

Avinashilingam Institute for Home Science and Higher Education for Women
Coimbatore-641 043
Master's Degree Examination –November 2017
I Semester

Class : I PG
Major: Mathematics

Max. Marks: 60
Time: 3 Hours

17MMAC05 Ordinary Differential Equations

Part A

10x½=5

Choose the correct Answer

1. In Non-homogeneous equation $L(y) = \dots$
(a) 0 (b) $b(x)$ (c) 1 (d) None of these
2. The roots of the auxiliary equation of the equation $y'' - 4y = 0$ are
(a) 1, 2 (b) 2, -2 (c) 2, -1 (d) None of these
3. The two μ_i solutions of the equation $y'' + y = 0$ are
(a) e^{ix}, e^{-ix} (b) e^x, e^{-x} (c) e^x, e^{ix} (d) None of these
4. The second independent solution of the equation $x^2 y'' - xy' + y = 0$, $\phi_2(x) = x$ is
(a) x (b) $\log x$ (c) $x \log x$ (d) None of these
5. The analytic function at x_0 of g is of the form
(a) $\sum_{k=0}^{\infty} (x-x_0)^k$ (b) $\sum_{k=0}^{\infty} C_k(x-x_0)^k$ (c) $\sum_{k=0}^{\infty} C_k(x-x_0)^k$ (d) None of these
6. $(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$ is called as
(a) Bessel equation (b) Euler equation (c) Legendre equation (d) None of these
7. The Legendre polynomial $P_1(x)$ is
(a) 1 (b) x (c) 0 (d) None of these
8. The solution of the equation $x^2 y'' + 2xy' - 6y = 0$ is
(a) $C_1 x^{-3} + C_2 x^2$ (b) $C_1 x^3 + C_2 x^2$ (c) $C_1 x + C_2 x^2$ (d) None of these
9. The regular singular point of the equation $x^2 y'' + (x+x^2)y' - y = 0$ is
(a) 0 (b) 1 (c) -1 (d) None of these
10. The value of the first approximation of the equation $y' = xy$ with $y(0) = 1$ is
(a) $X^2/2$ (b) $1+x$ (c) $1+x^2/2$ (d) None of these

PART -B

(5X4=20 MARKS)

Answer the following

Each answer should not exceed 200 words or one page.

11.a. Find all solutions of the equation $y^{11}-4y^1+5y = 0$.

(Or)

b. Consider the equation $y^{11}-4y^1 = 0$

(a) compute three linearly independent solutions

(b) compute the Wronskian of Solutions.

12.a. Find the particular solution of the equation $y^{11}+4y = \sin 2x$ by annihilator method.

(Or)

b. Find the second independent solution of the equation $y^{11}-2/x^2 y=0$, $\Phi_1(x)=x^2$.

13.a. Find all solutions of the equation $y^{11}-2/x^2 y=x$.

(Or)

b. Show that $\int_1^x P_n^z(x) dx = \frac{2}{2n+1}$

14.a. Find the solution of the equation $x^2y^{11}+xy^1+4y=1$.

(Or)

b. Find the singular point of the equation $x^2y^{11}+(x+x^2)y^1-y=0$.

15. a.(i) Define Gamma function

(ii) Prove that $\Gamma(z+1) = z \Gamma(z)$

(Or)

b. Show that $K_0^1(x) = -k_1(x)$.

PART -C

(5X7=35 MARKS)

Answer the following

Each answer should not exceed 600 words or three pages.

16.a. Prove that $\| \phi(x_0) \| e^{-k/x-x_0} \leq \| \phi(x) \| \leq \| \phi(x_0) \| e^{k/x-x_0}$ where ϕ be any solution

of $L(y) = y^{11} + a_1 y^1 + a_2 y = 0$ and $\| \phi(x) \| = [|\phi(x)|^2 + |\phi^1(x)|^2]^{1/2}$, $k=1+|a_1|+|a_2|$.

(Or)

b. If ϕ_1, ϕ_2 are two solutions of $L(y)=0$ on an interval I containing a point x_0 , then prove that

$$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)}$$

$$W(\phi_1, \phi_2)(x_0)$$

17.a. Find the particular solution of $y^{11}+y^{11}+y^1+y=0$

(Or)

b. Consider the equation $y^5 - y^4 - y^1 + y = 0$

(i) compute five linearly independent solutions

(ii) compute the wronskian of the solution.

18.a. If ϕ_1, \dots, ϕ_n are n solutions of $L(y)=0$ on an interval I , then prove that they are linearly independent if and only if $w(\phi_1, \phi_2, \dots, \phi_n)(x) \neq 0$ for all x in I .
(Or)

b. Derive n -th Legendre polynomial.

19.a. Find all solutions of the equation $x^2y^{111} + 2x^2y^1 - xy^1 + y = 0$
(Or)

b. Derive Bessel function of zero order of the first kind.

20. a. Determine the equation $(x^2 + xy) dx + xy dy = 0$ are exact and solve it.
(Or)

b. Prove that a function ϕ is a solution of the I.V.P $y' = f(x, y)$, $y(x_0) = y_0$ on I if and only if it is a solution of $y = y_0 + \int_{x_0}^x f(t, y) dt$.
