2}\right)\right)}{\left(\mu_0 + \lambda z_2\left(\frac{X(z_1)-1}{1-z_1}\right)\right)}\right]$$

further we notice that the first term $p(z)$ in the total *pgf* of the classical $M^X/M/1$ is

$$p_{M^X/M/1}(z) = \left[\frac{(1-p_0)(1-z)\mu_0}{\mu_0(1-z) - \lambda z(1-X(z))}\right] \text{ by [4]}$$

By writing $\frac{p_0(z)}{p_0(1)} = \frac{\eta(z-z_2)}{(1-z_2)(\lambda z(1-X(z)) + \mu_v(z-1) + \eta z)}$ (from (1.14))

and $$\frac{Q_0(z)}{Q_0(1)} = \left[\frac{\left(\mu_0 + \lambda z z_1\left(\frac{X(z_1)-X(z)}{z-z_1}\right)\right)}{\left(\mu_0 + \lambda z_1\left(\frac{X(z_1)-1}{1-z_1}\right)\right)}\right]$$, we have

$$p(z) = p_{M^X/M/1}(z)\,\frac{p_0(z)}{p_0(1)}\cdot\frac{Q_0(z)}{Q_0(1)} \qquad \dots (1.16)$$

Mean queue length:

If L denotes the mean system size of the model then by using equation (1.16) L can be written as

$$L = \frac{d}{dz}\left(P_{M^X/M/1}\right)_{z=1} + \frac{d}{dz}\left(\frac{p_0(z)}{p_0(1)}\right)_{z=1} + \frac{d}{dz}\left(\frac{Q_0(z)}{Q_0(1)}\right)_{z=1}$$

where $\frac{d}{dz}\left(P_{M^X/M/1}\right)_{z=1} = \frac{p_0}{1+p_0} = L_{M^X/M/1}$ where $\rho_0 = \frac{\lambda}{\mu E(X)}$. which is the expected system size of the model $M^X/M/1$ by [4]

Further $\frac{d}{dz}\left(\frac{p_0(z)}{p_0(1)}\right)_{z=1}$ is $\frac{(1-z_1)(\lambda-\mu_c-\eta)+\eta}{\eta(1-z_1)}$

and $$\frac{d}{dz}\left(\frac{Q_0(z)}{Q_0(1)}\right)_{z=1} = \frac{\lambda z_1}{\mu_0(1-z_1) + \lambda z_2(X(x_1)-1)}\left[\frac{z_1(1-X(z_1)) - (1-z_1)}{(1-z_1)}\right]$$

Hence we get the value of L as

$$L = \frac{\rho_0}{1-\rho_0} + \frac{(1-z_1)(\lambda-\mu_v-\eta)+\eta}{\eta(1-z_1)} + \frac{\lambda z_1}{\mu_0(1-z_1) + \lambda z_1(X(z_1)-1}\left[\frac{z_1(1-X(z_1)) - (1-z_1)}{(1-z_1)}\right] \qquad \dots (1.17)$$

Particular case:

Letting $x(z) = z$ and using the relationship $\lambda z_1 = \mu_v r_1$ between z_1 and $r_1 < 1$ the root of the equation $\mu_v z^2 - (\lambda + \mu_v + \mu) z + \lambda = 0$, we observe that expected system of the model $(L_{M^X/M/1/MWV})$ coincides with that of $L_{M/M/1/MWV}$

where
$$L_{M/M/1/MWV} = \frac{\rho_0}{1-\rho_0} + \frac{r_1}{1-r_1} - \frac{\mu_v r_1}{(\mu_0 - \mu_v r_1)} \text{ by [14].}$$

Other Performance Measure:

Let p_v and p_{busy} denote the probability that the server is on vacation and busy respectively then

$$p_v = \lim_{z \to 1} p_0(z) = \frac{\mu_v (1 - z_1)\, p_{00}}{\eta x_1}$$

$$p_{busy} = \lim_{z \to 1} p_1(z) = \frac{\lambda \mu_v [(z\,(X(z_1) - 1)) - (z_1 - 1)]\, p_{00}}{\mu_0 (1 - \rho_0)\, \eta z_1}$$ = probability that the server is busy with regular service rate.

Section 2: $M^X/M/1$ Single working vacation:

In this section we study the $M^X/M/1$ queue under **Single working vacation.** The server works in a vacation with the rate μ_v whenever the system becomes empty. After returning from a vacation if the server finds at least one customer in the system then he switches from the state μ_v to μ_0. On the other hand if the server returns from the vacation and finds the system is empty, then he joins the system and stays idle in the system until a batch of customers arrives.(*i.e.*) the server takes only a **single vacation after every busy period.**

The other assumption of the model of this section are same as in **section 1** and the inter arrival time, service time and working vacation are exponentially distributed random variables and are mutually independent of each other. The queue discipline is FIFO.

Let (n, j), $n \geq 0$ with $j = 0$ $(n \geq 1$ with $j = 1)$ denote the states that there are n customers in the system and the server is in working vacation (busy period) and $(0, 1)$ denote the state that the server is in idle-state and the system is empty. Thus the system is considered as a Markovian processes, on the state space $\{(n, 0); n \geq 0\} \cup \{(n, 1); n \geq 1\} \cup \{0, 1\}$.

By defining $p_{nj}(t)$ as the probability that the system is in state (n, j) and $P_{nj} = \lim_{t \to \infty} p_{nj}(t)$ be the corresponding steady state probabilities, the Kolmogrov equations satisfied by the steady state probabilities are given by

$$(\eta + \lambda)\; p_{00}^s = \mu_v p_{10}^s + \mu_0 p_{11}^s \qquad \dots (2.1)$$

$$(\lambda + \eta + \mu_v)\; p_{n_0}^s = \lambda \sum_{k=1}^{n} p_{n-k0}^s\, g_k + \mu_v p_{n+10}^s,\; n \geq 1 \qquad \dots (2.2)$$

13.

$$(\lambda + \mu_0)\; p_{11}^s = \mu_0 p_{21}^s + \eta p_{10}^s + \lambda p_{01}^s\, g_1 \qquad \dots (2.3)$$

$$(\lambda + \mu_0)\; p_{n1}^s = \lambda \sum_{k=1}^{n-1} p_{n-k1}^s\, g_k + \mu_0\, p_{n+11}^s + \eta\, p_{n0}^s + \lambda\, p_{01}^s\, g_n,\; n \geq 2 \qquad \dots (2.4)$$

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$$\lambda p_{01}^s = \eta p_{00}^s \qquad \text{... (2.5)}$$

The partial *pgf* of the model are given by

$$p_0^s(z) = \sum_{n=0}^{\infty} p_{n0}^z \, z^n \quad \text{and} \quad p_1^s(z) = \sum_{n=1}^{\infty} p_{n1}^z z^n$$

and $p(z) = p_0^s(s) + p_1^s(z) + p_{01}^s$ gives the total *pgf* of the model.

Equations (2.1) to (2.4) are similar to that of $M^X/M/1$ multiple working vacation model of ***section 1***. Hence proceeding as in section 1, we find that

$$(\lambda z\,(1 - x(z)) + \eta z + \mu_v\,(z-1))\; p_0^s(z) = (\mu_0 p_{11}^s)\, z + \mu_v (z-1)\; p_{00}^s \qquad \text{... (2.6)}$$

$$(\lambda z\,(1 - x(z)) + \mu_0\,(z-1))\; p_0^s(z) = \eta z\,(p_0^s(z) - (\eta p_{00}^s + \mu_0 p_{11}^s)\, z + \lambda z\; p_{01}^z\; x(z) \qquad \text{... (2.7)}$$

Since $0 < z_1 < 1$ be a root of the equation $(\lambda z\,(1 - x(z)) + \eta\, z + \mu_v\,(z - 1)) = 0$ by (1.10)

We have $(\mu_0\;\; p_{11}^s)\, z_1 + \mu_v\,(z_1 - 1)\;\; p_{00}^s = 0$

$$\Rightarrow \qquad p_{11}^s = \frac{\mu_v p_{00}^s (1 - z_1)}{\mu_0 z_1} \qquad \text{... (2.8)}$$

By substituting p_{11} in equation (2.6) we get $p_0(z)$ as

$$p_0^s(z) = \frac{\mu_v (z - z_1)\; p_{00}^s}{z_1 (\lambda z\,(1 - X(z)) + \mu_v (z-1) + \eta z)} \qquad \text{...(2.9)}$$

Further by using equations (2.8) and (2.9) in equation (2.7) we get

$$p_1^s(z) = \left[\frac{\mu_v \lambda z}{z_1 (\lambda z\; 1 - x(z)) + \eta z + \mu_v (z-1))}\left(\frac{(z-1)\; z_1 (X(z_1) - 1) - (z_1 - 1)\; z\,(X(z) - 1)}{(\lambda z\,(1 - X(z)) + \mu_0 (z-1))}\right)\right.$$

$$\left. - \frac{\eta z\,(1 - X(z))}{(\lambda z\,(1 - X(z)) + \mu_0 (z-1))}\right] p_{00}^s \qquad \text{... (2.10)}$$

Then the total *pgf* $p(z)$ can be obtained as

$$p(z) = p_0^s(z) + p_1^s(z) + p_{01}^s$$

by substituting $p_0^s(z)$, $p_1^s(z)$ and p_{01}^s from equations (2.9),(2.10) and (2.5) and after simplification we find that

$$p(z) = \left\{\frac{\mu_v (z-1)}{z_1 (\lambda z\,(1 - X(z)) + \mu_v (z-1) + \lambda z)}\left[\frac{\lambda z z_1\,(X(z_1) - X(z)) + \mu_0 (z - z_1)}{(\lambda z\,(1 - x(z)) + \mu_0 (z-1))}\right]\right.$$

$$\left. + \frac{\eta}{\lambda}\left[\frac{\mu_0 (z-1)}{(\lambda z\,(1 - X(z)) + \mu_0 (z-1))}\right]\right\} p_{00}^s \qquad \text{... (2.11)}$$

The value of p_{00}^s can be calculated by using the normalizing condition $p(1) = 1$.

i.e.,

$$p_{00}^{s}=\frac{(1-\rho_0)}{\frac{\eta}{\lambda}+\frac{\mu_v(\lambda z_1\,(X(z_1)-1)+\mu_0(1-z_1)}{(\eta z_1\mu_0)}}$$

Thus

$$p(z)=\left[\frac{(1-\rho_0)\,(1-z)\,\mu_0}{\mu_0(1-z)-\lambda z(1-X(z))}\right]\left\{\frac{\frac{\eta}{\lambda}+\frac{\mu_v}{\mu_0}\left[\frac{\lambda z z_1(X(z_1)-X(z))+\mu_0(z-z_1)}{z_1((\lambda z(1-X(z))+\mu_v(z-1)+\eta z))}\right]}{\frac{\eta}{\lambda}+\frac{\mu_v(\lambda z_1(X(z_1)-1)+\mu_0(1-z_1))}{(\eta z_1\mu_0)}}\right\} \qquad \ldots (2.12)$$

$$\text{i.e., } p(z)=p_{M^X/M/1}(z)\left\{\frac{\frac{\eta}{\lambda}+\frac{\mu_v}{\mu_0}\left[\frac{\lambda z z_1(X(z_1)-X(z))+\mu_0(z-z_1)}{z_1((\lambda z(1-X(z))+\mu_v(z-1)+\eta z))}\right]}{\frac{\eta}{\lambda}+\frac{\mu_v(\lambda z_1(X(z_1)-1)+\mu_0(1-z_1))}{(\eta z_1\mu_0)}}\right\} \qquad \ldots (2.13)$$

Mean queue length:

If L denotes the mean system size of the model then by using equation (2.13) L can be written as

$$L=\frac{dDZ}{}\left(p_{M^X/M/1}\right)_{z=1}+\frac{d}{dz}\left(\frac{\frac{\eta}{\lambda}+\frac{\mu_v}{\mu_0}\left[\frac{\lambda z z_1(X(z_1)-X(z))+\mu_0(z-z_1)}{z_1((\lambda z(1-X(z))+\mu_v(z-1)+\eta z))}\right]}{\frac{\eta}{\lambda}+\frac{\mu_v(\lambda z_1(X(z_1)-1)+\mu_0(1-z_1))}{(\eta z_1\mu_0)}}\right)_{z=1}$$

where

$$\frac{d}{dz}\left(\frac{\frac{\eta}{\lambda}+\frac{\mu_v}{\mu_0}\left[\frac{\lambda z z_1(X(z_1)-X(z))+\mu_0(z-z_1)}{z_1((\lambda z(1-X(z))+\mu_v(z-1)+\eta z))}\right]}{\frac{\eta}{\lambda}+\frac{\mu_v(\lambda z_1(X(z_1)-1)+\mu_0(1-z_1))}{(\eta z_1\mu_0)}}\right)_{z=1}$$

$$=\frac{1}{\left(\frac{\eta}{\lambda}\mu_0\,(1-\rho_v)+\mu_0+\lambda z_1\left(\frac{(X(z)_1)-1}{1-z_1}\right)\right)}$$

$$\left\{-\frac{\eta z_1\mu_0}{\mu_v}+\frac{\lambda z_1}{(1-z_1)}\left[\frac{z_1(1-X(z_1))-(1-z_1)}{(1-z_1)}\right]\right\}$$

Hence we get the value of L^s as

$$L^s:=\frac{\rho_0}{1-\rho_0}+\frac{(1-z_1)\,(\lambda-\mu_v-\eta)+\eta}{\eta(1-z_1)}+\frac{1}{\left(\frac{\eta}{\lambda}\mu_0(1-\rho_v)+\mu_0+\lambda z_1\left(\frac{(X(z_1))-1}{1-z_1}\right)\right)}$$

$$\left\{-\frac{\eta z_1\mu_0}{\mu_v}+\frac{\lambda z_1}{(1-z_1)}\left[\frac{z_1(1-X(z_1))-(1-z_1)}{(1-z_1)}\right]\right\}$$

REMARK

It is verified that when $x(z) = z$, the results coincides with the corresponding result of *M/M/1/SWV* by [5].

Other performance measure:

Let p_v, p_{busy} and p_I denote the probability that the server is on vacation, busy and idle respectively then

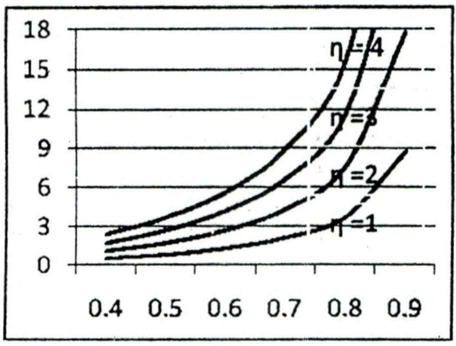
$$p_v = \lim_{z\to 1} p_0(z) = \frac{\mu_v(1-z_1)\, p_{00}}{\eta z_1}$$

$$p_{\text{busy}} = \lim_{z\to 1} p_1(z) = \left\{ \frac{\lambda\mu_v(z_1(X(z_1)-1))}{\mu_0(1-\rho_0)\,\eta z_1} + \frac{\eta}{\mu_0(1-\rho_0)} \right\} . p_{00}^{s}$$

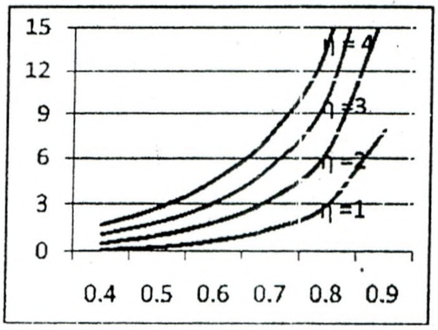
$$p_I = \lim_{z\to 1} p_{01}(z) = \frac{\eta}{\lambda}\, p_{00}^{s}$$

NUMERICAL EXAMPLES

We present numerical examples to explain the influence of different parameters on mean system length. Graph (1) and graph (2) show the fact that as η increases mean system length decreases.

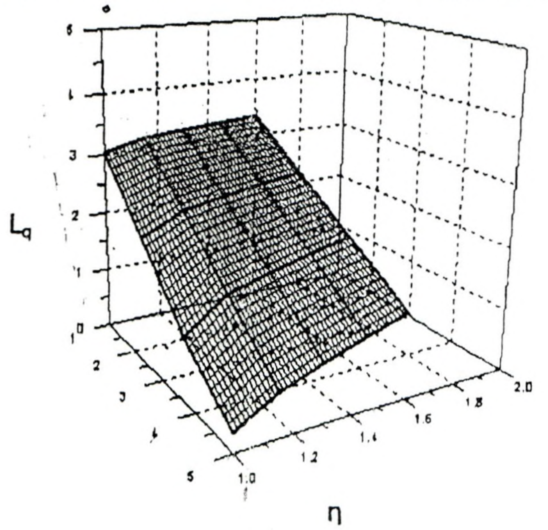


Graph (1) $M^X/M/1/mwv$

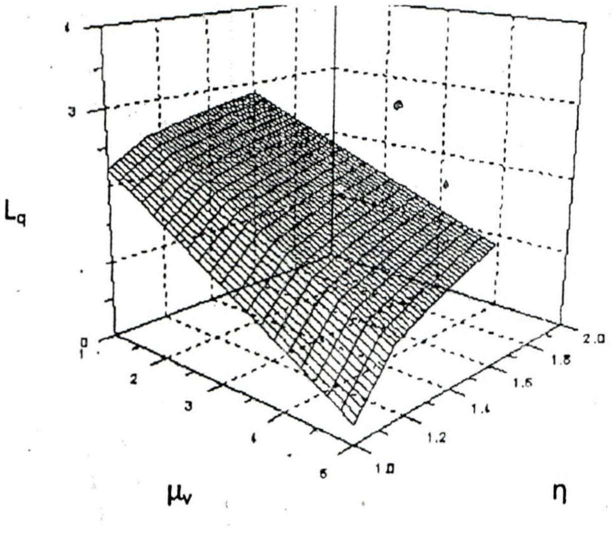


Graph (2) $M^X/M/1/swv$

Graph (3) and graph (4) indicate the effect of μ_v ,η and μ_0 on mean system length.



Graph (3) $M^X/M/1/mwv$



Graph (4) $M^x/M/1/swv$

CONCLUSION

Working vacation is introduced by Servi and Finn (9) is a new concept and the models discussed till now under the working vacation are $M/G/1$, $M/M/1$ and $G/M/1$ using matrix geometric method. In this paper we have made an attempt to discuss the $M^X/M/1$ queue under working vacation (both multiple and single) and derived the steady state results. We have obtained the results by writing the Kolmogrov equations and derived the pgf in a closed form so that various performance measures can be deduced from it. The research can be extended to more general models such as $M/M\ (a,\ b)/1$, and $GI/M\ (a,\ b)/1$ queuing models in future.

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GI/M/1 Queue with multiple working vacation

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Abstract

In this paper we analyze a GI/M/1 queue with working vacation, in which the server works at different rate rather than completely stopping the service during vacation periods. Baba investigated a GI/M/1 queue with multiple working vacations .They have formulated the queueing system as an embedded two dimensional Markov chain by choosing the arrival epoch as embedded points. Using the algorithmic approach Neutus and others derived the steady state distributions for the number of customers in the system both at arrival and arbitrary epochs and for the sojourn time for an arbitrary customer. Thus the purpose of this paper is to analyze GI/M/1 queueing model under single server **working vacation** and derived the equilibrium distribution of system size at pre arrival epoch by solving the difference equations using operator technique. Further the expected queue length and various performance measures are obtained in a closed form. Finally special cases are also deduced.

Key words : GI/M/1 Queue, Working vacation.

Mathematics Subject Classification : **90B22**.

1. Introduction

Over the past two decades, queueing systems with vacations have been studied by many researchers and have been applied to many situations , namely in computer systems, communication networks , production managing and so forth [2]. Servi and Finn [5] first studied queueing system with working vacations , where the server works at a lower rat rather than completely stopping service during vacation. That is during a working vacation, customers will undergo service at a lower rate and depart system , whereas, customers in the classical vacation will impossibly depart the system. Therefore the working vacation models have more complicated modalities and the analysis of this kind of models is more difficult than classical vacation queue.

Servi and Finn [5] studied an M/M/1 queue with multiple working vacations and obtained the p.g.f of the number of customers in the system and mean waiting time of customers, and applied the results to perform analysis of gateway router in fiber communication networks. Liu et al [4] discussed stochastic decomposition structures of stationary indices, derived the distribution of additional queue length and additional delay and obtained expected regular busy period and expected busy cycle. Kim, Choi and Chae [3], and Takagi [7] generalized the work of Servi and Finn's model to an M/G/1 queue with multiple working vacations . Baba [1] investigated a GI / M/1 queue with multiple working vacations and derived the steady state distributions for the number of customers in the system both at arrival and arbitrary epochs and for the sojourn time for an arbitrary customer.

Tian et al [6] analyzed an M/M/1 queue with single working vacation using quasi birth and death process and matrix geometric solution method, they derived the distributions for the number of customers and the virtual time in the system in steady state. Furthermore, they obtained the expected busy period, expected busy cycle, and got the stochastic decomposition structures of stationary indices.

In this paper, we have derived the steady state probabilities using embedded Markov chain technique and derived the expected queue length in a closed form. Further various performance measures are also deduced.

Model Formulation:

We consider GI/ M/1 queue in which the server begins an exponentially distributed working vacation, whenever the system becomes empty. During the working vacation the arriving customers are served at a mean rate μ_v. When the vacation ends, if there are customers in the queue, then server changes his service rate μ_v to μ_0 and regular busy period starts. Otherwise the server begins another working vacation. That is the server follows multiple (or) repeated vacation policy. Thus the service time during vacation and regular busy period and vacation follow exponential distributions with parameters μ_v, μ_0 and η respectively and they are independent of each other.

Embedded Markov chain of queue length:

Let τ_n denote the arrival epoch of the nth customer with $\tau_0 = 0$. The inter arrival times $\{ \tau_n , n \geq 1 \}$ are independent and identically distributed with a general distribution function denoted by A(t) with a mean $1/\lambda$ and a Laplace Stieltjes transform (LST) denoted by $A^*(\theta)$.

By considering the above assumptions and by utilizing the Embedded Markov Chain technique, the steady state probabilities , expected queue length and various performance measures are derived.

Queue length at pre arrival epochs:

τ_n , n = 1,2,3,....... ($\tau_0 = 0$) are the arrival epochs and we examine the system at the pre arrival epochs $\tau_n - 0$.

Let Q (τ) denote the number of customers in the system at time τ and $Q_n = Q(\tau_n - 0)$

and $\tau_n = \begin{cases} 0 & \text{if the nth arrival occurs during working vacation} \\ 1 & \text{if the nth arrival occurs during service period} \end{cases}$

Since the working vacation times, the service time during a regular busy period and working vacation are all exponentially distributed the process $\{ (Q_n, \tau_n), n \geq 1 \}$ defines a semi Markov chain with state space $\{ (n,0), n = 0,1,2,3,\ldots\} \cup \{ (n,1), n = 1,2,3,\ldots\ldots\}$

(i) Thus the state (n,0) ,($n \geq 1$) represents that there are n customers in the system and the server is on working vacation, whose service rate is μ_v.

(ii) The state (n,1) ,($n \geq 1$) denotes that the server is serving at regular rate μ_0 with n customers in the system.

(iii) (0,0) implies that the system is empty and the server is idle during vacation.

In order to obtain the steady state equations we first define the following probabilities :

1. Let $b_k = \int_0^\infty e^{-\mu_b t} \frac{(\mu_0 t)^k}{k!} dA(t)$ $k \geq 0$. Then b_k gives the probability that k customers are served at regular rate μ_0 in an inter arrival time .

2. Similarly $c_k = \int_0^\infty e^{-\eta t} \frac{(\mu_v t)^k}{k!} e^{-\mu_v t} dA(t)$ $k \geq 0$ implies the probability that the working vacation time is greater than the inter- arrival time and k service completions occur at rate μ_v during an inter arrival time . and

3. $d_k = \{\int_0^\infty \sum_{j=0}^{k} \int_0^t (\eta e^{-\eta x} \frac{(\mu_v x)^j}{j!} e^{-\mu_v x} \frac{(\mu_0 (t-x))^{k-j}}{(k-j)!} e^{-\mu_0 (t-x)})\, dx \;\; dA(t) \}$ gives the probability that the server who is in working vacation returns from vacation after a time x ($0 \leq x \leq t$) and k customers are served in an inter arrival time t, in such a way that j customers are served in time x at a rate μ_v and the remaining (k – j) customers are served in time (t-x) at rate μ_0.

Steady state equations:

By following the law of transition, the Markov chain $\{(Q_n, \tau_n) ; n \geq 1\}$ leads to the steady state equations satisfied by the limiting probabilities

$p_{nj} = \lim_{k\to\infty} pr(n_k = j_k, j_k = j)$, $n \geq 0$; $j = 0,1$

Thus by assuming the steady state exists the Chapman Kolmogrov equations satisfied by p_{nj} 's in the steady state are given by

$$p_{00} = \sum_{i=0}^{\infty} p_{i0}\left(1 - \sum_{k=0}^{i} c_k + d_k\right) + \sum_{i=1}^{\infty} p_{i1}\left(1 - \sum_{k=0}^{i} b_k\right) \tag{1}$$

$$p_{n0} = \sum_{j=0}^{\infty} p_{j+n-1}\, c_j \qquad n \geq 1 \tag{2}$$

$$p_{11} = \sum_{i=1}^{\infty} p_{i1}(b_i) + \sum_{i=0}^{\infty} p_{i0}\,(d_i) \tag{3}$$

$$p_{n1} = \sum_{j=0}^{\infty} p_{j+n-1\,1}\, b_j + \sum_{j=0}^{\infty} p_{j+n-1\,0}\, d_j \qquad n \geq 2 \tag{4}$$

Steady state solution:

To solve the steady state equations, we define the forward shifting operator E on p_{nj} by $E(p_{n0}) = p_{n+1\,0}$. Thus equation (2) can be written as

$$\left(E - \sum_{j=0}^{\infty} c_j E^j\right) p_{n0} = 0 \text{ for } n \geq 0 \tag{5}$$

The characteristic equation of the homogeneous difference equation (5) is

$$\phi(z) = z - \sum_{j=0}^{\infty} c_j z^j = 0$$

Let $C(z) = \sum_{j=0}^{\infty} c_j z^j$ be the p.g.f of the probabilities c_j's then $c(z) = A^{\bullet}(\eta + \mu_v(1-z))$.

Since C(z) is monotonically increasing and strictly convex , there exists a unique root $r_1 \in (0,1)$ of $\phi(z) = 0$ provided $\varphi'(1) > 0$ (Gross and Haris [2])

Since $\phi'(1) = 1 + \mu_v A^{\bullet}(\eta) > 0$,

the solution of the homogeneous difference equation is given by

$$p_{n0} = r_1^{\,n}\, p_{n0} \quad n \geq 0 \tag{6}$$

with $C(r_1) = r_1$ and $r_1 \in (0, 1)$

Similarly by defining the forward displacement operator E on p_{n1} ,

the equation (4) can be written as ,

$$\left(E - \sum_{j=0}^{\infty} b_j E^j\right) p_{n1} = \sum_{j=0}^{\infty} p_{n+j\,0}\, d_j \qquad n \geq 1 \tag{7}$$

$$= \sum_{j=0}^{\infty} d_j\, E^j (p_{n0})$$

Hence equation (7) is a non – homogeneous difference equation whose characteristic equation is given by $z = \sum_{j=0}^{\infty} b_j z^j = B(z) = A^{\bullet}(\mu_0(1-z))$.

Following the arguments mentioned earlier if $B'(1) = \frac{\mu_0}{\lambda} > 1$ then there exists an unique root $r_0 \in (0,1)$ for the characteristic equation $\varphi(z) = z - B(z) = 0$

$$i.e\ B(r_0) = r_0 \text{ and } 0 < r_0 < 1 \tag{8}$$

Thus the solution of the non- homogeneous difference equation (7) becomes

$$p_{n1} = A_1 r_0^{\,n} + \sum_{j=0}^{\infty} \frac{d_j r_1^{\,n+j} p_{00}}{r_1 - \sum_{j=0}^{\infty} b_j r^j} \text{ provided } r_1 \neq r_0 \text{ and } \rho_0 = \frac{\lambda}{\mu_0} < 1$$

Thus p_{nl} can be written as $p_{nl} = (A r_0^n + k(r_1)\, r_1^n)\, p_{00} \quad n \geq 1$

Where $A_1 = A\, p_{00}$ and $k(r_1) := \sum_{j=0}^{\infty} \frac{d_j r_1^{n+j} p_{00}}{r_1 - \sum_{j=0}^{\infty} b_j r^j}$ which can also be written as

$$k(r_1) = \frac{D(r_1)}{r_1 - B(r_1)} \qquad (9)$$

where $D(r_1) = \sum_{j=0}^{\infty} d_j r_1^j$

Hence the probabilities p_{n0}'s and p_{n1}'s are given by

$$\left.\begin{aligned} p_{n0} &= r_1^{\,n}\ p_{n0} \quad n \geq 0 \quad \text{and} \\ p_{nl} &= (A r_0^n + k(r_1)\, r_1^n)\, p_{00} \quad n \geq 1 \end{aligned}\right\} \qquad (10)$$

Thus the probabilities are expressed in terms of p_{00} and A . The constant A can be evaluated by using (1) or (3).

Substituting for p_{i0} ' s and p_{i1} 's from equation (10) in equation (3) we get

$$(A r_0 + k(r_1)\, r_1) = \sum_{i=1}^{\infty} (A r_0^i + k(r_1)\, r_1^i)\, b_i + \sum_{i=0}^{\infty} r_1^i d_i$$

$$A\,(r_0 - B(r_0)) = k(r_1)\,(B(r_1) - r_1) + D(r_1) - b_0\,(A + k(r_1)) \qquad (11)$$

Equation (8) and (9) imply that

$$B(r_0) = r_0 \quad \text{and} \quad k(r_1) = \frac{D(r_1)}{r_1 - B(r_1)}$$

The equation (11) can be simplified as

$$k(r_1)\,(B(r_1) - r_1) + D(r_1) - b_0\,(A + k(r_1)) = 0$$

$$\frac{D(r_1)}{r_1 - B(r_1)}\,(B(r_1) - r_1) + D(r_1) - b_0\,(A + k(r_1)) = 0$$

Thus $b_0\,(A + k(r_1)) = 0$, since $b_0 \neq 0$

we get $(A + k(r_1)) = 0 \Rightarrow A = -k(r_1)$

It is verified that equation (1) is also satisfied by noting that

$$\sum_{i=0}^{\infty} p_{i0} \sum_{k=0}^{i} c_k = \sum_{i=0}^{\infty} c_i \sum_{k=i}^{\infty} p_{k0}$$

$$\sum_{i=1}^{\infty} p_{i1} \sum_{k=1}^{i} b_k = \sum_{i=1}^{\infty} b_i \sum_{k=i}^{\infty} p_{k1}$$

Thus $p_{nl} = k(r_1)\,(r_1^n - r_0^n)\, p_{00} \quad n \geq 1 \qquad (12)$

Further the value of p_{00} can be calculated by using the normalizing condition

$$\sum_{n=0}^{\infty} p_{n\,0} + \sum_{n=1}^{\infty} p_{n\,1} = 1$$

Substituting for $p_{n\,0}$ *and* $p_{n\,1}$ from (10) we find that

$$\frac{1}{(1-r_1)} + k(r_1)\left(\frac{r_1}{1-r_1} - \frac{r_0}{1-r_0}\right) = p_{0\,0}^{-1}$$

Where $p_{00} = \dfrac{1-r_1}{1+k(r_1)\left(\frac{r_1-r_0}{1-r_0}\right)}$ (13)

Hence , $p_{n0} = r_1^{\,n}\ p_{n0} \quad n \geq 0$ and

$P_{n1} = k(r_1)\,(r_1^n - r_0^n)\,p_{00} \quad n \geq 1$ where p_{00} is given by equation (13) .

By using equation (14) the total pgf is given by

$$p(z) = \sum_{n=0}^{\infty} p_{n\,0}\, z^n + \sum_{n=1}^{\infty} p_{n\,1}\, z^n$$

$$p(z) = \sum_{n=0}^{\infty} r_1^n p_{0\,0}\, z^n + \sum_{n=1}^{\infty} (A r_0^n + k(r_1)\, r_1^n)\, p_{00}\, z^n$$

$$= p_{00}\,[(1-r_1 z)^{-1} + \sum_{n=1}^{\infty} -k(r_1) r_0^n z^n + k(r_1)\, r_1^n z^n]$$

$$= p_{00}\left[(1-r_1 z)^{-1} - k(r_1)\left\{\frac{r_0 z}{1-r_0 z} - \frac{r_1 z}{1-r_1 z}\right\}\right]$$

Mean queue length:

In this section we calculate the mean queue size of the model.

If L_q denotes the mean queue size for the model then it can be written as

$$L_q = \sum_{n=1}^{\infty} n\, p_{n1} + \sum_{n=0}^{\infty} n\, p_{n\,0}$$

By substituting the value of p_{n1} and p_{n0} from equation (10),

L_q is simplified as ,

$$L_q = \left[\frac{r_1}{(1-r_1)^2}\left(1 + \frac{D(r_1)}{r_1 - B(r_1)} - \frac{D(r_1)\, r_0}{(r_1 - B(r_1))(1-r_0)^2}\right)\right] p_{0\,0}$$

where $p_{00} = \dfrac{1-r_1}{1+k(r_1)\left(\frac{r_1-r_0}{1-r_0}\right)}$.

Other performance measures:

If p_v and p_b denote the probability that the server is on vacation and busy respectively then

$$p_v = \sum_{n=0}^{\infty} p_{n\,0} = \frac{p_{0\,0}}{(1-r_1)} \;;$$

$$p_b = \sum_{n=1}^{\infty} p_{n1} = \sum_{n=1}^{\infty} k(r_1)(r_1^n - r_0^n)p_{00}$$

by substituting for $k(r_1)$ and p_{00} we get

$$p_b = \frac{D(r_1)}{r_1 - B(r_1)} \left(\frac{r_1 - r_0}{(1-r_1)(1-r_0)}\right)$$

Particular cases:

If the arrival follows Poisson distribution the results of GI/M/1/MWV model Coincide with the corresponding results of M/M/1/MWV model of Servi and Finn [5].

Let $a(t) = \frac{dA(t)}{dt} = \lambda e^{-\lambda t}$

by applying the definition of Laplace transform for b_k we get $b_k = \frac{\lambda\mu_0^k}{(\lambda+\mu_0)^k}$

since $B(z) = \sum_{j=0}^{\infty} b_j z^j$ (from equation (7))

which in turn gives $B(z) = \frac{\lambda}{\lambda+\mu_v(1-z)}$

But from equation (8) we have $B(r_0) = r_0$

Hence we get $\mu_0 r_0^2 - (\lambda + \mu_0)r_0 + \lambda = 0$ as in M/M/1 where $r_0 = \rho_0$

Similarly by applying the definition of Laplace transform to c_k we get

$c_k = \frac{\lambda\mu_v^k}{(\lambda+\mu_v)^{k+1}}$, since $C(z) = \sum_{j=0}^{\infty} c_j z^j$ we get $C(z) = \frac{\lambda}{\lambda+\eta+\mu_v(1-z)}$

As $C(r_1) = r_1$, we obtain $\mu_v r_1^2 - (\lambda + \mu_v + \eta)r_1 + \lambda = 0$.

Again proceeding in the same way we get

$$d_k = \frac{\lambda\eta\mu_0^k}{(\lambda+\mu_v+\eta)(\lambda+\mu_0)} \sum_{j=0}^{k} \left(\frac{\mu_v(\lambda+\mu_0)}{\mu_0(\lambda+\eta+\mu_0)}\right)^k$$

Then $D(z) = \frac{\lambda\eta}{\lambda+\mu_0(1-z)\;\;\lambda+\eta+\mu_v(1-z)}$.

by substituting for B(z) and D(z) in equation (9) we get

$$k(r_1) = \frac{D(r_1)}{r_1 - B(r_1)} = \frac{-\eta r_1}{\mu_0(1-r_1)(\rho_0 - r_1)}$$ as in M/M/1/MWV

Thus we find that, $p_{n0} = r_1^n\ p_{n0}\quad n \geq 0$ and

$p_{n1} = k(r_1)(r_1^n - r_0^n)p_{00}\quad n \geq 1$ of GI/M/1/MWV

coincides with that of M/M/1/MWV of Servi and Finn [5].

Conclusion

In this paper we have developed the analytical steady state results for the GI/M/1 under multiple vacations in a closed form by solving its difference equations. Moreover we find that the results of M/M/1/MWV model of Servi and Finn [5] is deduced as a particular case.

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