Chapter 5

λ_g^α-Closed Maps and λ_g^α-Open Maps in Topological Spaces

5.1 Introduction

Over the course of years, several researchers, on introduction of closed and open sets gave various definitions of closed maps and open maps and studied their properties. These are significantly used in mathematics and related sciences. Initiation of Generalization to closed maps were conferred by Malghan [1982]. Following this many variations like α-open maps, rg-closed maps, wg-closed maps, w-closed maps, ω-closed maps, λ-closed maps, $rg\alpha$-closed maps were provided by Mashhour et al. [1983], Arockiarani [1997], Nagaveni [1999], Sheik John [2002], Pushpalatha [2000], Caldas et. al. [2008 a], Vadivel and Vairamanickam [2010] respectively. Their corresponding properties, theorems and results have been derived.

In a parallel consideration, we have defined λ_g^α-closed maps and λ_g^α-open maps in topological spaces. The association of the newly defined maps with the previously existing maps are also scrutinised. Basic theorems and properties related to the study have been derived sequentially. Later the various forms of λ_g^α-closed maps have been defined and analyzed namely quasi λ_g^α-closed maps, strongly λ_g^α-closed maps, contra λ_g^α-closed maps and strongly λ_g^α-closed maps. Fundamental properties and characterizations have also been examined.

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5.2 λ_g^α-Closed Maps

In this section, we have defined λ_g^α-closed maps in topological spaces. Their most vital properties, interdependencies and characterizations have been arranged accordingly.

Definition 5.2.1 A map $u:(M,\mu) \to (N,\nu)$ is called a **λ_g^α-closed map** if the image of each closed set in (M,μ) is λ_g^α-closed in (N,ν), i.e., if $u(T)$ is λ_g^α-closed in (N,ν) for every closed set T in (M,μ).

Example 5.2.2 Let $M = N = \{i,j,k\}$, $\mu = \{\phi,\{i\},M\}$ and $\nu = \{\phi,\{i,j\},N\}$. Let $u:(M,\mu) \to (N,\nu)$ be a map defined by $u(i) = j, u(j) = k$ and $u(k) = i$. Then u is a λ_g^α-closed map as the image of every closed set in (M,μ) is λ_g^α-closed in (N,ν).

Proposition 5.2.3 Every closed map $u:(M,\mu) \to (N,\nu)$ is a λ_g^α-closed map.

Proof: Let T be a closed set in (M,μ). Then $u(T)$ is a closed set in (N,ν). Using Proposition 2.2.5, $u(T)$ is a λ_g^α-closed set in (N,ν). Thus u is a λ_g^α-closed map.

The subsequent example shows that the converse of Proposition 5.2.3 may not hold good.

Example 5.2.4 Let $M = N = \{i,j,k,l,m\}$, $\mu = \{\phi,\{i\},\{j\},\{i,j\},\{j,k\},\{i,j,k\},\{j,k,l\},$ $\{i,j,k,l\},\{j,k,l,m\},M\}$ and $\nu = \{\phi,\{i\},\{j,k,l\},\{i,j,k,l\},N\}$. Let $u:(M,\mu) \to (N,\nu)$ be a map defined by $u(i) = k, u(j) = i, u(k) = j, u(l) = l$ and $u(m) = m$. Then u is a λ_g^α-closed map but not a closed map, since for the closed set $\{i\}$ in (M,μ), $u(\{i\}) = \{k\}$ is not a closed set in (N,ν).

Proposition 5.2.5 Every λ-closed map $u:(M,\mu) \to (N,\nu)$ is a λ_g^α-closed map.

Proof: Let T be a closed set in (M,μ). By Lemma 1.1.8, T is a λ-closed set in (M,μ). Since u is a λ-closed map, $u(T)$ is a λ-closed set in (N,ν). Using Proposition 2.2.3, $u(T)$ is a λ_g^α-closed set in (N,ν). Hence u is a λ_g^α-closed map.

The subsequent example shows that the converse of Proposition 5.2.5 may not hold good.

Example 5.2.6 Consider M, N, μ and ν as in Example 5.2.4. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = k, u(k) = j, u(l) = m$ and $u(m) = l$. Then u is a λ_g^α-closed map but not a λ-closed map, since for the λ-closed set $\{i, j\}$ in (M, μ), $u(\{i, j\}) = \{i, k\}$ is not a λ-closed set in (N, ν).

Definition 5.2.7 A map $u: (M, \mu) \to (N, \nu)$ is called a ***g*Λ-closed map** if the image of each closed set in (M, μ) is $g\Lambda$-closed in (N, ν), i.e., if $u(T)$ is $g\Lambda$-closed in (N, ν) for every closed set T in (M, μ).

Proposition 5.2.8 Every λ_g^α-closed map $u: (M, \mu) \to (N, \nu)$ is a $g\Lambda$-closed map.

Proof: Let T be a closed set in (M, μ). Then $u(T)$ is a λ_g^α-closed set in (N, ν). By Proposition 2.2.11, $u(T)$ is $g\Lambda$-closed in (N, ν). Hence u is a $g\Lambda$-closed map.

The subsequent example shows that the converse of Proposition 5.2.8 may not hold good.

Example 5.2.9 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{k\}, \{i, j\}, \{i, k\}, \{i, j, k\}, \{i, k, l\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = l, u(j) = k, u(k) = j$ and $u(l) = i$. Then u is a $g\Lambda$-closed map but not a λ_g^α-closed map, since for the closed set $\{j, l\}$ in (M, μ), $u(\{j, l\}) = \{i, k\}$ is not a λ_g^α-closed set in (N, ν).

Remark 5.2.10 g-closed maps and λ_g^α-closed maps are independent of each other as observed from the following examples.

Example 5.2.11 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{j\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = j, u(j) = i$ and $u(k) = k$. Then u is a g-closed map but not a λ_g^α-closed map, since for the closed set $\{j, k\}$ in (M, μ), $u(\{j, k\}) = \{i, k\}$ is not a λ_g^α-closed set in (N, ν).

Example 5.2.12 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, \{i, k\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be the identity map. Then u is a λ_g^α-closed map but not a g-closed map, since for the closed set $\{j\}$ in (M, μ), $u(\{j\}) = \{j\}$ is not a g-closed set in (N, ν).

Remark 5.2.13 α-closed maps and λ_g^α-closed maps are independent of each other as observed from the following examples.

Example 5.2.14 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{j, k\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be the identity map. Then u is a λ_g^α-closed map but not an α-closed map, since for the closed set $\{i\}$ in (M, μ), $u(\{i\}) = \{i\}$ is not an α-closed set in (N, ν).

Example 5.2.15 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j, k\}, M\}$ and $\nu = \{\phi, \{i\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = k, u(j) = l, u(k) = i$ and $u(l) = j$. Then u is an α-closed map but not a λ_g^α-closed map, since for the closed set $\{l\}$ in (M, μ), $u(\{l\}) = \{j\}$ is not a λ_g^α-closed set in (N, ν).

Remark 5.2.16 αg-closed maps and λ_g^α-closed maps are independent of each other as observed from the following examples.

Example 5.2.17 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{k\}, \{i, j\}, \{i, k\}, \{i, j, k\}, \{i, k, l\}, M\}$ and $\nu = \{\phi, \{i\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be the identity map. Then u is an αg-closed map but not a λ_g^α-closed map, since for the closed set $\{j\}$ in (M, μ), $u(\{j\}) = \{j\}$ is not a λ_g^α-closed set in (N, ν).

Example 5.2.18 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{j\}, \{i, j\}, \{i, j, k\}, \{i, j, l\}, M\}$ and $\nu = \{\phi, \{i\}, \{k\}, \{i, j\}, \{i, k\}, \{i, j, k\}, \{i, k, l\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = l, u(k) = k$ and $u(l) = j$. Then u is a λ_g^α-closed map but not an αg-closed map, since for the closed set $\{i, k, l\}$ in (M, μ), $u(\{i, k, l\}) = \{i, j, k\}$ is not an αg-closed set in (N, ν).

Remark 5.2.19 $g\alpha$-closed maps and λ_g^α-closed maps are independent of each other as observed from the following examples.

Example 5.2.20 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i, j, k\}, M\}$ and $\nu = \{\phi, \{i\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be the identity map. Then u is a $g\alpha$-closed map but not a λ_g^α-closed map, since for the closed set $\{l\}$ in (M, μ), $u(\{l\}) = \{l\}$ is not a λ_g^α-closed set in (N, ν).

Example 5.2.21 Let $M = N = \{i, j, k, l\}$, $\mu = \{\phi, \{i\}, \{k\}, \{i, j\}, \{i, k\}, \{i, j, k\}, \{i, k, l\}, M\}$ and $\nu = \{\phi, \{i, j, k\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = k, u(j) = k, u(k) = l$ and $u(l) = i$. Then u is a λ_g^α-closed map but not a $g\alpha$-closed map, since for the closed set $\{j\}$ in (M, μ), $u(\{j\}) = \{k\}$ is not a $g\alpha$-closed set in (N, ν).

Theorem 5.2.22 If $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-closed map then u is a g-closed map, whenever the co-domain (N, ν) is a partition space.

Proof: Let T be a closed set in (M, μ). Since u is a λ_g^α-closed map, $u(T)$ is a λ_g^α-closed set in (N, ν). As (N, ν) is a partition space and by Theorem 2.2.38, $u(T)$ is g-closed in (N, ν). Therefore u is a g-closed map.

Theorem 5.2.23 If $u: (M, \mu) \to (N, \nu)$ is a g-closed (resp. α-closed, αg-closed, g^*-closed) map and (N, ν) is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T_{1/2}^*$-space) then u is a λ_g^α-closed map.

Proof: Let T be a closed set in (M, μ). As u is a g-closed (resp. α-closed, αg-closed, g^*-closed) map, $u(T)$ is a g-closed (resp. α-closed, αg-closed, g^*-closed) set in (N, ν). Since (N, ν) is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T_{1/2}^*$-space), by Definitions 1.2.1 and 1.2.2, $u(T)$ is a closed set in (N, ν). By using Proposition 2.2.5, $u(T)$ is a λ_g^α-closed set in (N, ν). Therefore u is a λ_g^α-closed map.

Theorem 5.2.24 If a map $u: (M, \mu) \to (N, \nu)$ is α-irresolute and λ-closed then for every λ_g^α-closed set G of (M, μ), $u(G)$ is a λ_g^α-closed set in (N, ν).

Proof: Let G be a λ_g^α-closed set in (M, μ). Let S be an α-open set in (N, ν) such that $u(G) \subseteq S$ then $G \subseteq u^{-1}(S)$. As u is an α-irresolute map, $u^{-1}(S)$ is α-open in (M, μ). Since G is a λ_g^α-closed set and $u^{-1}(S)$ is an α-open set by definition of λ_g^α-closed set, $cl_\lambda(G) \subseteq u^{-1}(S)$ which implies $u(cl_\lambda(G)) \subseteq S$. As u is a λ-closed map, $u(cl_\lambda(G))$ is a λ-closed set. Now $cl_\lambda(u(G)) \subseteq cl_\lambda(u(cl_\lambda(G))) = u(cl_\lambda(G)) \subseteq S$. Therefore $u(G)$ is a λ_g^α-closed set in (N, ν).

Theorem 5.2.25 A map $u: (M, \mu) \to (N, \nu)$ is λ_g^α-closed if and only if for each subset G of (N, ν) and for each open set S of (M, μ) containing $u^{-1}(G)$, there exists a λ_g^α-open set T of (N, ν) such that $G \subseteq T$ and $u^{-1}(T) \subseteq S$.

Proof: (Necessity) Suppose that G is a subset of (N, ν) and S is an open set in (M, μ) such that $u^{-1}(G) \subseteq S$. Since u is a λ_g^α-closed map, $u(M \setminus S)$ is a λ_g^α-closed set in (N, ν) implies $N \setminus [u(M \setminus S)]$ is a λ_g^α-open set in (N, ν).

Let $T = N \setminus [u(M \setminus S)]$. Since $u^{-1}(G) \subseteq S, [M \setminus S] \subseteq M \setminus u^{-1}(G) = u^{-1}(N \setminus G) \Rightarrow u(M \setminus S) \subseteq (N \setminus G) \Rightarrow G \subseteq [N \setminus (u(M \setminus S))] = T$. Now $u(M \setminus S) \subseteq (N \setminus T) \Rightarrow (M \setminus S) \subseteq u^{-1}[(N \setminus T)] = M \setminus u^{-1}(T) \Rightarrow u^{-1}(T) \subseteq S$.

Sufficiency: Let G be a closed set in (M, μ). Then $u^{-1}(N \setminus u(G)) \subseteq M \setminus G$ and $M \setminus G$ is open. From the assumption, there exists a λ_g^α-open set T of (N, ν) such that $[N \setminus u(G)] \subseteq T$ and $u^{-1}(T) \subseteq [M \setminus G] \Rightarrow G \subseteq M \setminus u^{-1}(T)$. Hence $N \setminus T \subseteq u(G) \subseteq u\big(M \setminus u^{-1}(T)\big) = N \setminus T$, which implies $u(G) = N \setminus T$. Since $N \setminus T$ is λ_g^α-closed, $u(G)$ is λ_g^α-closed and thus u is a λ_g^α-closed map.

Theorem 5.2.26 If a map $u: (M, \mu) \to (N, \nu)$ is a λ_g^α-closed map then $\lambda_g^\alpha cl\big(u(A)\big) \subseteq u(cl(A))$ for every subset A of (M, μ).

Proof: Let u be a λ_g^α-closed map and $A \subseteq M$. As $cl(A)$ is closed in (M, μ), $u(cl(A))$ is λ_g^α-closed in (N, ν). Since $u(cl(A))$ is λ_g^α-closed by Proposition 2.5.5, $\lambda_g^\alpha cl(u\big(cl(A)\big)) = u(cl(A))$. From the fact that $u(A) \subseteq u(cl(A))$, we have $\lambda_g^\alpha cl\big(u(A)\big) \subseteq \lambda_g^\alpha cl(u\big(cl(A)\big)) = u(cl(A))$. Hence $\lambda_g^\alpha cl\big(u(A)\big) \subseteq u(cl(A))$.

The subsequent example shows that the converse of Proposition 5.2.26 may not hold good.

Example 5.2.27 Let $M = N = \{i, j, k, l\}, \mu = \{\phi, \{i, j, k\}, M\}$ and $\nu = \{\phi, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be the identity map. Then for every subset $G \subseteq M$ we have $\lambda_g^\alpha cl\big(u(G)\big) \subseteq u(cl(G))$ but u is not a λ_g^α-closed map, since for a closed set $\{l\}$ in (M, μ), $u(\{l\}) = \{l\}$ is not a λ_g^α-closed set in (N, ν).

Theorem 5.2.28 If $w:(M,\mu)\to(N,\nu)$ is a λ_g^α-closed map and P is a closed subset of (M,μ) then the restriction $w|_P:(P,\mu|_P)\to(N,\nu)$ is a λ_g^α-closed map.

Proof. Let $H\subseteq P$ be a closed set in $(P,\mu|_P)$, then $H=P\cap T$ for some closed set T of (M,μ). As P is closed in (M,μ), H is also closed in (M,μ). Since w is a λ_g^α-closed map, $w(H)=(w|_P)(H)$ is a λ_g^α-closed set in (N,ν). Hence $w|_P$ is a λ_g^α-closed map.

Theorem 5.2.29 Let H be an α-open and λ_g^α-closed set of (N,ν). If a bijective map $w:(M,\mu)\to(N,\nu)$ is λ-closed and $P=w^{-1}(H)$ then the restriction $w|_P:(P,\mu|_P)\to(N,\nu)$ is a λ_g^α-closed map.

Proof: Let $T\subseteq P$ be a closed set in $(P,\mu|_P)$, then $T=P\cap Q$ for some closed set Q of (M,μ). Since Q is closed in (M,μ), Q is also λ-closed in (M,μ). As w is a λ-closed map, $w(Q)$ is λ-closed in (N,ν). As H is α-open and $w(Q)$ is λ-closed, by Theorem 2.3.10, $w(Q)\cap H$ is λ_g^α-closed. Using the fact, $w|_P(T)=w(T)=w(P\cap Q)=w(w^{-1}(H)\cap Q)=H\cap w(Q)$ is λ_g^α-closed. Hence $w|_P:(P,\mu|_P)\to(N,\nu)$ is a λ_g^α-closed map.

5.3 λ_g^α-Open Maps

Most vital properties and characterizations of λ_g^α-open maps have been discussed in this section.

Definition 5.3.1 A map $u:(M,\mu)\to(N,\nu)$ is called a **λ_g^α-open map** if the image of each open set in (M,μ) is λ_g^α-open in (N,ν), i.e., if $u(T)$ is λ_g^α-open in (N,ν) for every open set T in (M,μ).

Example 5.3.2 Let $M=N=\{i,j,k\}$, $\mu=\{\phi,\{i\},\{j\},\{i,j\},M\}$ and $\nu=\{\phi,\{i\},\{j\},\{i,j\},\{i,k\},N\}$. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=j, u(j)=l$ and $u(k)=i$. Then u is a λ_g^α-open map as the image of every open set in (M,μ) is λ_g^α-open in (N,ν).

Proposition 5.3.3 Every open map is a λ_g^α-open map.

Proof: Similar to proof of Proposition 5.2.3.

Proposition 5.3.4 Every λ-open map is a λ_g^α-open map.

Proof: Similar to proof of Proposition 5.2.5

Definition 5.3.5 A map $u:(M,\mu)\to(N,\nu)$ is called a $\boldsymbol{g\Lambda}$**-open map** if the image of each open set in (M,μ) is $g\Lambda$-open in (N,ν), i.e., if $u(T)$ is $g\Lambda$-open in (N,ν) for every open set T in (M,μ).

Proposition 5.3.6 Every λ_g^α-open map is a $g\Lambda$-open map.

Proof: Similar to proof of Proposition 5.2.8.

Theorem 5.3.7 A bijection $u:(M,\mu)\to(N,\nu)$ is a λ_g^α-closed map if and only if u is a λ_g^α-open map.

Proof: Let $u:(M,\mu)\to(N,\nu)$ be a λ_g^α-closed map and T be an open set in (M,μ). Then T^c is closed in (M,μ). As u is a bijective map, $u(T^c)=(u(T))^c$, which is a λ_g^α-closed set in (N,ν). Hence $u(T)$ is a λ_g^α-open set in (N,ν). Thus u is a λ_g^α-open map.

Conversely, let $u:(M,\mu)\to(N,\nu)$ be a λ_g^α-open map and T be a closed set in (M,μ). Then T^c is open in (M,μ). As u is a bijective map, $u(T^c)=(u(T))^c$, which is a λ_g^α-open set in (N,ν). Hence $u(T)$ is a λ_g^α-closed set in (N,ν). Thus u is a λ_g^α-closed map.

Theorem 5.3.8 A map $u:(M,\mu)\to(N,\nu)$ is λ_g^α-open, then for every subset T of (M,μ) $u(int(T))\subseteq\lambda_g^\alpha int(u(T))$.

Proof: Let u be a λ_g^α-open map and T be any subset of (M,μ) such that $T\subseteq M$. Since $int(T)$ is open in (M,μ), $u\big(int(T)\big)$ is λ_g^α-open in (N,ν). Since $u\big(int(T)\big)$ is λ_g^α-open by Proposition 2.5.19, $\lambda_g^\alpha int(u\big(int(T)\big))=u(int(T))$. From the fact that $u\big(int(T)\big)\subseteq u(T)$, we have $u(int(T))=\lambda_g^\alpha int\left(u\big(int(T)\big)\right)\subseteq\lambda_g^\alpha int(u(T))$. Hence $u\big(int(T)\big)\subseteq\lambda_g^\alpha int(u(T))$.

Proposition 5.3.9 For any bijective map $u:(M,\mu)\to(N,\nu)$ the following statements are equivalent.

(i) $u^{-1}:(N,\nu)\to(M,\mu)$ is a λ_g^α-continuous map.

(ii) u is a λ_g^α-open map.

(iii) u is a λ_g^α-closed map.

Proof: (i) ⇒ (ii) Let S be an open set in (M,μ). As u^{-1} is a λ_g^α-continuous map and bijective, $(u^{-1})^{-1}(S)=u(S)$ is λ_g^α-open in (N,ν). Thus u is a λ_g^α-open map.

(ii) ⇒ (iii) Let T be a closed set in (M,μ), then $M\setminus T$ is open in (M,μ). Since u is a λ_g^α-open map, $u(M\setminus T)=N\setminus u(T)$ is a λ_g^α-open set in (N,ν) which implies $u(T)$ is a λ_g^α-closed set in (N,ν). Hence u is a λ_g^α-closed map.

(iii) ⇒ (i) Let T be a closed set in (M,μ). As u is a λ_g^α-closed map, $u(T)$ is a λ_g^α-closed set in (N,ν). Using the fact, $u(T)=((u^{-1})^{-1})(T)$ we have $(u^{-1})^{-1}(T)$ is a λ_g^α-closed set in (N,ν). Hence $u^{-1}:(N,\nu)\to(M,\mu)$ is a λ_g^α-continuous map as the inverse image of the closed set T in (M,μ) is a λ_g^α-closed set in (N,ν).

5.4 Quasi λ_g^α-Open Maps and Strongly λ_g^α-Open Maps

Some of the special forms of open maps called quasi λ_g^α-open maps and strongly λ_g^α-open maps have been defined in this section. Moreover, their associations with other maps and some fundamental characterizations have been discussed.

Definition 5.4.1 A map $u:(M,\mu)\to(N,\nu)$ is called **quasi λ_g^α-open** if the image of every λ_g^α-open set in (M,μ) is open in (N,ν) or if $u(T)$ is open in (N,ν) for each λ_g^α-open set T in (M,μ).

Example 5.4.2 Let $M=N=\{i,j,k\}$, $\mu=\{\phi,\{i\},M\}$ and $\nu=\{\phi,\{i\},\{j,k\},N\}$. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=i, u(j)=k$ and $u(k)=j$. Then u is a quasi λ_g^α-open map.

Theorem 5.4.3 A map $u:(M,\mu)\rightarrow(N,\nu)$ is quasi λ_g^α-open if for every λ_g^α-open set S of (M,μ), $u(\lambda_g^\alpha int(S))\subset int(u(S))$ and vice versa.

Proof: Let S be a λ_g^α-open set in (M,μ). Now $\lambda_g^\alpha int(S)$ is a λ_g^α-open set and $\lambda_g^\alpha int(S)\subset S$ implies $u(\lambda_g^\alpha int(S))\subset u(S)$. As u is quasi λ_g^α-open, $u(\lambda_g^\alpha int(S))$ is open. Therefore $u(\lambda_g^\alpha int(S))=int(u(\lambda_g^\alpha int(S)))\subset int(u(S))$. Thus, $u(\lambda_g^\alpha int(S))\subset int(u(S))$.

On the other side, assume that S is a λ_g^α-open set in (M,μ). Then $u(S)=u(\lambda_g^\alpha int(S))\subset int(u(S))$ but $int(u(S))\subset u(S)$. Consequently, $u(S)=int(u(S))$. Therefore $u(S)$ is open and hence u is quasi λ_g^α-open.

Theorem 5.4.4 If $u:(M,\mu)\rightarrow(N,\nu)$ is quasi λ_g^α-open then $\lambda_g^\alpha int\big(u^{-1}(P)\big)\subset u^{-1}(int(P))$, for every λ_g^α-open set P of (N,ν).

Proof: Let P be any λ_g^α-open set in (N,ν). Then $\lambda_g^\alpha int(u^{-1}(P))$ is a λ_g^α-open set in (M,μ). By Theorem 5.4.3, $u(\lambda_g^\alpha int\big(u^{-1}(P)\big))\subset int(u(u^{-1}(P)))=int(P)$. Thus $\lambda_g^\alpha int(u^{-1}(P))\subset u^{-1}(int(P))$.

Definition 5.4.5 A subset $\mathcal{A}$ is called $\boldsymbol{\lambda_g^\alpha}$**-neighbourhood** of a point m of (M,μ) if there exists a λ_g^α-open set S such that $m\in S\subset\mathcal{A}$.

Theorem 5.4.6 For the map $u:(M,\mu)\rightarrow(N,\nu)$, the following statements are equivalent.

(i) u is a quasi λ_g^α-open map

(ii) For every λ_g^α-open set S of (M,μ), $u(\lambda_g^\alpha int(S))\subset int(u(S))$

(iii) For each $m\in M$ and each λ_g^α-neighbourhood S of m in (M,μ), $\exists$ a neighbourhood T of $u(m)$ in (N,ν) such that $T\subset u(S)$

Proof: (i) $\Rightarrow$ (ii) Follows from Theorem 5.4.3.

(ii) ⇒ (iii) Let $m \in M$ and S be an arbitrary λ_g^α-neighbourhood of m in (M, μ). Then ∃ a λ_g^α-open set T in (M, μ) such that $m \in T \subset S$. Then by (ii), $u(T) = u(\lambda_g^\alpha int(T)) \subset int(u(T))$ which implies hence $u(T) \subset int(u(T))$ and hence $u(T) = int(u(T))$. Therefore $u(T)$ is open in (N, ν) such that $u(m) \in u(T) \subset u(S)$.

(iii) ⇒ (i) Let S be an arbitrary λ_g^α-open set in (M, μ). Then for each $s \in u(S)$, by (iii) ∃ a neighbourhood T_s of s in (N, ν) such that $T_s \subset u(S)$. As T_s is a neighbourhood of s, ∃ an open set R_s in (N, ν) such that $s \in R_s \subset T_s$. Thus $u(S) = \cup \{R_s : s \in u(S)\}$, is an open set in (N, ν). This implies that u is a quasi λ_g^α-open map.

Theorem 5.4.7 A map $u: (M, \mu) \to (N, \nu)$ is quasi λ_g^α-open if for any subset P of (N, ν) and for any λ_g^α-closed set T in (M, μ) containing $u^{-1}(P)$, ∃ a closed set Q of (N, ν) containing P such that $u^{-1}(Q) \subset T$ and vice versa.

Proof: Suppose u is a quasi λ_g^α-open map. Let $P \subset N$ and T be a λ_g^α-closed set in (M, μ) containing $u^{-1}(P)$. Let $Q = N \setminus [u(M \setminus T)]$ which follows that $u^{-1}(P) \subset T$ which implies $P \subset u(T) = Q$. Since u is quasi λ_g^α-open, Q is a closed set of (N, ν) and $u^{-1}(Q) \subset T$.

On the other side, let S be a λ_g^α-open set in (M, μ) and let $P = N \setminus (u(S))$. Now $M \setminus S$ is a λ_g^α-closed set in (M, μ). And $u^{-1}(P) \subseteq M \setminus S$. By hypothesis, ∃ a closed set T in (N, ν) such that $P \subset T$ and $u^{-1}(T) \subset M \setminus S$. Hence $u(S) \subset N \setminus T$. Now $P \subset T$ implies $N \setminus T \subset N \setminus P = u(S)$. Thus $u(S) = N \setminus T$ is open and hence u is a quasi λ_g^α-open map.

Theorem 5.4.8 A map $u: (M, \mu) \to (N, \nu)$ is a quasi λ_g^α-open if $u^{-1}\big(cl(P)\big) \subset \lambda_g^\alpha cl(u^{-1}(P))$, for every λ_g^α-closed set P of (N, ν) and vice versa.

Proof: Suppose u is a quasi λ_g^α-open map. For any λ_g^α-closed set P of (N, ν), $u^{-1}(P) \subset \lambda_g^\alpha cl(u^{-1}(P))$. Therefore, by Theorem 5.4.7, ∃ a closed set T in (N, ν) such that $P \subset T$ and $u^{-1}(T) \subset \lambda_g^\alpha cl(u^{-1}(P))$. Therefore $u^{-1}\big(cl(P)\big) \subset u^{-1}(T) \subset \lambda_g^\alpha cl(u^{-1}(P))$.

On the other side, let $P \subset N$ and T be λ_g^α-closed in (M, μ) containing $u^{-1}(P)$. Let $W = cl(P)$, then $P \subset W$ and W is closed and $u^{-1}(W) \subset \lambda_g^\alpha cl\left(u^{-1}(P)\right) \subset T$. Then by Theorem 8, u is a quasi λ_g^α-open map.

Definition 5.4.9 A map u is called **quasi λ_g^α-closed** if the image of every λ_g^α-closed set in (M, μ) is closed in (N, ν) or if $u(P)$ is closed in (N, ν) for every λ_g^α-closed set P in (M, μ).

Theorem 5.4.10 Every quasi λ_g^α-closed map is a closed map.

Proof: Follows from the Proposition 2.2.5 and 2.2.7.

The subsequent example shows that the converse of Proposition 5.4.10 may not hold good**.**

Example 5.4.11 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, M\}$ and $\nu = \{\phi, \{i\}, \{j\}, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = i, u(j) = k$ and $u(k) = j$. Then u is a closed map but not a quasi λ_g^α-closed map, since for the λ_g^α-closed set $\{i\}$ in (M, μ), $u(\{i\}) = \{i\}$ is not a closed set in (N, ν).

Theorem 5.4.12 A map $u: (M, \mu) \to (N, \nu)$ is quasi λ_g^α-closed if for every λ_g^α-closed set S of (M, μ), $cl\left(u(S)\right) \subset u(\lambda_g^\alpha cl(S))$ and vice versa.

Proof: Analogous to the proof of Theorem 5.4.3.

Theorem 5.4.13 A map $u: (M, \mu) \to (N, \nu)$ is quasi λ_g^α-closed if for any subset P of (N, ν) and for any λ_g^α-open set S in (M, μ) containing $u^{-1}(P)$, $\exists$ an open set Q of (N, ν) containing P such that $u^{-1}(Q) \subseteq S$ and vice versa.

Proof: Analogous to the proof of Theorem 5.4.7.

Theorem 5.4.14 For any two topological spaces (M, μ) and (N, ν), $u: (M, \mu) \to (N, \nu)$ is quasi λ_g^α-closed if $u(M)$ is closed in (N, ν) and $u(P) \setminus [u(M \setminus P)]$ is open in $u(M)$ whenever P is λ_g^α-open in (M, μ) and vice versa.

Proof: Suppose u is a quasi λ_g^α-closed map. Since M is λ_g^α-closed, $u(M)$ is closed in (N,ν) and $u(P) \setminus [u(M \setminus P)] = u(P) \cap u(M) \setminus [u(M \setminus P)]$ is open in $u(M)$ when P is λ_g^α-open in (M,μ).

On the other side, suppose $u(M)$ is closed in (N,ν), $u(P) \setminus [u(M \setminus P)]$ is open in $u(M)$ when P is λ_g^α-open in (M,μ) and let W be λ_g^α-closed in (M,μ). Then $u(W) = u(M) \setminus [u(M \setminus W) \setminus u(W))]$ is closed in $u(M)$ and hence closed in (N,ν).

Corollary 5.4.15 Let (M,μ) and (N,ν) be topological spaces. Then a surjective map $u: (M,\mu) \to (N,\nu)$ is quasi λ_g^α-closed if $u(P) \setminus u(M \setminus P)$ is open in (N,ν) whenever P is λ_g^α-open in (M,μ) and vice versa.

Proof: Obvious.

Theorem 5.4.16 For the topological spaces (M,μ) and (N,ν), let $u: (M,\mu) \to (N,\nu)$ be a λ_g^α-continuous and quasi λ_g^α-closed surjective map. Then the topology on (N,ν) is $\{u(P) \setminus [u(M \setminus P)]: P \text{ is } \lambda_g^\alpha\text{-open in } (M,\mu)\}$.

Proof: Let W be an open set in (N,ν). Then $u^{-1}(W)$ is λ_g^α-open in (M,μ) and $u[u^{-1}(W)] \setminus [u(M \setminus u^{-1}(W)] = W$. Hence all open sets in (N,ν) are of the form $u(P) \setminus [u(M \setminus P)]$, P is λ_g^α-open in (M,μ). On the other hand, all sets of the form $u(P) \setminus [u(M \setminus P)]$, P is λ_g^α-open in (M,μ) are open in (N,ν) from Corollary 5.4.15.

Definition 5.4.17 A map $u: (M,\mu) \to (N,\nu)$ is called **strongly $\boldsymbol{\lambda_g^\alpha}$-closed** if the image of every λ_g^α-closed set in (M,μ) is λ_g^α-closed in (N,ν) or if $u(T)$ is λ_g^α-closed in (N,ν) for every λ_g^α-closed set T in (M,μ).

Example 5.4.18 Let $M = N = \{i,j,k,l\}, \mu = \{\phi, \{i,j\}, M\}, \nu = \{\phi, \{i,j,k\}, N\}$. Let $u: (M,\mu) \to (N,\nu)$ be a map defined by $u(i) = j$, $u(j) = k$, $u(k) = l$ and $u(l) = i$. Then u is a strongly λ_g^α-closed map.

Proposition 5.4.19 Every strongly λ_g^α-closed map $u: (M,\mu) \to (N,\nu)$ is a λ_g^α-closed map.

Proof: Follows from Proposition 2.2.5.

The subsequent example shows that the converse of Proposition 5.4.19 may not hold good.

Example 5.4.20 Let $M = N = \{i, j, k, l\}, \mu = \{\phi, \{i, j, k\}, M\}, \nu = \{\phi, \{i, j\}, N\}$. Let $u: (M, \mu) \to (N, \nu)$ be a map defined by $u(i) = l$, $u(j) = j$, $u(k) = k$ and $u(l) = i$. Then u is a λ_g^α-closed map but not a strongly λ_g^α-closed map since for the λ_g^α-closed set $\{k\}$ in $(M, \mu), u(\{k\}) = \{k\}$ is not λ_g^α-closed in (N, ν).

Proposition 5.4.21 Every quasi λ_g^α-closed map $u: (M, \mu) \to (N, \nu)$ is a strongly λ_g^α-closed map.

Proof: Evident from the Proposition 2.2.5.

The subsequent example shows that the converse Proposition 5.4.21 may not hold good.

Example 5.4.22 Let $M = N = \{i, j, k\}$, $\mu = \{\phi, \{i\}, \{i, j\}, M\}$ and $\nu = \{\phi, \{i\}, \{j\}, \{i, j\}, N\}$. Then the identity map $u: (M, \mu) \to (N, \nu)$ is a strongly λ_g^α-closed map but not a quasi λ_g^α-closed map, since for the λ_g^α-closed set $\{i, j\}$ in (M, μ), $u(\{i, j\}) = \{i, j\}$ is not a closed set in (N, ν).

5.5 Compositions of λ_g^α-Closed Maps and λ_g^α-Open Maps

Compositions are the basic relationship between two maps. Those compositions related to λ_g^α-closed maps, λ_g^α-open maps and their special forms are derived and proved in this section.

Proposition 5.5.1 If a map $u: (M, \mu) \to (N, \nu)$ is a g-closed (resp. α-closed, αg-closed, g^*-closed) map, (N, ν) is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T_{1/2}^*$-space) and $w: (N, \nu)$` $\to (K, \kappa)$ is a closed map then $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ_g^α-closed map.

Proof: Let T be a closed set in (M, μ). Since u is a g-closed (resp. α-closed, αg-closed, g^*-closed) map, $u(T)$ is a g-closed (resp. α-closed, αg-closed, g^*-closed) set in (N, ν). As (N, ν) is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T_{1/2}^*$-space), by Definitions 1.2.1 and 1.2.2,

$u(T)$ is a closed set in (N,ν). Since $w:(N,\nu)\to(K,\kappa)$ is a closed map, $w\big(u(T)\big)=(w\circ u)(T)$ is a closed set in (K,κ). Using Proposition 2.2.5, $(w\circ u)(T)$ is a λ_g^α-closed set in (K,κ). Thus $(w\circ u)$ is a λ_g^α-closed map.

Proposition 5.5.2 If a map $u:(M,\mu)\to(N,\nu)$ is a g-closed (resp. α-closed, αg-closed, g^*-closed) map, (N,ν) is a $T_{1/2}$-space (resp. α-space, ${}_\alpha T_b$-space, $T_{1/2}^*$-space) and $w:(N,\nu)\to(K,\kappa)$ is a λ_g^α-closed map then $(w\circ u):(M,\mu)\to(K,\kappa)$ is a λ_g^α-closed map.

Proof: Similar to the proof of Proposition 5.5.1.

Proposition 5.5.3 The composition of two closed (resp. open) maps is a λ_g^α-closed (resp. λ_g^α-open) map.

Proof: Follows from Proposition 2.2.5 and 2.4.3.

Remark 5.5.4 The composition of two λ_g^α-closed (resp. λ_g^α-open) maps need not be a λ_g^α-closed (resp. λ_g^α-open) map as observed from the following example.

Example 5.5.5 Let $M=N=K=\{i,j,k\}$, $\mu=\{\phi,\{i\},M\}$, $\nu=\{\phi,\{i,j\},N\}$ and $\kappa=\{\phi,\{i\},\{i,j\},K\}$. Let $u:(M,\mu)\to(N,\nu)$ be a map defined by $u(i)=j, u(j)=i$ and $u(k)=k$ and $w:(N,\nu)\to(K,\kappa)$ be the identity map. Then the maps u and w are both λ_g^α-closed maps, but their composition $(w\circ u):(M,\mu)\to(K,\kappa)$ is not a λ_g^α-closed map, since for the closed set $\{j,k\}$ in (M,μ), $(w\circ u)(\{j,k\})=\{i,k\}$ is not a λ_g^α-closed set in (K,κ). Also the maps u and w are both λ_g^α-open, but their composition $(w\circ u):(M,\mu)\to(K,\kappa)$ is not a λ_g^α-open map, since for an open set $\{i\}$ in (M,μ), $(w\circ u)(\{i\})=\{j\}$ is not a λ_g^α-open set in (K,κ).

Remark 5.5.6 If $u:(M,\mu)\to(N,\nu)$ is a λ_g^α-closed map and $w:(N,\nu)\to(K,\kappa)$ is a closed map then their composition $(w\circ u):(M,\mu)\to(K,\kappa)$ need not be a λ_g^α-closed map as observed from the following example.

Example 5.5.7 Consider $M, N, K, \mu, \nu, \kappa$ and the maps u, w as in Example 5.5.5. Here u is a λ_g^α-closed map and w is a closed map, but their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is not a λ_g^α-closed map, since for a closed set $\{j, k\}$ in (M, μ), $(w \circ u)(\{j, k\}) = \{i, k\}$ is not a λ_g^α-closed set in (K, κ).

Proposition 5.5.8 If $u: (M, \mu) \to (N, \nu)$ is a closed map and $w: (N, \nu) \to (K, \kappa)$ is a λ_g^α-closed map then their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ_g^α-closed map.

Proof: Let T be a closed set in (M, μ). As u is a closed map, $u(T)$ is closed in (N, ν). Since w is a λ_g^α-closed map, $w\big(u(T)\big) = (w \circ u)(T)$ is a λ_g^α-closed set in (K, κ). Therefore $(w \circ u)$ is a λ_g^α-closed map.

Proposition 5.5.9 If $u: (M, \mu) \to (N, \nu)$ is a closed map and $w: (N, \nu) \to (K, \kappa)$ is a λ-closed map then their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ_g^α-closed map.

Proof: Let T be a closed set in (M, μ). Then $u(T)$ is a closed set in (N, ν). Using Lemma 1.1.8, $u(T)$ is a λ-closed set in (N, ν). Since w is a λ-closed map, $(w \circ u)(T) = w(u(T))$ is a λ-closed set in (K, κ). Using Proposition 2.2.3, $w(u(T))$ is a λ_g^α-closed set in (K, κ). Hence $(w \circ u)$ is a λ_g^α-closed map.

Proposition 5.5.10 If $u: (M, \mu) \to (N, \nu)$ is a continuous map and $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ_g^α-closed map then $w: (N, \nu) \to (K, \kappa)$ is a λ_g^α-closed map.

Proof: Let T be a closed set in (N, ν). As u is continuous, $u^{-1}(T)$ is a closed set in (M, μ). Since $(w \circ u)$ is a λ_g^α-closed map, $(w \circ u)(u^{-1}(T)) = w(T)$ is a λ_g^α-closed set in (K, κ). Hence w is a λ_g^α-closed map.

Proposition 5.5.11 Let $u: (M, \mu) \to (N, \nu)$ be any map and $w: (N, \nu) \to (K, \kappa)$ be an injective and λ_g^α-irresolute map. If their composition $(w \circ u): (M, \mu) \to (K, \kappa)$ is a λ_g^α-closed map then u is a λ_g^α-closed map.

Proof: Let T be any closed set in (M,μ). Since $(w \circ u)$ is a λ_g^α-closed map, $(w \circ u)(T)$ is a λ_g^α-closed set in (K,κ). As w is λ_g^α-irresolute and injective, $w^{-1}[(w \circ u)(T)] = u(T)$, which is a λ_g^α-closed set in (N,ν). Hence u is a λ_g^α-closed map.

Proposition 5.5.12 Let $u: (M,\mu) \to (N,\nu)$ be any map and $w: (N,\nu) \to (K,\kappa)$ be an injective and λ_g^α-closed map. If their composition $(w \circ u): (M,\mu) \to (K,\kappa)$ is a λ_g^α-irresolute map then u is a λ_g^α-continuous map.

Proof: Let T be any closed set in (N,ν). Since w is a λ_g^α-closed map, $w(T)$ is a λ_g^α-closed set in (K,κ). As $(w \circ u)$ is λ_g^α-irresolute, $(w \circ u)^{-1}\big(w(T)\big)$ is a λ_g^α-closed set in (M,μ). Also, since w is injective $w^{-1}\big(w(T)\big) = T$. Hence $u^{-1}(w^{-1}\big(w(T)\big) = u^{-1}(T)$ is a λ_g^α-closed set in (M,μ). Hence u is a λ_g^α-continuous map.

Proposition 5.5.13 Let $u: (M,\mu) \to (N,\nu)$ be any map and $w: (N,\nu) \to (K,\kappa)$ be an injective and λ_g^α-irresolute map. If their composition $(w \circ u): (M,\mu) \to (K,\kappa)$ is a λ_g^α-open map then u is a λ_g^α-open map.

Proof: Let S be any open set in (M,μ). As $(w \circ u)$ is a λ_g^α-open map, $(w \circ u)(S)$ is a λ_g^α-open set in (K,κ). Since w is λ_g^α-irresolute and injective $w^{-1}[(w \circ u)(S)] = u(S)$ is a λ_g^α-open set in (N,ν). Hence u is a λ_g^α-open map.

Proposition 5.5.14 Let $u: (M,\mu) \to (N,\nu)$ and $w: (N,\nu) \to (K,\kappa)$ be two maps and $(w \circ u): (M,\mu) \to (K,\kappa)$ is a quasi λ_g^α-open map. If w is a continuous and injective map, then u is a quasi λ_g^α-open map.

Proof: Let S be a λ_g^α-open set in (M,μ). Then $(w \circ u)(S)$ is open in (K,κ), as $(w \circ u)$ is quasi λ_g^α-open. Since w is an injective continuous map, $w^{-1}\big((w \circ u)(S)\big) = u(S)$ is open in (N,ν). Hence u is a quasi λ_g^α-open map.

Proposition 5.5.15 If $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ are quasi λ_g^α-open (resp. quasi λ_g^α-closed) maps then $(w \circ u): (M, \mu) \to (K, \kappa)$ is also a quasi λ_g^α-open (resp. quasi λ_g^α-closed) map.

Proof: Let S be any λ_g^α-open (resp. λ_g^α-closed) set in (M, μ). As u is quasi λ_g^α-open (resp. quasi λ_g^α-closed), $u(S)$ is open (resp. closed) in (N, ν). Since every open (resp. closed) set is λ_g^α-open (resp. λ_g^α-closed), $u(S)$ is λ_g^α-open (resp. λ_g^α-closed) set in (N, ν). Since w is quasi λ_g^α-open (resp. quasi λ_g^α-closed), $(w \circ u)(S) = w(u(S))$ is open (resp. closed) in (K, κ). Thus $(w \circ u)$ is a quasi λ_g^α-open (resp. quasi λ_g^α-closed) map.

Proposition 5.5.16 Let $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$ be any two maps such that $(w \circ u): (M, \mu) \to (K, \kappa)$ is a quasi λ_g^α-closed map. If u is a λ_g^α-irresolute surjective map, then w is a closed map.

Proof: Let T be a closed set in (N, ν). Using Proposition 2.2.5 and since u is a λ_g^α-irresolute map, $u^{-1}(T)$ is a λ_g^α-closed set in (M, μ). Since $(w \circ u)$ is a quasi λ_g^α-closed map and u is surjective, $(w \circ u)(u^{-1}(T)) = w(T)$ is closed in (K, κ). Hence w is a closed map.

Proposition 5.5.17 Let $(w \circ u): (M, \mu) \to (K, \kappa)$ be the composition of any two maps $u: (M, \mu) \to (N, \nu)$ and $w: (N, \nu) \to (K, \kappa)$.

(i) If u is quasi λ_g^α-closed map and w is a λ_g^α-closed map then $(w \circ u)$ is a strongly λ_g^α-closed map.

(ii) If u is strongly λ_g^α-closed map and w is a quasi λ_g^α-closed map then $(w \circ u)$ is a quasi λ_g^α-closed map.

(iii) If u and W are quasi λ_g^α-closed maps then $(w \circ u)$ is a strongly λ_g^α-closed map.

Proof: (i) Let T be a λ_g^α-closed set in (M, μ). Since u is quasi λ_g^α-closed, $u(T)$ is closed in (N, ν). Then $w(u(T)) = (w \circ u)(T)$ is λ_g^α-closed in (K, κ) as w is λ_g^α-closed. Hence $(w \circ u)$ is a strongly λ_g^α-closed map.

(ii) Let T be a λ_g^α-closed set in (M,μ). Since u is strongly λ_g^α-closed, $u(T)$ is λ_g^α-closed in (N,ν). Then $w(u(T)) = (w \circ u)(T)$ is closed in (K,κ), as w is quasi λ_g^α-closed. Hence $(w \circ u)$ is a quasi λ_g^α-closed map.

(iii) Evident from the Proposition 2.2.5.

Proposition 5.5.18 Let $u: (M,\mu) \to (N,\nu)$ and $w: (N,\nu) \to (K,\kappa)$ be any two maps such that their composition $(w \circ u): (M,\mu) \to (K,\kappa)$ is a quasi λ_g^α-closed map. If w is λ_g^α-continuous and injective map then u is a strongly λ_g^α-closed map.

Proof: Let T be a λ_g^α-closed set in (M,μ). Since $(w \circ u)$ is quasi λ_g^α-closed, $(w \circ u)(T)$ is closed in (K,κ). Since w is λ_g^α-continuous and injective, $w^{-1}((w \circ u)(T)) = u(T)$ is λ_g^α-closed in (N,ν). Hence u is strongly λ_g^α-closed map.

Remark 5.5.19 Proposition 5.5.18 is true if w is continuous (resp. λ-continuous). Since every closed (resp.λ-closed) set is λ_g^α-closed.

Proposition 5.5.20 The composition of two strongly λ_g^α-closed maps is a strongly λ_g^α-closed map.

Proof: Let $u: (M,\mu) \to (N,\nu)$ and $w: (N,\nu) \to (K,\kappa)$ be strongly λ_g^α-closed maps. Let T be a λ_g^α-closed set in (M,μ). Then $u(T)$ is a λ_g^α-closed set in (N,ν). Since w is a strongly λ_g^α-closed map, $w(u(T))$ is λ_g^α-closed in (K,κ). Hence $(w \circ u): (M,\mu) \to (K,\kappa)$ is a strongly λ_g^α-closed map.