

CHAPTER II

DESIGNING AND ANALYZING CONSUMER PROTECTION OF GENERALIZED TIGHTENED THREE LEVEL CONTINUOUS SAMPLING PLAN

In this chapter, the generalized tightened three level continuous sampling plan having general clearance numbers and general sampling rates of inspection is proposed. Operating procedure of the plan and derivation of performance measures by simplified Markov-chain approach are presented. Designing of generalized tightened three level continuous sampling plan using the quality indices (AQL, AOQL) and (LQL, AOQL) are outlined. The protection provided to the consumer is analyzed by designing, consumer oriented three level plan is developed for the given LQL with consumer risk of 0.10 and f . Examples are presented to illustrate the selection of plans. The method of construction of tables is also indicated.

Dodge (1943) introduced continuous sampling plan, CSP-1 to monitor the continuous flow of products from a manufacturing process. A number of single level continuous sampling plans like CSP-2, CSP-3, CSP-V are considered as modifications of CSP-1. Following that as an extension of CSP-1, Lieberman and Solomon (1955) considered multilevel plan which allows for smoother transition between sampling inspection and screening inspection. Derman, Littauer and Solomon (1957) presented three generalizations to the plan of Lieberman and Solomon (1955). The tightest of the three plans is designated as MLP-T.

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This plan provides for an infinite number of sampling inspection levels with only same clearance number and geometrical reduction in sampling rates.

Infinite number of levels introduce difficulty in application of the plan by the unskilled workers. This necessitated the investigator to propose the generalized tightened three level continuous sampling plan which carry simultaneous reduction of clearance numbers and sampling rates while switching from lower level to a next higher level of sampling inspection. This three level plan has three levels of sampling inspection with general clearance numbers and general sampling rates. This plan is designated as tightened MLP-3(G).

The proposed plan introduces (i) the reduction in the proportion of items to be inspected both in screening and sampling inspection, when there is a good indication that the process has been running satisfactorily (ii) permits a faster release of the screening crew to return for the production duties

Consumer protection is a chief factor in the construction of acceptance sampling plans. Standard table MIL-STD-1235C (1988), provides the tables and procedures for applying the continuous sampling plans without considering the aspect of consumer protection. This motivated the investigator to estimate the consumer protection of the proposed plan in terms of LQL and f by estimating i , average fraction inspected and AOQL.

Operating procedure of tightened MLP-3(G) plan

The operating procedure of generalized tightened three level continuous sampling plan have the following steps. When units are offered for inspection in the order of production

Step 1 : At the outset, start with the screening inspection

Step 2 : After i consecutive units in succession are found to be conforming, switch to sampling inspection at rate f_1 .

- Step 3 : If i_1 units inspected at rate f_1 are found to be consecutively conforming switch to second level of sampling inspection at rate f_2
- Step 4 : If i_2 units inspected at rate f_2 are found to be consecutively conforming switch to sampling inspection at rate f_3 and continue with sampling inspection in the same level
- Step 5 : On finding a nonconforming unit at any sampling level revert to screening inspection

During sampling inspection, sample units are selected at random one at a time from the flow of products so as to assure an unbiased sample. All nonconforming units found are replaced with conforming units. Therefore Tightened MLP-3(G) plan is identified with six parameters i, i_1, i_2, f_1, f_2 and f_3

The **chief features** of the tightened MLP-3(G) plan are

- i. The number of sampling levels is fixed at three
- ii. The sampling rates f_1, f_2 and f_3 satisfy the relation $f_3 < f_2 < f_1$
- iii. The clearance numbers i, i_1 and i_2 satisfy the relation $i_2 \leq i_1 \leq i$
- iv. The reversion of screening inspection takes place on finding a nonconforming unit at any sampling inspection level

Derivation of performance measures

The simplified Markov chain approach for deriving the performance measures due to Brugger (1972), has the following steps

1. defining various phases
2. developing the transitional probability matrix
3. identifying the expected phase lengths and
4. forming the working table

The various phases of tightened MLP-3(G) plan are

SC : Screening inspection phase

SA1 : First sampling inspection phase at rate f_1

SA2 : Second sampling inspection phase at rate f_2

SA3 : Third sampling inspection phase at rate f_3

The transitional probability matrix of different phases for tightened MLP-3(G) plan is

$$\begin{array}{c}
 \text{From} \\
 \text{SC} \\
 \text{SA1} \\
 \text{SA2} \\
 \text{SA3}
 \end{array}
 \begin{array}{c}
 \text{To} \\
 \text{SC} \quad \text{SA1} \quad \text{SA2} \quad \text{SA3} \\
 \left(\begin{array}{cccc}
 0 & 1 & 0 & 0 \\
 1 - q^{i_1} & 0 & q^{i_1} & 0 \\
 1 - q^{i_2} & 0 & 0 & q^{i_2} \\
 1 & 0 & 0 & 0
 \end{array} \right)
 \end{array}$$

The steady state probabilities in terms of any one state (P''_{SA1}) are

$$P''_{SC} = P''_{SA1}$$

$$P''_{SA1} = P''_{SA1}$$

$$P''_{SA2} = q^{i_1} P''_{SA1}$$

$$P''_{SA3} = q^{i_1+i_2} P''_{SA1}$$

The expected lengths for each phase are

| <u>State</u> | <u>Expected Length</u> |
|--------------|------------------------|
| SC | $(1 - q^i)/pq^i$ |
| SA1 | $(1 - q^i)/f_1p$ |
| SA2 | $(1 - q^i)/f_2p$ |
| SA3 | $1/f_3p$ |

Working table for tightened MLP-3(G) plan

| Phase | Expected Length | Simplification of column 2 | Coe. in terms of P_{SA1}'' | AFI | |
|-------|------------------------|----------------------------|------------------------------|--------------------------------|-----------------------------------|
| | | | | Denominator | Numerator |
| SC | $(1 - q^i)/pq^i$ | $f_1f_2f_3(1 - q^i)$ | 1 | $f_1f_2f_3(1 - q^i)$ | $f_1f_2f_3(1 - q^i)$ |
| SA1 | $(1 - q^{i_1})/(f_1p)$ | $f_2f_3q^i(1 - q^{i_1})$ | 1 | $f_2f_3q^i(1 - q^{i_1})$ | $f_1f_2f_3q^i(1 - q^{i_1})$ |
| SA2 | $(1 - q^{i_2})/(f_2p)$ | $f_1f_3q^i(1 - q^{i_2})$ | q^{i_1} | $f_1f_3q^{i+i_1}(1 - q^{i_2})$ | $f_1f_2f_3q^{i+i_1}(1 - q^{i_2})$ |
| SA3 | $1/(f_3p)$ | $f_1f_2q^i$ | $q^{i_1+i_2}$ | $f_1f_2q^{i+i_1+i_2}$ | $f_1f_2f_3q^{i+i_1+i_2}$ |

Using the working table one may derive the performance measures of the plan.

Performance measures

The five most common performance measures to evaluate the plan are

1. The average number of units inspected under the screening inspection is

$$U = (1 - qi)/(pq^i) \tag{2.1}$$

2. The average number of units passed under the sampling inspection is

$$V = \{f_2 f_3 (1 - q^{i_1}) + f_1 f_3 q^{i_1} (1 - q^{i_2}) + f_1 f_2 q^{i_1 + i_2}\} / \{f_1 f_2 f_3 p\} \quad (2.2)$$

3. The average fraction of total produced units inspected in the long run is

$$F = f_1 f_2 f_3 / D \quad (2.3)$$

4. The average outgoing quality when nonconforming units are replaced with conforming unit is

$$AOQ = p \{f_2 f_3 q^i (1 - f_1) + f_3 q^{i+i_1} (f_1 - f_2) + f_1 q^{i+i_1+i_2} (f_2 - f_3)\} / D \quad (2.4)$$

5. The average fraction of total production accepted on a sampling basis is

$$P_a = \{f_2 f_3 q^i (1 - q^{i_1}) + f_1 f_3 q^{i+i_1} (1 - q^{i_2}) + f_1 f_2 q^{i+i_1+i_2}\} / D \quad (2.5)$$

$$\text{where } D = \{f_1 f_2 f_3 (1 - q^i) + f_2 f_3 q^i (1 - q^{i_1}) + f_1 f_3 q^{i+i_1} (1 - q^{i_2}) + f_1 f_2 q^{i+i_1+i_2}\}$$

$$(2.6)$$

Designing of plans

Tables are constructed to design tightened MLP-3(G) plan for the specified AOQL and AQL with $P_a = 0.95$ and AOQL and LQL with $P_a = 0.10$ by assuming $i_1 = i_2 = i/2$ and $f_1 = f$, $f_2 = f/2$, $f_3 = f/4$.

The designing procedure described in Chapter I may be followed to design the tightened MLP-3(G) plan.

Tables 2.1 and 2.2 may be used to select generalized tightened three level continuous sampling plan for given (AQL, AOQL) and (LQL, AQQL) respectively.

Table values indicate that

- i. increase in AQL decreases i
- ii. increase in the ratio $AOQL/AQL$ decreases i
- iii. increase in LQL decreases i and
- iv. increase in $LQL/AOQL$ decreases i

Selection for given AQL and AOQL

- (i) Suppose that one needs the generalized tightened three level plan satisfying the conditions $AQL = 0.25\%$ with $P_a = 0.95$ and $AOQL = 0.55\%$. Then compute the ratio $AOQL/AQL = 2.2$. Locate under the column $AOQL/AQL$, the ratio which is less than or equal to the computed ratio. Corresponding to the specified values, from the table one gets $i = 140$, $f = 1/3$. Therefore the required generalized tightened MLP-3(G) plan $(i, i_1, i_2, f_1, f_2, f_3)$ is $(140, 70, 70, 1/3, 1/6, 1/12)$.
- (ii) Suppose that one requires the generalized tightened three level plan satisfying the conditions $AQL = 0.65\%$ with $P_a = 0.95$ and $AOQL = 0.975\%$. Then compute the ratio $AOQL/AQL = 1.5$. Locate under the column $AOQL/AQL$, the ratio which is less than or equal to the computed ratio. Corresponding to the specified values, from the table one gets $i = 178$, $f = 1/20$. Therefore the required generalized tightened MLP-3(G) plan $(i, i_1, i_2, f_1, f_2, f_3)$ is $(178, 89, 89, 1/20, 1/40, 1/80)$.

Selection for given LQL and AOQL

- (i) Suppose that one wishes to use the generalized tightened three level plan satisfying the conditions $LQL = 0.4\%$ with $P_a = 0.10$ and $AOQL = 0.1\%$. Then compute the ratio $LQL/AOQL = 4$. Under the column captioned with $LQL/AOQL$ in table 1.2 locate the ratio which is greater than or equal to the computed ratio. The tabulated ratio is 4. Associated parameters are $i = 1008$,

$f = 1/5$. Therefore, the required tightened MLP-3(G) continuous sampling plan $(i, i_1, i_2, f_1, f_2, f_3)$ is (1008, 504, 504, 1/5, 1/10, 1/20).

- (ii) Suppose that one requires generalized tightened three level plan satisfying the conditions $LQL = 1.5\%$ with $P_a = 0.10$ and $AOQL = 0.54\%$. Then compute the ratio $LQL/AOQL = 2.8$. Under the column captioned with $LQL/AOQL$ in table 2.2 locate the ratio which is greater than or equal to the computed ratio. The tabulated ratio is 2.8. Associated parameters are $i = 384$, $f = 1/34$. Therefore the required tightened MLP-3(G) plan $(i, i_1, i_2, f_1, f_2, f_3)$ is (384, 192, 192, 1/34, 1/68, 1/136).

Consumer protection in tightened MLP-3(G) plan

Tables 2.3 and 2.4 for i , AOQL and AFI values at the incoming quality corresponding to AOQL indexed by f and LQL with consumer's risk of 0.10 are constructed. The analysis of protection provided to the consumer by tightened MLP-3(G) plan is carried out numerically using

- (i) AFI to estimate the cost of inspection
- (ii) AOQL to study the quality of the outgoing product

Numerical values in the tables 2.3 and 2.4 signify that

- i. increase in LQL decreases i for any f
- ii. decrease in f increases i for any LQL
- iii. increase in f decreases i

Selection of plan

Tables 2.3 and 2.4 are constructed to enable the selection of generalized tightened three level continuous sampling plan with the consumer orientation on the basis of LQL and f , corresponding to $i_1 = i_2 = i/2$ and $i_1 = i_2 = i$.

Suppose that one opts for the tightened MLP-3(G) plan for a given LQL of 12.5% and sampling rate f , $1/200$ with $i_1 = i_2 = i/2$ table 2.3, gives the values as $i = 56$, AOQL = 5.56% and AFI = 0.2111. The required plan tightened MLP-3(G) is (56,28,28,1/200,1/400 1/800)

The illustration of the application of the selected plan

Start with the screening inspection, inspect each and every unit in the order of production from the manufacturing process. If 56 consecutive units are found to be free of defects switch to sampling inspection at rate $f_1 = 1/200$ and continue the sampling inspection until 28 units in succession are found to be defect free. Switch to second level of sampling inspection at rate $f_2 = 1/400$. If 28 units in succession are found to be conforming in second level of sampling inspection switch to third level of sampling inspection at rate $f_3 = 1/800$ and continue with the sampling inspection. If a nonconforming unit is found at any level of sampling inspection then revert to screening inspection.

For the selected plan the worst outgoing quality is 0.556 and the amount of inspection at the worst quality is 0.2111.

Likewise, the following plans may be selected to monitor the production process for the stated condition.

- (i) For a given LQL of 6.5% and sampling rate $f = 1/7$ with $i_1 = i_2 = i/2$. table 2.3, gives $i = 64$, AOQL = 1.65% and AFI = 0.3772 . The plan is (64,32,32,1/7,1/14,1/28)
- (ii) For a given LQL of 0.8% and sampling rate $f = 1/50$ having $i_1 = i_2 = i/2$. From table 2.3 $i = 74$, AOQL = 2.9% and AFI = 0.2740. The required MLP-3(G) plan is (74,37,37,1/3,1/6,1/12)
- (iii) For a given LQL of 2% and sampling rate $f = 1/3$ with $i_1 = i_2 = i$, one finds $i = 162$, AOQL = 0.38% and AFI = 0.5297 from table 1.4. The relevant plan is (162,162,162,1/3,1/6,1/12)

- (iv) For a given LQL of 20% and sampling rate $f = 1/15$ having $i_1 = i_2 = i$ from table 1.4, one gets $i = 22$, AOQL = 6.09% and AFI = 0.3657. The required tightened MLP-3(G) plan is (22,22,22,1/15,1/30,1/60)

Construction of tables 2.1 to 2.4

For tightened MLP-3(G) continuous sampling plan the operating characteristic (OC) function representing the average fraction of total production accepted on a sampling basis is given in (2.5).

From the OC function the parameter f is obtained as

$$f = \{(1 - P_a)q^i(1 + q^{i_1} + 2q^{i+i_1+i_2})\} / P_a(1 - q^i) \quad (2.7)$$

by assuming $f_1 = f, f_2 = f/2, f_3 = f/4$. This expresses f in terms of i, i_1, i_2 and P_a .

(2.5) and (2.7) can also be expressed in the form

$$P_a = A/(A + fB) \quad (2.8)$$

$$f = A(1 - P_a)/(BP_a) \quad (2.9)$$

$$\text{with } A = q^i(1 + q^{i_1} + 2q^{i+i_1+i_2})$$

$$\text{and } B = (1 - q^i)$$

In the construction of Tables 2.1 and 2.2 for generalized tightened three level continuous plan $i = 2i_1 = 2i_2$ is assumed.

The following **six step procedure** is used to construct the tables.

- (i) If the AQL condition is specified take $p = \text{AQL}$ and $P_a = 0.95$; if the LQL condition is specified take $p = \text{LQL}$ and $P_a = 0.95$
- (ii) Find f using (1.9) if the specified condition is AQL or LQL

- (iii) Take $n = \text{int}(1/f + 0.9999)$ if AQL is specified, $n = \text{int}(1/f)$ if LQL is specified
- (iv) Take the revised f as $f = 1/n$ (this makes f an integer)
- (v) Use search procedure and find $\text{AOQL}(p_L)$, which is the maximum of AOQ given in (2.3) with p takes values from 0 to 1 in steps of 0.0001
- (vi) For the desired AOQL, find i and f for which Difference = desired AOQL $-p_L$ is non-negative and minimum. These i and $1/f$ values are tabulated

The method presented for construction of tables may be adopted to accommodate

- lower ratios
- primary and secondary indices other than the tabulated values and
- values of i_1 and i_2 other than the tabulated value

Tables 2.3 and 2.4 are constructed for LQL values taken from consumer oriented plans of CSP-1 by Stephens (1981), and f values from MIL-STD-1235 C(1988) by assuming $i_1 = i_2 = i/2$ and $i_1 = i_2 = i$ with the following procedure.

- (i) The average fraction of total units accepted on a sampling basis, P_a is given in (2.5) for generalized tightened three level continuous sampling plan. By using the equation and following search procedure with i varying between (2, 2000), the values of i which is closest to the 10% probability of acceptance for a given LQL and f is determined.
- (ii) With the values of i and f , using search procedure AOQL, the maximum of AOQ given in (2.4) is computed for generalized tightened three level continuous sampling plan when p takes the values from 0 to 1 in steps of 0.0001.
- (iii) With the values of i , f , AOQL and p at which AOQL is attained, the value of average fraction inspected in the long run (AFI) is computed for generalized tightened three level continuous sampling plan using (2.3).

Table 2.1 Tightened MLP-3(G) plan indexed by AQL at $P_a = 0.95$ and AOQL/AQL

| <u>AOQL</u> | AQL IN PERCENT | | | | | | | | | | | | | | | | | | | |
|-------------|----------------|-------------|----------|-------------|----------|-------------|----------|-------------|----------|-------------|----------|-------------|----------|-------------|----------|-------------|----------|-------------|----------|-------------|
| | 0.065 | | 0.1 | | 0.15 | | 0.25 | | 0.4 | | 0.65 | | 1 | | 1.5 | | 2.5 | | 4 | |
| | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> | <i>i</i> | 1/ <i>f</i> |
| 3.2 | 288 | 2 | 188 | 2 | 126 | 2 | 76 | 2 | 48 | 2 | 30 | 2 | 20 | 2 | 12 | 2 | 8 | 2 | 6 | 2 |
| 3 | 308 | 2 | 200 | 2 | 134 | 2 | 80 | 2 | 50 | 2 | 32 | 2 | 20 | 2 | 14 | 2 | 8 | 2 | 6 | 2 |
| 2.8 | 330 | 2 | 214 | 2 | 144 | 2 | 86 | 2 | 54 | 2 | 34 | 2 | 22 | 2 | 14 | 2 | 8 | 2 | 6 | 2 |
| 2.6 | 356 | 2 | 230 | 2 | 154 | 2 | 92 | 2 | 58 | 2 | 36 | 2 | 30 | 3 | 20 | 3 | 12 | 3 | 6 | 2 |
| 2.4 | 496 | 3 | 322 | 3 | 214 | 3 | 130 | 3 | 62 | 2 | 50 | 3 | 32 | 3 | 22 | 3 | 16 | 4 | 6 | 2 |
| 2.2 | 540 | 3 | 410 | 4 | 234 | 3 | 140 | 3 | 102 | 4 | 64 | 4 | 42 | 4 | 28 | 4 | 16 | 4 | 10 | 4 |
| 2 | 776 | 5 | 504 | 5 | 336 | 5 | 202 | 5 | 126 | 5 | 78 | 5 | 50 | 5 | 30 | 4 | 20 | 5 | 14 | 6 |
| 1.9 | 888 | 6 | 530 | 5 | 384 | 6 | 212 | 5 | 132 | 5 | 88 | 6 | 58 | 6 | 38 | 6 | 22 | 5 | 14 | 6 |
| 1.8 | 1004 | 7 | 652 | 7 | 434 | 7 | 224 | 5 | 152 | 6 | 100 | 7 | 66 | 7 | 46 | 8 | 22 | 5 | 16 | 7 |
| 1.65 | 1214 | 9 | 856 | 11 | 548 | 10 | 316 | 9 | 206 | 10 | 126 | 10 | 86 | 11 | 56 | 11 | 32 | 10 | 18 | 8 |
| 1.6 | 1442 | 13 | 910 | 12 | 606 | 12 | 374 | 13 | 220 | 11 | 144 | 13 | 90 | 12 | 58 | 11 | 36 | 12 | 22 | 12 |
| 1.55 | 1528 | 14 | 1018 | 15 | 694 | 16 | 416 | 16 | 254 | 15 | 152 | 14 | 96 | 13 | 66 | 14 | 40 | 15 | 24 | 14 |
| 1.5 | 1690 | 17 | 1142 | 19 | 748 | 18 | 456 | 19 | 274 | 17 | 178 | 20 | 114 | 19 | 74 | 18 | 42 | 16 | 26 | 16 |
| 1.45 | 1962 | 24 | 1258 | 23 | 850 | 24 | 510 | 24 | 322 | 25 | 198 | 25 | 128 | 24 | 84 | 24 | 50 | 24 | 30 | 22 |
| 1.4 | 2224 | 32 | 1432 | 31 | 954 | 31 | 572 | 31 | 364 | 33 | 226 | 34 | 166 | 46 | 96 | 32 | 56 | 31 | 34 | 29 |
| 1.35 | 2564 | 46 | 1666 | 46 | 954 | 31 | 572 | 31 | 408 | 43 | 252 | 44 | 168 | 47 | 108 | 43 | 64 | 42 | 38 | 38 |

Table 2.2 Tightened MLP-3(G) plan indexed by LQL at $P_a = 0.10$ and LQL/AOQL

| LQL — AOQL | LQL IN PERCENT | | | | | | | | | | | | | | | | | |
|------------------|----------------|-----|------|-----|------|-----|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|-----|
| | 0.15 | | 0.25 | | 0.4 | | 0.65 | | 1 | | 1.5 | | 4 | | 6.5 | | 10 | |
| | i | 1/f | i | 1/f | i | 1/f | i | 1/f | i | 1/f | i | 1/f | i | 1/f | i | 1/f | i | 1/f |
| 4.4 | 2766 | 6 | 1484 | 3 | 992 | 4 | 570 | 3 | 372 | 3 | 248 | 3 | 92 | 3 | 54 | 3 | 34 | 3 |
| 4.2 | 2942 | 8 | 1516 | 4 | 992 | 4 | 584 | 4 | 380 | 4 | 254 | 4 | 94 | 4 | 58 | 4 | 34 | 3 |
| 4 | 3140 | 11 | 1612 | 5 | 1008 | 5 | 620 | 5 | 404 | 5 | 268 | 5 | 100 | 5 | 58 | 4 | 36 | 4 |
| 3.9 | 3290 | 14 | 1710 | 6 | 1070 | 6 | 658 | 6 | 404 | 5 | 268 | 5 | 100 | 5 | 60 | 5 | 36 | 4 |
| 3.8 | 3430 | 17 | 1710 | 6 | 1070 | 6 | 658 | 6 | 416 | 6 | 278 | 6 | 104 | 6 | 60 | 5 | 38 | 5 |
| 3.7 | 3494 | 19 | 1738 | 7 | 1086 | 7 | 668 | 7 | 434 | 7 | 290 | 7 | 108 | 7 | 62 | 6 | 40 | 6 |
| 3.6 | 3588 | 22 | 1788 | 8 | 118 | 8 | 688 | 8 | 448 | 8 | 298 | 8 | 108 | 7 | 62 | 6 | 40 | 6 |
| 3.5 | 3700 | 26 | 1822 | 9 | 1140 | 9 | 720 | 9 | 456 | 9 | 304 | 9 | 110 | 8 | 66 | 8 | 40 | 6 |
| 3.4 | 3756 | 29 | 1912 | 11 | 1154 | 10 | 710 | 10 | 462 | 10 | 308 | 10 | 112 | 9 | 68 | 9 | 42 | 8 |
| 3.3 | 3908 | 36 | 1922 | 12 | 1202 | 12 | 740 | 12 | 480 | 12 | 320 | 12 | 116 | 11 | 70 | 10 | 44 | 10 |
| 3.2 | 4000 | 42 | 2022 | 15 | 1234 | 14 | 758 | 14 | 494 | 14 | 328 | 14 | 120 | 13 | 72 | 12 | 46 | 12 |
| 3 | 4264 | 63 | 2158 | 22 | 1330 | 21 | 818 | 21 | 532 | 21 | 354 | 21 | 130 | 20 | 78 | 19 | 48 | 15 |
| 2.9 | 4390 | 77 | 2224 | 27 | 1392 | 27 | 856 | 27 | 556 | 27 | 366 | 26 | 134 | 24 | 80 | 22 | 50 | 19 |
| 2.8 | 4538 | 97 | 2344 | 36 | 1456 | 35 | 888 | 34 | 576 | 34 | 384 | 34 | 140 | 31 | 84 | 29 | 52 | 24 |
| 2.7 | 4696 | 124 | 2430 | 46 | 1518 | 46 | 934 | 46 | 604 | 45 | 402 | 45 | 146 | 40 | 88 | 38 | 54 | 30 |
| 2.6 | 4880 | 164 | 2552 | 63 | 1596 | 63 | 978 | 62 | 634 | 62 | 420 | 60 | 154 | 56 | 92 | 51 | 58 | 47 |

Table 2.3 Tightened MLP-3(G) plan indexed by LQL at $P_a=0.10$ and f for $i_1 = i_2 = i/2$

| f | | LQL | | | | | | | | | | | | |
|------|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | 0.005 | 0.008 | 0.013 | 0.020 | 0.032 | 0.050 | 0.065 | 0.080 | 0.100 | 0.125 | 0.150 | 0.200 | 0.320 |
| 1/2 | i | 600 | 380 | 234 | 152 | 94 | 60 | 46 | 38 | 30 | 24 | 20 | 14 | 8 |
| | AOQL | 0.007 | 0.0012 | 0.0020 | 0.0030 | 0.0049 | 0.0077 | 0.0099 | 0.0120 | 0.0151 | 0.0189 | 0.0225 | 0.0318 | 0.5393 |
| | AFI | 0.5018 | 0.4789 | 0.4909 | 0.5093 | 0.5128 | 0.5292 | 0.5238 | 0.5150 | 0.5226 | 0.5131 | 0.5156 | 0.5121 | 0.5003 |
| 1/3 | i | 680 | 426 | 264 | 170 | 106 | 68 | 52 | 42 | 34 | 26 | 22 | 16 | 8 |
| | AOQL | 0.009 | 0.0015 | 0.0024 | 0.0038 | 0.0061 | 0.0094 | 0.0123 | 0.0152 | 0.0187 | 0.0243 | 0.0285 | 0.0388 | 0.0713 |
| | AFI | 0.4354 | 0.4064 | 0.4219 | 0.4983 | 0.4831 | 0.4648 | 0.4768 | 0.4682 | 0.4704 | 0.4648 | 0.4438 | 0.4632 | 0.4543 |
| 1/4 | i | 730 | 456 | 280 | 182 | 114 | 72 | 56 | 44 | 36 | 28 | 24 | 16 | 10 |
| | AOQL | 0.0010 | 0.0017 | 0.0028 | 0.0043 | 0.0068 | 0.0108 | 0.0138 | 0.0175 | 0.0213 | 0.0273 | 0.0317 | 0.0468 | 0.0728 |
| | AFI | 0.3887 | 0.5135 | 0.4649 | 0.4513 | 0.4282 | 0.4363 | 0.4325 | 0.4403 | 0.4426 | 0.4404 | 0.402 | 0.4287 | 0.4243 |
| 1/5 | i | 770 | 482 | 296 | 192 | 120 | 76 | 58 | 48 | 38 | 30 | 24 | 18 | 10 |
| | AOQL | 0.0011 | 0.0018 | 0.0030 | 0.0047 | 0.0074 | 0.0117 | 0.0152 | 0.0184 | 0.0231 | 0.0291 | 0.0361 | 0.0476 | 0.0826 |
| | AFI | 0.3559 | 0.4405 | 0.3904 | 0.3763 | 0.4027 | 0.3849 | 0.3772 | 0.3844 | 0.3904 | 0.3861 | 0.3870 | 0.3849 | 0.3761 |
| 1/7 | i | 832 | 520 | 320 | 208 | 130 | 82 | 64 | 52 | 40 | 32 | 26 | 20 | 12 |
| | AOQL | 0.0012 | 0.0020 | 0.0033 | 0.0051 | 0.0082 | 0.0129 | 0.0165 | 0.0203 | 0.0262 | 0.0326 | 0.0398 | 0.0512 | 0.0828 |
| | AFI | 0.3115 | 0.4405 | 0.3904 | 0.3763 | 0.4027 | 0.3849 | 0.3772 | 0.3844 | 0.3904 | 0.3861 | 0.3870 | 0.3849 | 0.3761 |
| 1/10 | i | 900 | 562 | 346 | 221 | 140 | 88 | 68 | 56 | 44 | 34 | 28 | 20 | 12 |
| | AOQL | 0.0014 | 0.0022 | 0.0036 | 0.0054 | 0.0090 | 0.0142 | 0.0183 | 0.0222 | 0.0281 | 0.0362 | 0.0437 | 0.0603 | 0.0971 |
| | AFI | 0.2703 | 0.3987 | 0.3481 | 0.4673 | 0.3579 | 0.3643 | 0.3654 | 0.3566 | 0.3586 | 0.3650 | 0.3598 | 0.354 | 0.3510 |

Continuation of Table 2.3

| f | | LQL | | | | | | | | | | | | |
|-------|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | 0.005 | 0.008 | 0.013 | 0.020 | 0.032 | 0.050 | 0.065 | 0.080 | 0.100 | 0.125 | 0.150 | 0.200 | 0.320 |
| 1/15 | i | 976 | 610 | 374 | 244 | 152 | 96 | 74 | 60 | 48 | 38 | 30 | 22 | 14 |
| | AOQL | 0.0015 | 0.0024 | 0.0040 | 0.0061 | 0.0098 | 0.0155 | 0.0199 | 0.0246 | 0.0306 | 0.0384 | 0.0482 | 0.0648 | 0.0989 |
| | AFI | 0.2280 | 0.3543 | 0.3013 | 0.3564 | 0.3562 | 0.3503 | 0.3436 | 0.3381 | 0.3310 | 0.3283 | 0.3338 | 0.3259 | 0.3166 |
| 1/25 | i | 1074 | 670 | 412 | 268 | 166 | 106 | 82 | 66 | 52 | 40 | 34 | 24 | 14 |
| | AOQL | 0.0016 | 0.0027 | 0.0044 | 0.0067 | 0.0108 | 0.0169 | 0.0217 | 0.0269 | 0.0340 | 0.0439 | 0.0513 | 0.0715 | 0.1179 |
| | AFI | 0.4486 | 0.3017 | 0.3562 | 0.3037 | 0.2970 | 0.3010 | 0.2970 | 0.3039 | 0.3110 | 0.2917 | 0.3022 | 0.2959 | 0.2867 |
| 1/50 | i | 1208 | 754 | 464 | 300 | 188 | 118 | 92 | 74 | 58 | 46 | 38 | 28 | 16 |
| | AOQL | 0.0010 | 0.0029 | 0.0048 | 0.0074 | 0.0112 | 0.0189 | 0.0242 | 0.0299 | 0.0379 | 0.0475 | 0.0571 | 0.0764 | 0.1283 |
| | AFI | 0.3866 | 0.2403 | 0.2935 | 0.3076 | 0.4317 | 0.2866 | 0.2756 | 0.2740 | 0.2842 | 0.2662 | 0.2632 | 0.2654 | 0.2565 |
| 1/100 | i | 1346 | 840 | 516 | 336 | 208 | 132 | 102 | 82 | 64 | 52 | 42 | 30 | 18 |
| | AOQL | 0.0020 | 0.0032 | 0.0053 | 0.0081 | 0.0130 | 0.0204 | 0.0263 | 0.326 | 0.0415 | 0.0508 | 0.0624 | 0.0859 | 0.1377 |
| | AFI | 0.3302 | 0.1888 | 0.2371 | 0.2560 | 0.2256 | 0.2348 | 0.2390 | 0.2504 | 0.2363 | 0.2447 | 0.2401 | 0.2317 | 0.2267 |
| 1/200 | i | 1480 | 924 | 568 | 368 | 228 | 146 | 112 | 90 | 72 | 56 | 46 | 34 | 20 |
| | AOQL | 0.0021 | 0.0034 | 0.0056 | 0.0087 | 0.0140 | 0.0218 | 0.0283 | 0.0351 | 0.0436 | 0.0557 | 0.0472 | 0.0896 | 0.1461 |
| | AFI | 0.2747 | 0.3156 | 0.1879 | 0.2016 | 0.2143 | 0.2152 | 0.2294 | 0.2147 | 0.2157 | 0.2111 | 0.213 | 0.2095 | 0.2053 |

Table 2.4 Tightened MLP-3(G) plan indexed by indexed by LQL at $P_a = 0.10$ and $f i_1 = i_2 = i$

| f | | LQL | | | | | | | | | | | |
|------|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | 0.005 | 0.008 | 0.013 | 0.020 | 0.032 | 0.050 | 0.065 | 0.080 | 0.100 | 0.125 | 0.150 | 0.200 |
| 1/2 | i | 580 | 362 | 222 | 144 | 90 | 57 | 44 | 35 | 28 | 22 | 18 | 13 |
| | AOQL | 0.0008 | 0.0012 | 0.0020 | 0.0031 | 0.0050 | 0.0079 | 0.0102 | 0.0128 | 0.0160 | 0.0202 | 0.0246 | 0.0337 |
| | AFI | 0.6127 | 0.5823 | 0.5929 | 0.5492 | 0.5421 | 0.5339 | 0.5346 | 0.5414 | 0.5430 | 0.5396 | 0.5352 | 0.5376 |
| 1/3 | i | 653 | 407 | 250 | 162 | 101 | 64 | 49 | 40 | 32 | 25 | 21 | 15 |
| | AOQL | 0.0009 | 0.0015 | 0.0024 | 0.0038 | 0.0060 | 0.0095 | 0.0123 | 0.1510 | 0.0188 | 0.0239 | 0.0283 | 0.0892 |
| | AFI | 0.5320 | 0.4995 | 0.5112 | 0.5297 | 0.4981 | 0.5014 | 0.5064 | 0.4977 | 0.5060 | 0.5022 | 0.4945 | 0.4976 |
| 1/4 | i | 706 | 441 | 271 | 176 | 109 | 69 | 53 | 43 | 34 | 27 | 22 | 16 |
| | AOQL | 0.0010 | 0.0016 | 0.0027 | 0.0042 | 0.0067 | 0.0105 | 0.0137 | 0.0168 | 0.212 | 0.0265 | 0.0323 | 0.0439 |
| | AFI | 0.4803 | 0.4483 | 0.4605 | 0.4804 | 0.4849 | 0.4733 | 0.4741 | 0.4749 | 0.4838 | 0.4802 | 0.4784 | 0.4706 |
| 1/5 | i | 748 | 467 | 287 | 186 | 116 | 174 | 56 | 45 | 36 | 29 | 24 | 17 |
| | AOQL | 0.0011 | 0.0018 | 0.0029 | 0.0045 | 0.0071 | 0.0112 | 0.0147 | 0.0183 | 0.0227 | 0.0281 | 0.0337 | 0.0470 |
| | AFI | 0.4430 | 0.4108 | 0.4230 | 0.4419 | 0.4500 | 0.4677 | 0.4744 | 0.4473 | 0.4590 | 0.4604 | 0.4561 | 0.4347 |
| 1/7 | i | 813 | 507 | 312 | 202 | 126 | 80 | 61 | 49 | 39 | 31 | 26 | 19 |
| | AOQL | 0.0012 | 0.0019 | 0.0032 | 0.0049 | 0.0079 | 0.0123 | 0.0161 | 0.0200 | 0.0250 | 0.0313 | 0.0371 | 0.0502 |
| | AFI | 0.3915 | 0.4560 | 0.4692 | 0.4533 | 0.4379 | 0.4389 | 0.4437 | 0.4284 | 0.4312 | 0.4308 | 0.4289 | 0.4297 |
| 1/10 | i | 882 | 551 | 338 | 219 | 136 | 87 | 66 | 53 | 42 | 34 | 28 | 20 |
| | AOQL | 0.0013 | 0.0021 | 0.0035 | 0.0053 | 0.0086 | 0.0134 | 0.177 | 0.0219 | 0.0275 | 0.338 | 0.0408 | 0.0563 |
| | AFI | 0.3410 | 0.4681 | 0.4178 | 0.4022 | 0.440 | 0.4150 | 0.4110 | 0.4075 | 0.4143 | 0.4067 | 0.3999 | 0.4006 |

Continuation of Table 2.4

| f | | LQL | | | | | | | | | | | |
|-------|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | | 0.005 | 0.008 | 0.013 | 0.020 | 0.032 | 0.050 | 0.065 | 0.080 | 0.100 | 0.125 | 0.150 | 0.200 |
| 1/15 | I | 961 | 600 | 368 | 239 | 149 | 94 | 72 | 58 | 46 | 37 | 30 | 22 |
| | AOQL | 0.0014 | 0.0023 | 0.0038 | 0.0058 | 0.0094 | 0.0148 | 0.0193 | 0.0238 | 0.0299 | 0.0370 | 0.0453 | 0.0609 |
| | AFI | 0.2893 | 0.4153 | 0.3642 | 0.3499 | 0.3741 | 0.3826 | 0.3782 | 0.3728 | 0.3770 | 0.3736 | 0.3714 | 0.3657 |
| 1/25 | I | 1062 | 663 | 407 | 264 | 164 | 104 | 80 | 64 | 51 | 40 | 33 | 24 |
| | AOQL | 0.0015 | 0.0026 | 0.0042 | 0.0065 | 0.0104 | 0.0163 | 0.0211 | 0.0263 | 0.0328 | 0.0415 | 0.0500 | 0.0678 |
| | AFI | 0.2323 | 0.3541 | 0.3038 | 0.3531 | 0.3511 | 0.3481 | 0.3404 | 0.3431 | 0.3439 | 0.3407 | 0.3333 | 0.3356 |
| 1/50 | I | 11999 | 748 | 460 | 298 | 185 | 118 | 90 | 73 | 58 | 45 | 37 | 27 |
| | AOQL | 0.0017 | 0.0029 | 0.0047 | 0.0072 | 0.0116 | 0.0181 | 0.0236 | 0.0290 | 0.0364 | 0.0465 | 0.0561 | 0.0758 |
| | AFI | 0.4210 | 0.2792 | 0.3311 | 0.2785 | 0.3175 | 0.3032 | 0.3043 | 0.2917 | 0.3007 | 0.2953 | 0.2981 | 0.2916 |
| 1/100 | I | 1337 | 834 | 512 | 332 | 206 | 131 | 100 | 81 | 64 | 50 | 42 | 30 |
| | AOQL | 0.0019 | 0.0031 | 0.0051 | 0.0079 | 0.0127 | 0.0199 | 0.0260 | 0.0320 | 0.0402 | 0.0511 | 0.0604 | 0.0832 |
| | AFI | 0.3550 | 0.2158 | 0.2636 | 0.2789 | 0.2509 | 0.2614 | 0.2568 | 0.2563 | 0.2684 | 0.2592 | 0.2629 | 0.2576 |
| 1/200 | I | 1475 | 921 | 565 | 366 | 228 | 145 | 110 | 89 | 71 | 56 | 46 | 33 |
| | AOQL | 0.0021 | 0.0033 | 0.0055 | 0.0086 | 0.0137 | 0.0215 | 0.0282 | 0.0347 | 0.0432 | 0.0544 | 0.0657 | 0.0900 |
| | AFI | 0.2944 | 0.1641 | 0.2068 | 0.2204 | 0.2371 | 0.2328 | 0.2382 | 0.2292 | 0.2281 | 0.2232 | 0.2271 | 0.2240 |