CHAPTER I

CHAPTER I
SEMI STAR GENERALIZED CLOSED SETS IN BITOPOLOGICAL SPACES

In this chapter the notion of $\tau_1\tau_2$-s^*g closed sets, pairwise s^*g continuous functions, pairwise S^*GO- connected space , pairwise S^*GO-compact space due to Kannan, Narasimhan, Chandrasekhara Rao[17] and $(\tau_1 , \tau_2)^*$ - s^*g closed (open) sets, $(1,2)^*$ -s^*g continuous functions and pairwise s^*g T_S –spaces due to Kannan, Narasimhan, Chandrasekhara Rao and Sundararaman [18] are discussed. Properties, characterizations and applications are analysed.

SECTION 1.1
PRELIMINARIES

Definition: 1.1.1

A set A of bitopological space (X, τ_1 , τ_2) is called **τ_1 τ_2- open** if $A \subseteq \tau_1 \subseteq \tau_2$

Definition: 1.1.2

A set A of bitopological space (X, τ_1 , τ_2) is called **τ_1 τ_2- closed** if A^C is τ_1 τ_2- open.

Definition: 1.1.3

A set A of a bitopological space (X, τ_1 ,τ_2) is called **τ_1 τ_2- semi open** if there exists an τ_1 - open set U such that $U \subseteq A \subseteq \tau_2$ -cl(U).

Definition: 1.1.4

A set A of a bitopological space (X, τ_1 ,τ_2) is called **τ_1 τ_2- semi closed** if there exists an τ_1 – closed set F such that τ_2 -int(F) $\subseteq A \subseteq F$.

Definition: 1.1.5

A set A of a bitopological space (X, τ_1, τ_2) is called **τ_1 τ_2- generalized closed** (τ_1 τ_2-g closed) if τ_2 -cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_1 -open in X.

Definition: 1.1.6

A set A of a bitopological space (X, τ_1, τ_2) is called **τ_1 τ_2- generalized open** (τ_1 τ_2-g open) if X-A is τ_1 τ_2-g closed.

Definition: 1.1.7

A set A of a bitopological space (X, τ_1, τ_2) is called **τ_1 τ_2- semi generalized closed** (τ_1 τ_2-sg closed) if τ_2 -scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_1 - semi open in X.

Definition:1.1.8

A set A of a bitopological space (X, τ_1, τ_2) is called **τ_1 τ_2- semi generalized open** (τ_1 τ_2-sg open) if X-A is τ_1 τ_2-sg closed.

Definition:1.1.9

A set A of a bitopological space (X, τ_1, τ_2) is called **τ_1 τ_2- generalized semi closed** (τ_1 τ_2-gs closed) if τ_2 -scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_1 - open in X.

Definition:1.1.10

A set A of a bitopological space (X, τ_1, τ_2) is called **τ_1 τ_2- generalized semi open** (τ_1 τ_2-gs open) if X-A is τ_1 τ_2-gs closed.

Definition: 1.1.11

Let A be a subset of a bitopological space (X, τ_1, τ_2) is called **τ_1 τ_2- closure(A)** (τ_1 τ_2-cl(A)) is defined as the intersection of all τ_1 τ_2- closed sets containing A.

Definition: 1.1.12

Let A be a subset of a bitopological space (X, τ_1, τ_2) is called **$\tau_1 \tau_2$- interior(A)** ($\tau_1 \tau_2$-int(A)) is defined as the union of all $\tau_1 \tau_2$- open sets contained in A.

Definition: 1.1.13

A set A of a bitopological space (X, τ_1, τ_2) is called **$(\tau_1, \tau_2)^*$ -semi open** if $A \subseteq \tau_1 \tau_2$ -cl[$\tau_1 \tau_2$ -int(A)].

Definition: 1.1.14

A set A of a bitopological space (X, τ_1, τ_2) is called **$(\tau_1, \tau_2)^*$ -semi closed** if X – A is $(\tau_1, \tau_2)^*$ -semi open.

Definition: 1.1.15

A set A of a bitopological space (X, τ_1, τ_2) is called **$(\tau_1, \tau_2)^*$ -generalized closed** {$(\tau_1, \tau_2)^*$ -g closed } if $\tau_1 \tau_2$ -cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_1 \tau_2$ -open in X.

Definition: 1.1.16

A set A of a bitopological space (X, τ_1, τ_2) is called **$(\tau_1, \tau_2)^*$ -generalized open** {$(\tau_1, \tau_2)^*$ -g open } if X – A is $(\tau_1, \tau_2)^*$ -g closed.

Definition: 1.1.17

A set A of a bitopological space (X, τ_1, τ_2) is called **$(\tau_1, \tau_2)^*$ -semi generalized closed** {$(\tau_1, \tau_2)^*$ -sg closed } if $(\tau_1, \tau_2)^*$ -scl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$ -semi open in X.

Definition: 1.1.18

A set A of a bitopological space (X, τ_1, τ_2) is called **$(\tau_1, \tau_2)^*$ -semi generalized open** {$(\tau_1, \tau_2)^*$ -sg open} if X – A is $(\tau_1, \tau_2)^*$ -sg closed.

Definition: 1.1.19

A set A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)^*$ **-generalized semi closed** $\{(\tau_1, \tau_2)^*$ -gs closed $\}$ if $(\tau_1, \tau_2)^*$ -scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_1 \tau_2$ -open in X.

Definition: 1.1.20

A set A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)^*$ **-generalized semi open** $\{(\tau_1, \tau_2)^*$ -gs open$\}$ if X – A is $(\tau_1, \tau_2)^*$ -gs closed.

Definition: 1.1.21

Let A be a subset of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)^*$ **-semi closure(A)** $\{ (\tau_1, \tau_2)^*$ -scl(A) $\}$ is defined as the intersection of all $(\tau_1, \tau_2)^*$ -semi closed sets containing A.

Definition: 1.1.22

Let A be a subset of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)^*$ **-semi interior(A)** $\{ (\tau_1, \tau_2)^*$ -sint(A)$\}$ is defined as the union of all $(\tau_1, \tau_2)^*$ -semi open sets contained in A.

Definition: 1.1.23

A map f : X →Y is called **(1, 2)* -continuous** if the inverse image of $\sigma_1 \sigma_2$ - closed set in Y is $\tau_1 \tau_2$ -closed in X.

Definition: 1.1.24

A map f : X →Y is called **(1, 2)* -semi continuous** if the inverse image of $\sigma_1 \sigma_2$ - closed set in Y is $(\tau_1, \tau_2)^*$ - semi closed in X.

Definition: 1.1.25

A map f : X → Y is called **(1, 2)* -g continuous** if the inverse image of $\sigma_1\sigma_2$- closed set in Y is $(\tau_1, \tau_2)^*$ -g closed in X.

Definition: 1.1.26

A map $f : X \rightarrow Y$ is called **(1, 2)* -sg continuous** if the inverse image of $\sigma_1 \sigma_2$ - closed set in Y is $(\tau_1, \tau_2)^*$ -sg closed in X.

Definition: 1.1.27

A map $f : X \rightarrow Y$ is called **(1, 2)* -gs continuous** if the inverse image of $\sigma_1\sigma_2$- closed set in Y is $(\tau_1, \tau_2)^*$ -gs closed in X.

Definition: 1.1.28

A map $f : X \rightarrow Y$ is called **(1, 2)*-g closed** if the image of $\tau_1 \tau_2$ - closed set in X is $(\sigma_1, \sigma_2)^*$ -g closed in Y .

Definition: 1.1.29

A map $f : X \rightarrow Y$ is called **(1, 2)* -sg closed** if the image of $\tau_1\tau_2$- closed set in X is $(\sigma_1, \sigma_2)^*$ -sg closed in Y.

Definition: 1.1.30

A map $f : X \rightarrow Y$ is called **(1, 2)* -gs closed** if the image of $\tau_1\tau_2$- closed set in X is $(\sigma_1, \sigma_2)^*$ -gs closed in Y .

Definition: 1.1.31

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is **pairwise g-continuous** if $f^{-1}(U)$ is $\tau_i \tau_j$ -g closed for each σ_j -closed set U in Y , $i \neq j$ and $i, j = 1, 2$.

Definition: 1.1.32

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is **pairwise sg-continuous** if $f^{-1}(U)$ is $\tau_i \tau_j$ -sg closed for each σ_j -closed set U in Y , $i \neq j$ and $i, j = 1, 2$.

Definition: 1.1.33

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is **pairwise gs-continuous** if $f^{-1}(U)$ is $\tau_i \tau_j$ -gs closed for each σ_j -closed set U in Y, $i \neq j$ and $i, j = 1, 2$.

Definition: 1.1.34

A map f : X → Y is called **(1, 2) * -sgc irresolute** if the inverse image of $(\sigma_1, \sigma_2)^*$ -sg closed set in Y is $(\tau_1, \tau_2)^*$ -sg closed in X.

Definition: 1.1.35

A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is **pairwise g-irresolute** if $f^{-1}(U)$ is $\tau_i \tau_j$ -g closed for each $\sigma_i \sigma_j$ –g closed set U in Y , $i \neq j$ and $i, j = 1, 2$.

Definition: 1.1.36

A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is **pairwise sg- irresolute** if $f^{-1}(U)$ is $\tau_i \tau_j$ -sg closed for each $\sigma_i \sigma_j$ –sg closed set U in Y , $i \neq j$ and $i, j = 1, 2$.

Definition: 1.1.37

A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is **pairwise gs- irresolute** if $f^{-1}(U)$ is $\tau_i \tau_j$ -gs closed for each $\sigma_i \sigma_j$ -gs closed set U in Y , $i \neq j$ and $i, j = 1, 2$.

Definition: 1.1.38

A map f : X → Y is called **(1, 2) * -gc irresolute** if the inverse image of $(\sigma_1, \sigma_2)^*$ -g closed set in Y is $(\tau_1, \tau_2)^*$ -g closed in X.

Definition: 1.1.39

A map f : X → Y is called **(1, 2) * -gsc irresolute** if the inverse image of $(\sigma_1, \sigma_2)^*$ -gs closed set in Y is $(\tau_1, \tau_2)^*$ -gs closed in X.

Definition: 1.1.40

A space (X, τ) is called **$T_{1/2}$ space** if every g – closed set is closed.

Definition: 1.1.41

A space (X, τ_1, τ_2) is called **pairwise $T_{1/2}$ space** if every τ_1g – closed set is τ_2 closed, and τ_2g – closed set is τ_1 closed.

SECTION-1.2

$\tau_1\tau_2$ –S*G CLOSED SETS AND S*G CONTINUOUS FUNCTIONS

In this section $\tau_1\tau_2$ -semi star generalized closed ($\tau_1\tau_2$-s^*g closed) and s^*g-continuous functions are studied. Properties, characterizations and applications are analysed.

Definition: 1.2.1

A set A of a bitopological space (X, τ_1, τ_2) is called **τ_1 τ_2 -semi star generalized closed (τ_1 τ_2 –s^*g closed)** if τ_2 -cl(A) $\subseteq$ U whenever A $\subseteq$U and U is τ_1 -semi open in X.

Example: 1.2.2

Let X = {a, b, c}, τ_1 = {φ, X, {a}, {b, c}}, τ_2 = {φ, X, {a}, {b},{a, b}}. Then {a, b} is τ_1 τ_2 – s^*g closed and {a} is not τ_1 τ_2 – s^*g closed.

Proposition:1.2.3

(i) Every τ_i τ_j–s^*g closed set is τ_i τ_j -g closed, i $\neq$ j and i, j = 1, 2.

(ii) Every τ_i τ_j -s^*g closed set is τ_i τ_j -sg closed, i $\neq$ j and i, j = 1, 2.

(iii) Every τ_i τ_j–s^*g closed set is τ_i τ_j -gs closed, i $\neq$ j and i, j = 1, 2.

But none of the above is reversible.

Proposition: 1.2.4

The arbitrary union of τ_1 τ_2 –s^*g closed sets A_i , i$\in$ I in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$ –s^*g closed if the family {A_i , i$\in$I} is τ_2 –locally finite.

Proof:

Let {A_i , i$\in$I} be τ_2 -locally finite and A_i is $\tau_1\tau_2$ –s^*g closed in X for each i$\in$I.

To prove:

Y A_i is $\tau_1\tau_2$ –s^*g closed in X

Let Y A_i $\subseteq$U and U is τ_1 -semi open in X.

Then, A_i $\subseteq$ U and U is τ_1 –semi open in X for each i.

Since A_i is $\tau_1\tau_2$ –s^*g closed in X for each i$\in$I,τ_2 -cl(A_i)$\subseteq$U .

Hence, Y [τ_2 -cl(A_i)] $\subseteq$U .

Since the family {A_i , i$\in$I} is τ_2 -locally finite, τ_2 -cl[Y (A_i)] = Y [τ_2 -cl(A_i)]$\subseteq$U .

Therefore, Y A_i is $\tau_1\tau_2$ –s^*g closed in X.

Proposition: 1.2.5

The arbitrary intersection of $\tau_1\tau_2$ –s^*g open sets A_i , i$\in$I in a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$ –s*g open if the family {A_i^C , i$\in$I} is τ_2 –locally finite.

Proof:

Let {A_i^C, i$\in$I} be a τ_2 -locally finite.

Since A_i is $\tau_1\tau_2$ –s^*g -open in X for each i$\in$I.

Then A_i^C is $\tau_1\tau_2$ –s^*g -closed in X for each i$\in$I.

As arbitrary union of $\tau_1\tau_2$ –s^*g closed sets is $\tau_1\tau_2$ –s^*g closed, $\cup$[A_i^C] is $\tau_1\tau_2$ –s^*g -closed in X.

Hence, {$\cap$(A_i)}^{C} is $\tau_1\tau_2$ –s^*g -closed in X.

Therefore, $\cap$(A_i) is $\tau_1\tau_2$ –s^*g -open in X.

Definition:1.2.6

A function is **pairwise s*g -continuous** if f^{-1}(U) is $\tau_i\tau_j$ – s^*g closed for each σ_j -closed set U in Y , i $\neq$ j and i, j = 1, 2.

Example: 1.2.7

Let X = Y = {a, b, c, d}, τ_1 = {φ, X, {a}, {a, b}}, τ_2 = {φ, X, {a, b}, {a, b, c}, {a, b, d}}, σ_1 = {φ, Y, {a}}, σ_2 = {φ, Y, {a, b}, {a, b, c}}.

Let f : (X, τ_1 , τ_2) →(Y, σ_1 , σ_2) be a function defined by f (φ) = φ, f (X) = Y , f (a) = {a, b, d}, f (b)= {c}, f (c) = {b}, f (d) = {d},f (a, b) = {a, c}, f (a, c) = {a, b}, f (a, d) = {b, c}, f (b, c) = {a, d}, f (b, d) = {a, b, c}, f (c, d) = {c, d}, f (a, b, c) = {b, d}, f (a, b, d) = {a},f(a,c,d) = {b,c,d},f(b,c,d) = {a,c,d}.Then f is pairwise s^*g-continuous.

Propoistion:1.2.8

Every pairwise continuous function is pairwise s^*g-continuous.

Remark: 1.2.9

The converse of the above theorem need not be true.

Example:1.2.10

In Example 1.2.7 {a} is σ_1 -open in Y . But $f^{-1}(a)$ = {a, b, d} is not τ_1 -open in X. Therefore, f is pairwise s^*g-continuous but not pairwise continuous.

Proposition: 1.2.11

(i) Every pairwise s^*g-continuous function is pairwise g-continuous.

(ii) Every pairwise s^*g- continuous function is pairwise sg-continuous.

(iii) Every pairwise s^*g- continuous function is pairwise gs-continuous.

But none of the above is reversible.

Example:1.2.12

(i) Let X = Y = {a, b, c, d},τ_1 = {φ, X, {a}, {a, b}}, τ_2 = {φ, X, {a, b}, {a, b, c}, {a, b, d}}, σ_1 = {φ, Y, {a}},σ_2 = {φ, Y, {a, b}, {a, b, c}}. Let f : (X, τ_1 , τ_2) → (Y, σ_1 , σ_2) be a function defined by f (φ) = φ, f (X) = Y , f (a) ={b}, f (b) = {a} f (c) = {a, b}, f (d) = {a, c, d}, f (a, b) = {c}, f (a, c) = {a, d},f (a, d) = {a, c}, f (b, c) = {b, d}, f (b, d) = {b, c}, f (c, d) = {c, d}, f (a, b, c) ={a, b, d}, f (a, b, d) = {a, b, c}, f (a, c, d) = {d}, f (b, c, d) = {b, c, d}.Then f is pairwise g-continuous but not pairwise s^*g-continuous.

Let X = Y = {a, b, c, d},τ_1 = {φ, X, {a}, {a, b}}, τ_2 = {φ, X, {a, b}, {a,b,c}, {a, b, d}}, σ_1 = {φ, Y, {a}},σ_2 = {φ, Y, {a, b}, {a, b, c}}. Let f : (X, τ_1 , τ_2) →(Y, σ_1 , σ_2) be a function defined by f (φ) = φ, f (X) = Y , f (a) = {b}, f (b) = {a},f (c) = {a, b}, f (d) = {d}, f (a, b) = {c}, f (a, c) = {a, d},f (a, d) = {a, c}, f (b, c) = {a, b, c}, f (b, d) = {b, c, d}, f (c, d) = {c, d}, f (a, b, c) ={b, c}, f (a, b, d) = {a, c, d}, f (a, c, d) = {b, c, d}, f (b, c, d) = {a, b, d}. Then f is both pairwise gs-continuous and pairwise sg-continuous but not pairwise s^*g-continuous.

Proposition: 1.2.13

The following are equivalent for a function f : (X, τ_1 , τ_2) → (Y, σ_1 , σ_2).

a) f is pairwise s^*g-continuous.

b) $f^{-1}(U)$ is $\tau_i \tau_j$ -s^*g open for each σ_i -open set U in Y , $i \neq j$ and i, j = 1, 2.

Proof:

(a)$\Rightarrow$(b):

Assume that f is pairwise s^*g-continuous.

To prove:

$f^{-1}(U)$ is $\tau_i \tau_j$ -s^*g open for each σ_i -open set U in Y , $i \neq j$ and i, j = 1, 2.

Let A be σ_j –open in Y .

Then A^C is σ_j -closed in Y.

Since f is pairwise s^*g-continuous, $f^{-1}(A^C)$ is $\tau_i \tau_j$ -s^*g closed in X, $i \neq j$ and i, j = 1,2.

(i.e.) ,$(f^{-1}(A))^C$ is $\tau_i \tau_j$ -s^*g closed in X,

Hence $f^{-1}(A)$ is $\tau_i \tau_j$ -s^*g open in X.

(b)$\Rightarrow$(a)

Assume that $f^{-1}(U)$ is $\tau_i \tau_j$-s^*g open for each σ_i -open set U in Y, $i \neq j$ and i, j = 1,2.

To prove:

f is pairwise s^*g-continuous

Let V be σ_j -closed in Y .

Then V^C is σ_j -open in Y .

By hypothesis, $f^{-1}(V^C)$ is $\tau_i \tau_j$ -s^*g open in X, $i \neq j$ and i, j = 1,2.

(i.e.), $(f^{-1}(V))^C$ is $\tau_i \tau_j$ -s^*g open in X.

Hence $f^{-1}(V)$ is $\tau_i \tau_j$ -s^*g closed in X

Therefore f is pairwise s^*g-continuous.

Definition:1.2.14

A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is **pairwise s^*g - irresolute** if $f^{-1}(U)$ is $\tau_i \tau_j$ - s^*g closed for each $\sigma_i \sigma_j$ - s^*g closed set U in Y , $i \neq j$ and i, j = 1, 2.

Example: 1.2.15

Let X = Y = {a, b, c, d},τ_1 = {φ, X, {a}, {a, b}}, τ_2 = {φ, X, {a, b}, {a, b, c}, {a, b, d}}, σ_1 = {φ, Y, {a}},σ_2 = {φ, Y, {a, b}, {a, b, c}}.

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function defined by $f(\varphi) = \varphi, f(X) = Y$, $f(a) = \{b\}$, $f(b) = \{a\}$, $f(c) = \{c\}$, $f(d) = \{d\}$, $f(a, b) = \{a, c\}$, $f(a, c) = \{a, b\}$, $f(a, d) = \{b, c\}$, $f(b, c) = \{a, d\}$, $f(b, d) = \{b, d\}$, $f(c, d) = \{c, d\}$, $f(a, b, c) = \{a, b, d\}$, $f(a, b, d) = \{a, b, c\}$, $f(a, c, d) = \{a, c, d\}$, $f(b, c, d) = \{b, c, d\}$. Then f is pairwise s^*g-irresolute.

Proposition: 1.2.16

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ be two functions. Then

(a) If f and g are pairwise s^*g-irresolute, then $g \circ f$ is pairwise s^*g- irresolute.

(b) If f is pairwise s^*g-irresolute and g is pairwise s^*g-continuous, then $g \circ f$ is pairwise s^*g-continuous.

(c) If f is pairwise g-irresolute and g is pairwise s^*g-continuous, then $g \circ f$ is pairwise g-continuous.

(d) If f is pairwise sg-irresolute and g is pairwise s^*g-continuous, then $g \circ f$ is pairwise sg-continuous.

(e) If f is pairwise gs-irresolute and g is pairwise s^*g-continuous, then $g \circ f$ is pairwise gs-continuous.

(f) If f is pairwise s^*g-continuous and g is pairwise continuous, then $g \circ f$ is pairwise s^*g-continuous.

Proof:

Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ be two functions.

(a) Assume f and g are pairwise s^*g-irresolute.

To prove: $g \circ f$ is pairwise s^*g- irresolute.

Let U be $\mu_i \mu_j$ -s^*g closed set in Z, $i \neq j$ and $i, j = 1,2$.

Since g is pairwise s^*g-irresolute, $g^{-1}(U)$ is $\sigma_i \sigma_j$ -s^*g closed in Y .

Since f is pairwise s^*g-irresolute, $(g \circ f)^{-1} = f^{-1}[g^{-1}(U)]$ is $\tau_i \tau_j$ -s^*g closed in X.

Therefore, $g \circ f$ is pairwise s^*g-irresolute.

(b) Assume f is pairwise s^*g-irresolute and g is pairwise s^*g-continuous,

To prove: $g \circ f$ is pairwise s^*g-continuous.

Since g is pairwise s^*g-continuous, $g^{-1}(U)$ is $\sigma_i\,\sigma_j$ -s^*g closed for each μ_j -closed set U in Z $i \neq j$ and $i, j = 1, 2$.

Since f is pairwise s^*g-irresolute, $(g\circ f)^{-1} = f^{-1}[g^{-1}(U)]$ is $\tau_i\,\tau_j$ -s^*g closed in X.

Therefore, $g\circ f$ is pairwise s^*g-continous.

(c) Assume f is pairwise g-irresolute and g is pairwise s^* g-continuous.

To prove: $g\circ f$ is pairwise g-continuous.

Let U be $\mu_i\,\mu_j$ - closed set in Z, $i \neq j$ and $i, j = 1,2$.

Since g is pairwise s^* g-continuous, $g^{-1}(U)$ is $\sigma_i\,\sigma_j$ -s^*g closed in Y.

Since every $\sigma_i\,\sigma_j$ -s^*g closed set is $\sigma_i\,\sigma_j$ - g closed and f is pairwise g-irresolute, $(g\circ f)^{-1}(U) = f^{-1}[g^{-1}(U)]$ is $\tau_i\,\tau_j$ - g closed in X.

Therefore, $g\circ f$ is pairwise g-continuous.

(d) Assume f is pairwise sg-irresolute and g is pairwise s^*g-continuous.

To prove: $g\circ f$ is pairwise sg-continuous.

Let U be $\mu_i\,\mu_j$ - closed set in Z, $i \neq j$ and $i, j = 1,2$.

Since g is pairwise s^*g-continuous, $g^{-1}(U)$ is $\sigma_i\,\sigma_j$ -s^*g closed in Y.

Since every $\sigma_i\,\sigma_j$ -s^*g closed set is $\sigma_i\,\sigma_j$ - sg closed and f is pairwise sg-irresolute, $(g\circ f)^{-1}(U) = f^{-1}[g^{-1}(U)]$ is $\tau_i\,\tau_j$ - sg closed in X.

Therefore, $g\circ f$ is pairwise sg-continuous.

(e) Assume f is pairwise gs-irresolute and g is pairwise s^*g-continuous.

To prove: $g\circ f$ is pairwise gs-continuous.

Let U be $\mu_i\,\mu_j$ - closed set in Z, $i \neq j$ and $i, j = 1,2$.

Since g is pairwise s^*g-continuous, $g^{-1}(U)$ is $\sigma_i\,\sigma_j$ -s^*g closed in Y.

Since every $\sigma_i\,\sigma_j$ -s^*g closed set is $\sigma_i\,\sigma_j$ - gs closed and f is pairwise gs-irresolute, $(g\circ f)^{-1}(U) = f^{-1}[g^{-1}(U)]$ is $\tau_i\,\tau_j$ - sg closed in X.

Therefore, $g\circ f$ is pairwise gs-continuous.

(f) Assume f is pairwise s^*g-continuous and g is pairwise continuous.

To prove: $g\circ f$ is pairwise s^*g -continuous.

Let U be $\mu_i\,\mu_j$ - closed set in Z, $i \neq j$ and $i, j = 1,2$.

Since g is pairwise -continuous, $g^{-1}(U)$ is $\sigma_i\,\sigma_j$ -closed in Y.

Since f is pairwise s*g-continuous, $(g\circ f)^{-1}(U) = f^{-1}[g^{-1}(U)]$ is $\tau_i\,\tau_j$ - s^*g closed in X.

Therefore, $g\circ f$ is pairwise s^*g -continuous.

Remark:1.2.18

The composition of two pairwise s^*g-continuous functions need not be a pairwise s^*g-continuous function.

Example: 1.2.19

Let X = Y = Z = {a, b, c, d}, τ_1 = {φ, X, {a}, {a, b}}, τ_2 = {φ, X, {a, b}, {a, b, c}, {a, b, d}}, σ_1 = {φ, Y, {a}}, σ_2 = {φ, Y, {a, b}, {a, b, c}}, μ_1 = {φ, Z, {a}}, μ_2 = {φ, X, {a}, {b, c}}.Let f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function defined by f (φ) = φ, f (X) = Y , f (a) = {a, b, d}, f (b) = {c}, f (c) = {b}, f (d) = {d}, f (a, b) = {a, c}, f (a, c) ={a, b}, f (a, d) = {b, c}, f (b, c) = {a, d},f (b, d) = {a, b, c}, f (c, d) = {c, d}, f (a, b, c)= {b, d}, f (a, b, d) = {a},f (a, c, d) = {b, c, d}, f (b, c, d) = {a, c, d}. Then f is pairwise s^*g-continuous.Let g : $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ be a function defined by g(φ) = φ,g(Y) =Z, g(a) = {b}, g(b) = {a}, g(c) = {d}, g(d) = {c}, g(a, b) = {a, c}, g(a, c) ={a, b}, g(a, d) = {a, d}, g(b, c) = {b, d}, g(b, d) = {b, c}, g(c, d) = {a, b, c}, g(a, b, c)= {c, d}, g(a, b, d) = {a, c, d}, g(a, c, d) = {a, b, d}, g(b, c, d) = {b, c, d}. Then g is pairwise s^*g-continuous. But $(g \circ f)^{-1}$ ({b, c, d}) = {a, c, d} is not $\tau_1 \tau_2 - s^*g$ closed in X. Hence g o f is not pairwise s^*g-continuous.

Definition: 1.2.20

A function f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is **pairwise pre s^*g-continuous** if f^{-1} (U) is $\tau_i \tau_j$ -s^*g closed for each $\sigma_i \sigma_j$ -semi closed set U in Y , $i \neq j$ and i, j = 1,2.

Example: 1.2.21

Let X = Y = {a, b, c},τ_1 = {φ, X, {a}, {b, c}}, τ_2 = {φ, X, {a}}, σ_1 = {φ, Y, {c}, {a, b}}, σ_2 = {φ, Y, {c}}. Let f : $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function defined by f (φ) = φ, f (X) = Y , f (a) = {a, c}, f (b) = {b},f (c) = {c}, f (a, b) ={a, b},f (a, c) = {a}, f (b, c) = {a, b}. Then f is pairwise pre s^*g-continuous.

Proposition: 1.2.22

Every pairwise pre s^*g-continuous function is pairwise s^*g-continuous. But converse not true. It is shown in the following example.

Example: 1.2.23

In example 1.2.16, f is pairwise s^*g-continuous but not pairwise pre s^*g-continuous.

Theorem: 1.2.24

Let Y be a pairwise semi $T_{1/2}$ -space. A function f : (X, τ_1 , τ_2) → (Y, σ_1, σ_2) is pairwise s^*g-irresolute if it is pairwise pre s^*g-continuous.

Proof:

Let f : (X, τ_1 , τ_2) → (Y, σ_1 , σ_2) is pairwise pre s^*g-continuous.

To prove: f is pairwise pre s^*g-irresolute.

Let A be $\sigma_i\, \sigma_j$ -s^*g closed in Y , i, j = 1, 2 and $i \neq j$.

Since every $\sigma_i\, \sigma_j$ -s^*g closed set is $\sigma_i\, \sigma_j$ –sg closed, A is $\sigma_i\, \sigma_j$ -sg closed in Y .

Since Y is pairwise semi $T_{1/2}$ -space and every $\sigma_i\, \sigma_j$ -sg closed set is σ_j -semi closed.

Therefore A is $\sigma_i\, \sigma_j$ -semi closed.

Since f is pairwise pre s^*g – continuous, $f^{-1}(A)$ is $\tau_i\, \tau_j$ -s^*g closed in X.

Hence f is pairwise s^*g-irresolute.

Definition: 1.2.25

A function is f : (X, τ_1 , τ_2) → (Y, σ_1 , σ_2) **pairwise s^*g-closed** if f (U) is $\sigma_i\, \sigma_j$ -s^*g closed for each τ_j - closed set U in X ,$i \neq j$ and i, j = 1,2.

Definition:1.2.27

A function f : (X, τ_1 , τ_2) → (Y, σ_1 , σ_2) is **pairwise pre s^*g-closed** if f (U) is $\sigma_i\, \sigma_j$ -s^*g closed for each $\tau_i\, \tau_j$ -semi closed set U in X, $i \neq j$ and i, j = 1,2.

Example: 1.2.28

Let X = Y = {a, b, c},τ_1 = {φ, X, {a}, {b, c}}, τ_2 = {φ, X, {a},σ_1= {φ, Y, {c}, {a, b}}, σ_2 = {φ, Y, {c}}. Let f : (X, τ_1 , τ_2) → (Y, σ_1 , σ_2) be a function defined by f (φ) = φ, f (X) = Y , f (a) = {a}, f (b) = {b}, f (c) = {a, c}, f (a, b) ={b, c},f (a, c) = {c}, f (b, c) = {a, b}. Then f is pairwise pre s^*g-closed.

SECTION -1.3

PAIRWISE S *GO-CONNECTED SPACE AND S *GO-COMPACT SPACE

Definition:1.3.1

A bitopological space (X, τ_1, τ_2) is **pairwise S *GO -connected** if X cannot be expressed as the union of two nonempty disjoint sets A and B such that $[A\cap\tau_1\text{-}s^*gcl(B)] \cup [\tau_2\text{ -}s^*gcl(A) \cap B] = \varphi$.

Definition:1.3.2

A bitopological space (X, τ_1, τ_2) is **pairwise S *GO -disconnected** if X can be expressed as the union of two nonempty disjoint sets A and B such that $[A\cap\tau_1\text{-}s^*gcl(B)] \cup [\tau_2\text{ -}s^*gcl(A) \cap B] = \varphi$ and $X = A\backslash B$ and call this pairwise S^*GO -separation of X.

Example: 1.3.3

(a) Let $X = \{a, b, c, d\}, \tau_1 = \{\varphi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\varphi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then (X, τ_1, τ_2) is pairwise S^*GO- connected.

(a) $Y = \{a, b, c, d\}$, $\sigma_1 = \{\varphi, Y, \{a\}\}$, $\sigma_2 = \{\varphi, Y, \{a, b\}, \{a, b, c\}\}$.

Then (Y, σ_1, σ_2) is pairwise S *GO -connected.

Theorem:1.3.4

The following conditions are equivalent for any bitopological space.

(a) X is pairwise S *GO -connected.

(b) X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 -s^*g open and B is τ_2 -s^*g open.

(c) X contains no nonempty proper subset which is both τ_1 -s^*g open and τ_2 -s^*g closed.

Proof:

(a) $\Rightarrow$ (b)

Assume that X is pairwise S *GO -connected.

To prove:

X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 -s^*g open and B is τ_2 -s^*g open.

Assume that X can be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 -s^*g open and B is τ_2 -s^*g open.

Then $A \cap B = \varphi$.

Hence, $A \subseteq B^C$.

Then τ_2 -$s^*gcl(A) \subseteq \tau_2$ -$s^*gcl(B^C) = B^C$.

Therefore, τ_2 -$s^*gcl(A) \cap B = \varphi$.

Similarly, $A \cap \tau_1$ -$s^*gcl(B) = \varphi$.

Hence $[A \cap \tau_1$ -$s^*gcl(B)] \cup [\tau_2$ -$s^*gcl(A) \cap B] = \varphi$.

This is a contradiction to the fact that X is pairwise S^*GO -connected. Therefore, X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 -s^*g open and B is τ_2 -s^*g open.

(b)$\Rightarrow$(c)

Assume that X cannot be expressed as the union of two nonempty disjoint sets A and B such that A is τ_1 -s^*g open and B is τ_2 -s^*g open. Assume that X contains a nonempty proper subset A which is both τ_1 -s^*g open and τ_2 -s^*g closed.

Then $X = A \cup A^C$ where A is τ_1 -s^*g open, A^C is τ_2 -s^*g open and A, A^C are disjoint.

This is the contradiction to our assumption.

Therefore, X contains no nonempty proper subset which is both τ_1 -s^*g open and τ_2 -s^*g closed.

(c)$\Rightarrow$(a)

Assume that X contains no nonempty proper subset which is bothτ_1 -s^*g open and τ_2 -s^*g closed.

Suppose that X is pairwise S^*GO -disconnected.

Then X can be expressed as the union of two nonempty disjoint sets A and B such that $[A \cap \tau_1$ -$s^*gcl(B)] \cup [\tau_2$ -$s^*gcl(A) \cap B] = \varphi$.

Since $A \cap B = \varphi$, $A = B^C$ and $B = A^C$.

Since τ_2 -$s^*gcl(A) \cap B = \varphi$, τ_2 -$s^*gcl(A) \subseteq B^C$.

Hence τ_2 -$s^*gcl(A) \subseteq A$.

Therefore, A is τ_2 -s^*g closed.

Similarly, B is τ_1 -s^*g closed.

Since $A = B^C$, A is τ_1 -s^*g open.

Therefore, there exists a nonempty proper set A which is both τ_1 -s^*g open and τ_2 -s^*g closed.

This is the contradiction to our assumption.

Therefore, X is pairwise S^*GO -connected.

Theorem: 1.3.5

If A is pairwise S^*GO -connected subset of a bitopological space (X, τ_1, τ_2) which has the pairwise S^*GO -separation $X = C\backslash D$, then $A \subseteq C$ or $A \subseteq D$.

Proof:

Suppose that (X, τ_1 , τ_2) has the pairwise S^*GO -separation $X = C\backslash D$.

Then $X = C \cup D$ where C and D are two nonempty disjoint sets such that

$[C \cap \tau_1\text{-}s^*gcl(D)] \cup [\tau_2\text{-}s^*gcl(C) \cap D] = \varphi$.

Since $C \cap D = \varphi$, $C = D^C$ and $D = C^C$.

Now,

$$[(C \cap A) \cap \tau_1\text{-}s^*gcl(D \cap A)]\ Y\ [\tau_2\text{-}s^*gcl(C \cap A) \cap (D \cap A)]$$

$$\subseteq [C \cap \tau_1\text{-}s^*g\,cl(D)]\ Y\ [\tau_2\text{-}s^*gcl(C) \cap D]$$

$$= \varphi.$$

Hence $A = (C \cap A)\backslash(D \cap A)$ is pairwise S^*GO -separation of A.

Since A is pairwise S^*GO -connected, either $(C \cap A) = \varphi$ or $(D \cap A) = \varphi$.

Hence, $A \subseteq C^C$ or $A \subseteq D^C$.

Therefore, $A \subseteq C$ or $A \subseteq D$.

Theorem: 1.3.6

If A is pairwise S^*GO -connected and $A \subseteq B \subseteq \tau_1\text{-}s^*gcl(A) \cap \tau_2\text{-}s^*gcl(A)$ then B is pairwise S^*GO -connected.

Proof:

Let A be pairwise S^*GO -connected and $A \subseteq B \subseteq \tau_1\text{-}s^*gcl(A) \cap \tau_2\text{-}s^*gcl(A)$

...

To prove:

B is pairwise S^*GO -connected.

Suppose that B is not pairwise S^*GO -connected.

Then $B = C \cup D$ where C and D are two nonempty disjoint sets such that

$[C \cap \tau_1\text{-}s^*gcl(D)] \cup [\tau_2\text{-}s^*gcl(C) \cap D] = \varphi$.

Since A is pairwise S^*GO -connected, $A \subseteq C$ or $A \subseteq D$.

Suppose $A \subseteq C$.

Then $D \subseteq D \cap B \subseteq D \cap \tau_2\text{-}s^*gcl(A) \subseteq D \cap \tau_2\text{-}s^*gcl(C) = \varphi$. Therefore, $\varphi \subseteq D \subseteq \varphi$.

Hence, $D = \varphi$.

Similarly, we can prove $C = \varphi$ if $A \subseteq D$ {by Theorem 1.3.5}

This is the contradiction to the fact that C and D are nonempty.

Therefore, B is pairwise S^*GO -connected.

Theorem:1.3.7

The union of any family of pairwise S^*GO -connected sets having a nonempty intersection is pairwise S^*GO -connected.

Proof:

Let $A = \bigcup_{i \in I} A_i$ where each A_i is pairwise S^*GO connected with $\cap A_i \neq \varphi$, and I is an index set.

Assume that A is not a pairwise S^*GO -connected set.

Then $A = C \cup D$, where C and D are two nonempty disjoint sets such that

$[C \cap \tau_1\text{-}s^*gcl(D)] \cup [\tau_2\text{-}s^*gcl(C) \cap D] = \varphi$.

Since A_i is pairwise S^*GO -connected and $A_i \subseteq A$, $A_i \subseteq C$ or $A_i \subseteq D$

Therefore $\bigcup (A_i) \subseteq C$ or $\cup (A_i) \subseteq D$.

Hence, $A \subseteq C$ or $A \subseteq D$.

Since $\cap A_i \neq \varphi$, let $x \in \cap A_i$.

Therefore, $x \in A_i$ for all i.

Hence, $x \in A$. Therefore, $x \in C$ or $x \in D$.

Suppose $x \in C$.

Since $C \cap D = \varphi$, $x \notin D$.

Therefore, $A \not\subset D$. Therefore, $A \subseteq C$.

Therefore, A is not pairwise S^*GO -connected. This shows that A is pairwise S^*GO -connected.

Definition: 1.3.8

A nonempty collection $\zeta = \{A_i , i \in I$, an index set$\}$ is called a **pairwise s^*g-open cover** of a bitopological space (X, τ_1, τ_2) if $X = \bigcup A_i$ and $\zeta \subseteq \tau_1 - S^*GO(X, \tau_1, \tau_2) \cup \tau_2 - S^*GO(X, \tau_1, \tau_2)$ and ζ contains at least one member of $\tau_1 - S^*GO(X, \tau_1, \tau_2)$ and one member of $\tau_2 - S^*GO(X, \tau_1, \tau_2)$.

Definition: 1.3.9

A bitopological space (X, τ_1, τ_2) is **pairwise S^*GO -compact** if every pairwise s^*g-open cover of X has a finite subcover.

Definition: 1.3.10

A set A of a bitopological space (X, τ_1, τ_2) is **pairwise S^*GO compact** relative to X if every pairwise s^*g-open cover of B by has a finite subcover as a subspace.

Example: 1.3.11

Let $X = \{a, b, c, d\}, \tau_1 = \{\varphi, X, \{a\}, \{a, b\}\}, \tau_2 = \{\varphi, X, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Let $\zeta = \{\{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}$. Then (X, τ_1, τ_2) is pairwise S^*GO -compact.

Theorem: 1.3.12

Every pairwise s^*g-compact space is pairwise compact.

Proof:

Let (X, τ_1, τ_2) be pairwise S^*GO -compact.

Let $\zeta = \{A_i , i \in I$, an index set$\}$ be a pairwise open cover of X.

Then $X = \bigcup A_i$ and $\zeta \subseteq \tau_1 \cup \tau_2$ and ζ contains at least one member of τ_1 and one member of τ_2.

Since every τ_i -open set is τ_i -s^*g open, X = $\bigcup A_i$ and $\zeta \subseteq \tau_1$ - S^*GO (X, τ_1 , τ_2) $\cup \tau_2$ - S^*GO (X, τ_1 , τ_2) and ζ contains at least one member of τ_1 - S^*GO (X, τ_1 , τ_2) and one member of τ_2 -S^*GO (X, τ_1 , τ_2).

Therefore, ζ is the pairwise s^*g-open cover of X.

Since X is pairwise S^*GO -compact, ζ has the finite subcover.

Therefore X is pairwise compact.

Theorem: 1.3.13

Let f : (X, τ_1 , τ_2) $\rightarrow$ (Y, σ_1 , σ_2) be a pairwise continuous, bijective and pairwise pre semi closed. Then the image of a pairwise S^*GO -compact space under f is pairwise S^* GO -compact.

Proof:

Let f : (X, τ_1 , τ_2) $\rightarrow$ (Y, σ_1 , σ_2) be pairwise continuous surjection and pairwise pre semi closed.

Let X be pairwise S^*GO -compact.

Let ζ = {A_i , i$\in$I, an index set} be a pairwise s^*g-open cover of Y .

Then Y =$\bigcup A_i$ and $\zeta \subseteq \sigma_1$ -S^*GO (Y, σ_1 , σ_2) $\cup \sigma_2$ - S^*GO (Y, σ_1 , σ_2) and ζ contains at least one member of σ_1 -S^*GO (Y, σ_1 , σ_2) and one member of σ_2 - S^*GO (Y, σ_1 , σ_2).

Therefore, X = f^{-1} [$\bigcup$(A_i)] = $\bigcup f^{-1}$ (A_i) and

f^{-1} (ζ) $\subseteq \tau_1$ - S^*GO (X, τ_1 , τ_2) $\cup \tau_2$ - S^*GO (X, τ_1 , τ_2) and f^{-1} (ζ) contains at least one member of τ_1 - S^*GO (X, τ_1 , τ_2) and one member of τ_2 - S^* GO (X, τ_1 , τ_2).

Therefore f^{-1} (ζ) is the pairwise s^* g-open cover of X.

Since X is pairwise S^*GO -compact, X = $\bigcup f^{-1}(A_i)$, i = 1 to n.

$\Rightarrow$ Y = f (X) =$\bigcup$(A_i), i = 1 to n.

Hence ζ has the finite subcover.

Therefore Y is pairwise S^*GO -compact.

Theorem: 1.3.14

If Y is τ_1 -s^* g closed subset of a pairwise S^*GO -compact space (X, τ_1 , τ_2), then Y is τ_2 - S^* GO compact

Proof:

Let (X, τ_1, τ_2) be a pairwise S^*GO -compact space.

Let $\zeta = \{A_i, i \in I$, an index set$\}$ be a τ_2 -s^*g open cover of Y .

Since Y is τ_1 -s^*g closed subset, Y^C is τ_1 -s^*g open.

Hence $\zeta \cup Y^C = Y^C \cup \{A_i, i \in I$, an index set$\}$ will be a pairwise s^*g-open cover of X.

Since X is pairwise S^* GO -compact, $X = Y^C \cup A_1 \cup ...... \cup A_n$.

Hence $Y = A_1 \cup .... \cup A_n$.

Therefore Y is τ_2 - S^*GO compact.

SECTION -1.4
$(\tau_1, \tau_2)^*$ -SEMI STAR GENERALIZED CLOSED SETS AND OPEN SETS

Definition: 1.4.1

A set A of a bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)^*$ **-semi star generalized closed** { $(\tau_1, \tau_2)^*$ -s^*g closed } if $\tau_1 \tau_2$ -cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$ -semi open in X.

Example:1.4.2

Let X = {a, b, c}, τ_1 = { φ , X, {a}}, τ_2 ={ φ , X, {b}}. Then φ , X, {c}, {a, c}, {b, c} are $(\tau_1, \tau_2)^*$ -s^*g closed and {a}, {b}, {a, b} are not $(\tau_1, \tau_2)^*$ -s^*g closed.

Theorem:1.4.3

Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. Then the following statements are true.

(a) If A is $\tau_1 \tau_2$ -closed, then A is $(\tau_1, \tau_2)^*$ -s^*g closed.

(b) If A is $(\tau_1, \tau_2)^*$ -semi open and $(\tau_1, \tau_2)^*$ -s^*g closed, then A is $\tau_1 \tau_2$-closed .

(c) If A is $(\tau_1, \tau_2)^*$ -s^*g closed, then A is $(\tau_1, \tau_2)^*$ -g closed.

(d) If A is $(\tau_1, \tau_2)^*$ -s^*g closed, then A is $(\tau_1, \tau_2)^*$ -sg closed.

(e) If A is $(\tau_1, \tau_2)^*$ -s^*g closed, then A is $(\tau_1, \tau_2)^*$ -gs closed.

Proof:

(a) Assume A is $\tau_1\,\tau_2$-closed.

To prove: A is $(\tau_1 , \tau_2)^*$ -s *g closed

Let $A \subseteq U$ and U is $(\tau_1 , \tau_2)^*$ -semi open in X.

Since A is $\tau_1\,\tau_2$-closed, $\tau_1\,\tau_2$-cl(A) = $A \subseteq U$.

Therefore A is $(\tau_1 , \tau_2)^*$ -s *g closed.

(b) Assume A is $(\tau_1 , \tau_2)^*$ -semi open and $(\tau_1 , \tau_2)^*$ -s *g closed.

To prove: A is $\tau_1\,\tau_2$ -closed

Since $A \subseteq A$, A is $(\tau_1 , \tau_2)^*$ -semi open.

And A is $(\tau_1 , \tau_2)^*$ -s *g closed, $\tau_1\,\tau_2$-cl(A)$\subseteq$A.

Always A$\subseteq\tau_1\,\tau_2$-cl(A).

Therefore $\tau_1\,\tau_2$-cl(A) = A.

(c) Assume A is $(\tau_1 , \tau_2)^*$ -s *g closed.

To prove: A is $(\tau_1 , \tau_2)^*$ -g closed.

Let $A \subseteq U$ and U is $\tau_1\,\tau_2$-open in X.

Since U is $\tau_1\,\tau_2$-open in X, U is $(\tau_1 , \tau_2)^*$ -semi open in X.

Since A is $(\tau_1 , \tau_2)^*$ -s* g closed, $\tau_1\,\tau_2$ -cl(A)$\subseteq$U .

Hence A is $(\tau_1 , \tau_2)^*$ -g closed.

(d) Assume A is $(\tau_1 , \tau_2)^*$ -s *g closed.

To prove: A is $(\tau_1 , \tau_2)^*$ -sg closed.

Let $A \subseteq U$ and U is $(\tau_1 , \tau_2)^*$ -semi open in X.

Then, $\tau_1\,\tau_2$-cl(A)$\subseteq$U .

Hence $\tau_1\,\tau_2$-scl(A)$\subseteq$U .

Therefore A is $(\tau_1 , \tau_2)^*$ -sg closed.

(e) Assume A is $(\tau_1 , \tau_2)^*$ -s *g closed.

To prove: A is $(\tau_1 , \tau_2)^*$ -gs closed.

Let $A \subseteq U$ and U is $\tau_1\,\tau_2$-open in X.

Since U is $\tau_1\,\tau_2$ -open in X, U is $(\tau_1 , \tau_2)^*$ -semi open in X.

Since A is $(\tau_1 ,\tau_2)^*$ -s *g closed, $\tau_1\,\tau_2$-cl(A)$\subseteq$U .

Hence , $\tau_1\,\tau_2$-scl(A)$\subseteq\tau_1\tau_2$-cl(A)

$\tau_1\,\tau_2$-scl(A) $\subseteq \tau_1\,\tau_2$-cl(A) $\subseteq$ U .

Hence A is $(\tau_1 , \tau_2)^*$ -gs closed.

Remark: 1.4.4

The converses of the above theorem need not true.

Example: 1.4.5

In example 1.4.2, {c} is $(\tau_1 , \tau_2)^*$ -s *g closed but not $\tau_1\,\tau_2$-closed. {a, b} is $(\tau_1 , \tau_2)^*$ -g closed, $(\tau_1 , \tau_2)^*$ -gs closed but not $(\tau_1 , \tau_2)^*$ -s *g closed. {b} is $(\tau_1 , \tau_2)^*$ -sg closed, $(\tau_1 , \tau_2)^*$ -gs but not $(\tau_1 , \tau_2)^*$ -s *g closed.

Remark: 1.4.6

In a bitopological space (X, τ_1 , τ_2) , $(\tau_1 , \tau_2)^*$ -s *g closed sets and $(\tau_1 , \tau_2)^*$ -semi closed sets are independent.

Example: 1.4.7

In example 1.4.2, {b} is $(\tau_1 , \tau_2)^*$ -semi closed, but not $(\tau_1 , \tau_2)^*$ -s *g closed.

Example: 1.4.8

Let X = {a, b, c}, τ_1 = { φ , X, {a, b}}, τ_2 ={ φ , X}. Then {a, c} is $(\tau_1 , \tau_2,)^*$ -s *g closed , but not $(\tau_1 , \tau_2)^*$ -semi closed in (X, τ_1 , τ_2).

Remark: 1.4.9

$$
\begin{array}{ccc}
\tau_1\,\tau_2\text{– closed} & \Rightarrow & (\tau_1 , \tau_2)^* \text{-semi closed} \\
\Downarrow & & \Downarrow \\
(\tau_1 , \tau_2)^* \text{-s}^*\text{g closed} & \Rightarrow & (\tau_1 , \tau_2)^* \text{-sg closed} \\
\Downarrow & & \Uparrow \\
(\tau_1 , \tau_2)^* \text{-g closed} & \Rightarrow & (\tau_1 , \tau_2)^* \text{-gs closed}
\end{array}
$$

Theorem 1.4.10

If A is $(\tau_1 , \tau_2)^*$ -s *g closed in X and $A \subseteq B \subseteq \tau_1\,\tau_2$ -cl(A), then B is $(\tau_1 , \tau_2)^*$ -s *g closed.

Proof :

Assume that A is $(\tau_1, \tau_2)^*$ -s^*g closed in X and $A \subseteq B \subseteq \tau_1 \tau_2$ -cl(A).

To prove: B is $(\tau_1, \tau_2)^*$ -s^*g closed.

Let $B \subseteq U$ and U is $(\tau_1, \tau_2)^*$ -semi open in X.

Since $A \subseteq B$ and $B \subseteq U$, $A \subseteq U$ and as A is $(\tau_1, \tau_2)^*$ -s^*g closed . $\tau_1 \tau_2$ -cl(A)$\subseteq U$.

Since $B \subseteq \tau_1 \tau_2$ -cl(A), $\tau_1 \tau_2$ -cl(B)$\subseteq \tau_1 \tau_2$ -cl(A)$\subseteq U$.

Therefore, B is $(\tau_1, \tau_2)^*$ -s^*g closed.

Proposition: 1.4.11

If A and B are $(\tau_1, \tau_2)^*$ -s^*g closed sets then so is $A \cup B$.

Proof:

Let A and B are $(\tau_1, \tau_2)^*$ -s^*g closed sets.

To prove: $A \cup B$ is $(\tau_1, \tau_2)^*$ -s^*g closed set.

Let U be $(\tau_1, \tau_2)^*$ -semi open in X and $A \cup B \subseteq U$.

Then $A \subseteq U$ and $B \subseteq U$.

Since U is $(\tau_1, \tau_2)^*$ -semi open in X and A and B are $(\tau_1, \tau_2)^*$ -s^*g closed sets,

$\tau_1 \tau_2$ -cl(A)$\subseteq U$ and $\tau_1 \tau_2$ -cl(B)$\subseteq U$.

Therefore { $\tau_1 \tau_2$ -cl(A)} $\cup$ { $\tau_1 \tau_2$ -cl(B)} $\subseteq U$.

Hence $\tau_1 \tau_2$ -{cl($A \cup B$)} $\subseteq U$.

Therefore $A \cup B$ is $(\tau_1, \tau_2)^*$ -s^*g closed.

Remark: 1.4.12

The converse of the above proposition need not true.

Example: 1.4.13

In example 1.4.2, Let A = {b} and B = {c}. Then $A \cup B$ = {b, c} is $(\tau_1, \tau_2)^*$ -s^*g closed , B is $(\tau_1, \tau_2)^*$ -s^*g closed but A is not $(\tau_1, \tau_2)^*$ s^*g closed.

Remark: 1.4.14

The following example shows that $A \cap B$ is $(\tau_1, \tau_2)^*$ -s^*g closed even if A and B are not $(\tau_1, \tau_2)^*$ -s^*g closed.

Example: 1.4.15

In example 1.4.2, let A = {a} and B = {b}. Then $A \cap B = \varphi$ is $(\tau_1, \tau_2)^*$ -s^*g closed but A and B are not $(\tau_1, \tau_2)^*$ -s^*g closed.

Theorem: 1.4.16

A set A is $(\tau_1, \tau_2)^*$ -s^*g closed in X if and only if $\tau_1 \tau_2$-cl(A)−A contains no nonempty $(\tau_1, \tau_2)^*$ -semi closed set.

Proof:

Let A be $(\tau_1, \tau_2)^*$ -s^*g closed in X.

To prove: $\tau_1 \tau_2$-cl(A)−A contains no nonempty $(\tau_1, \tau_2)^*$ -semi closed set.

Let F be $(\tau_1, \tau_2)^*$ -semi closed such that $F \subseteq \tau_1 \tau_2$-cl(A) − A.

Since F is $(\tau_1, \tau_2)^*$ -semi closed, F^C is $(\tau_1, \tau_2)^*$ -semi open.

Since $F \subseteq \tau_1 \tau_2$-cl(A) − A, $A \subseteq F^C$ and $F \subseteq \tau_1 \tau_2$-cl(A) (1)

Since A is $(\tau_1, \tau_2)^*$ -s^*g closed in X, $\tau_1 \tau_2$-cl(A) $\subseteq F^C$

Therefore, $F \subseteq \{\tau_1 \tau_2\text{-cl}(A)\}^C$ (2)

From (1) and (2),

$F \subseteq \varphi$ implies that $F = \varphi$.

Hence $\tau_1 \tau_2$-cl(A) − A contains no nonempty $(\tau_1, \tau_2)^*$ -semi closed set.

Conversely,

Assume that $\tau_1 \tau_2$-cl(A) − A contains no nonempty $(\tau_1, \tau_2)^*$ -semi closed set.

Let $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$ -semi open in X.

Suppose that $\tau_1 \tau_2$-cl(A) $\not\subset$ U.

Then $\tau_1 \tau_2$-cl(A) $\cap U^C \neq \varphi$.

Since $A \subseteq U$, $U^C \subseteq A^C$.

Therefore, $\tau_1 \tau_2$-cl(A) $\cap U^C \subseteq \tau_1 \tau_2$-cl(A) $\cap A^C = \tau_1 \tau_2$-cl(A) − A.

Since U is $(\tau_1, \tau_2)^*$ -semi open in X, U^C is $(\tau_1, \tau_2)^*$ -semi closed in X.

Hence $\tau_1 \tau_2$-cl(A) $\cap U^C$ is $(\tau_1, \tau_2)^*$ -semi closed in X.

This is the contradiction to the fact that $\tau_1 \tau_2$ -cl(A) − A contains no nonempty

$(\tau_1 , \tau_2)^*$ -semi closed set.

Therefore, $\tau_1 \tau_2$ -cl(A) $\subseteq$ U .

Hence A is $(\tau_1 , \tau_2)^*$ -s*g closed in X.

Theorem: 1.4.17

If A is $(\tau_1 , \tau_2)^*$ -s*g closed and A $\subseteq$ B$\subseteq$ $\tau_1 \tau_2$ -cl(A) then $\tau_1 \tau_2$ -cl(B) $-$ B contains no nonempty $(\tau_1 , \tau_2)^*$ -semi closed set.

Proof:

Let A be $(\tau_1 , \tau_2)^*$ -s*g closed and A $\subseteq$ B $\subseteq$ $\tau_1 \tau_2$ -cl(A).

By theorem 1.4.11, B is $(\tau_1 , \tau_2)^*$ -s*g closed.

By theorem 1.4.17, $\tau_1 \tau_2$ -cl(B) $-$ B contains no nonempty $(\tau_1 , \tau_2)^*$ -semi closed set.

Theorem: 1.4.18

For each $x \in X$, the singleton {x} is either $(\tau_1 , \tau_2)^*$ -semi closed or $(\tau_1 , \tau_2)^*$ -s*g open.

Proof:

Let $x \in X$

Suppose that {x} is not $(\tau_1 , \tau_2)^*$ -semi closed.

Then X $-$ {x} is not $(\tau_1 , \tau_2)^*$ -semi open.

Hence X is the only $(\tau_1 , \tau_2)^*$ -semi open set containing the set X $-$ {x}.

Therefore, X $-$ {x} is $(\tau_1 , \tau_2)^*$ -s*g closed.

Definition: 1.4.19

A set A is called $(\tau_1 , \tau_2)^*$ **-semi star generalized open** {$(\tau_1 , \tau_2)^*$ s*g open} if and only if A^C is $(\tau_1 , \tau_2)^*$ -s*g closed.

Example: 1.4.20

In example 1.4.2, φ , X, {a}, {b}, {a, b} are $(\tau_1 , \tau_2)^*$ -s*g open and {c}, {a, c}, {b, c} are not $(\tau_1 , \tau_2)^*$ -s*g open.

Theorem: 1.4.21

A set A is $(\tau_1, \tau_2)^*$ -s *g open if and only if $F \subseteq \tau_1 \tau_2$ -int(A) whenever F is $(\tau_1, \tau_2)^*$ -semi closed and $F \subseteq A$.

Proof:

Let A be $(\tau_1, \tau_2)^*$ -s *g open.

Let F be $(\tau_1, \tau_2)^*$ -semi closed and $F \subseteq A$.

To prove: $F \subseteq \tau_1 \tau_2$ -int(A)

As F^C is $(\tau_1, \tau_2)^*$ -semi open and $A^C \subseteq F^C$.

Since A^C is $(\tau_1, \tau_2)^*$ -s *g closed, $\tau_1 \tau_2$ -cl(A^C) $\subseteq F^C$

Since $\tau_1 \tau_2$ -cl(A^C) = $[\tau_1 \tau_2$ -int(A)$]^C$,

$[\tau_1 \tau_2$ -int(A)$]^C \subseteq F^C$.

Hence $F \subseteq \tau_1 \tau_2$ -int(A).

Conversely,

Assume that $F \subseteq \tau_1 \tau_2$ -int(A) whenever F is $(\tau_1, \tau_2)^*$ -semi closed and $F \subseteq A$.

To prove: A is $(\tau_1, \tau_2)^*$ - s *g open.

As $A^C \subseteq F^C$ and F^C is $(\tau_1, \tau_2)^*$ -semi open and

Since $F \subseteq \tau_1 \tau_2$ -int(A), $[\tau_1 \tau_2$ -int(A)$]^C \subseteq F^C$.

Since $\tau_1 \tau_2$ -cl(A^C) = $[\tau_1 \tau_2$ -int(A)$]^C$, $\tau_1 \tau_2$ -cl(A^C) $\subseteq F^C$.

Therefore, A^C is $(\tau_1, \tau_2)^*$ -s *g closed.

Hence A is $(\tau_1, \tau_2)^*$ -s *g open

Remark: 1.4.22

Every $\tau_1\tau_2$ -open set is $(\tau_1, \tau_2)^*$ -s *g open but the converse need not be true.

Example: 1.4.23

In example 1.4.2, {a, b} is $(\tau_1, \tau_2)^*$ -s *g open, but not $\tau_1 \tau_2$ -open.

Remark: 1.4.24

$(\tau_1, \tau_2)^*$ -s *g open sets and $(\tau_1, \tau_2)^*$ -semi open sets are independent.

Example: 1.4.25

In example 1.4.2, {a, c} is $(\tau_1, \tau_2)^*$ -semi open, but not $(\tau_1, \tau_2)^*$ -s *g open.

Theorem: 1.4.26

If A and B are $(\tau_1, \tau_2)^*$ -s *g open sets such that $\tau_1 \tau_2$ -cl(A) $\cap$ B = A $\cap \tau_1 \tau_2$ -cl(B) = φ then $A \cup B$ is $(\tau_1, \tau_2)^*$ -s *g open.

Proof:

Let F be $(\tau_1, \tau_2)^*$ -semi closed and $F \subseteq A \cup B$.

Then, $F \cap \tau_1 \tau_2$ -cl(A) $\subseteq (A \cup B) \cap \tau_1 \tau_2$ -cl(A)

$= \{A \cap \tau_1 \tau_2 \text{-cl}(A)\} \cup \{B \cap \tau_1 \tau_2 \text{-cl}(A)\}$

$= A \cup \varphi$

$= A.$

Therefore, $F \cap \tau_1 \tau_2$ -cl(A) $\subseteq$ A.

Similarly, $F \cap \tau_1 \tau_2$ -cl(B) $\subseteq$ B.

Since F is $(\tau_1, \tau_2)^*$ -semi closed, $F \cap \tau_1 \tau_2$ -cl(A) and $F \cap \tau_1 \tau_2$ -cl(B) are $(\tau_1, \tau_2)^*$ -semi closed.

Since A and B are $(\tau_1, \tau_2)^*$ -s*g open,

$F \cap \tau_1 \tau_2$ -cl(A) $\subseteq \tau_1 \tau_2$ -int(A) and $F \cap \tau_1 \tau_2$ -cl(B) $\subseteq \tau_1 \tau_2$ -int(B).

$F = F \cap (A \cup B)$

$= (F \cap A) \cup (F \cap B)$

$\subseteq \{F \cap \tau_1 \tau_2 \text{-cl}(A)\} \cup \{F \cap \tau_1 \tau_2 \text{-cl}(B)\}$

$\subseteq \{\tau_1 \tau_2 \text{-int}(A)\} \cup \{\tau_1 \tau_2 \text{-int}(B)\}.$

Therefore, $F \subseteq \tau_1 \tau_2$ -int($A \cup B$).

Hence $A \cup B$ is $(\tau_1, \tau_2)^*$ -s*g open.

Theorem: 1.4.27

If A is $(\tau_1, \tau_2)^*$ -s*g open in X and $\tau_1 \tau_2$ -int(A) $\subseteq$ B $\subseteq$ A, then B is $(\tau_1, \tau_2)^*$ -s*g open.

Proof :

Let A be $(\tau_1, \tau_2)^*$ -s *g open in X and $\tau_1 \tau_2$ -int(A) $\subseteq$ B $\subseteq$ A.

To prove: B is $(\tau_1, \tau_2)^*$-s^*g open.

Then A^C is $(\tau_1, \tau_2)^*$-s^*g closed in X and $A^C \subseteq B^C\ \tau_1\tau_2$-cl$(A^C)$.

Let $B^C \subseteq U$ and U is $(\tau_1, \tau_2)^*$-s^*g open in X.

Since $A^C \subseteq B^C$ and $B^C \subseteq U$, $A^C \subseteq U$

Therefore, $\tau_1\tau_2$-cl$(A^C) \subseteq U$ (since A^C is $(\tau_1, \tau_2)^*$-s^*g closed)

Since $\tau_1\tau_2$-cl$(B^C) \subseteq \tau_1\tau_2$-cl$(A^C) \subseteq U$, B^C is $(\tau_1, \tau_2)^*$-s^*g closed.

Therefore, B is $(\tau_1, \tau_2)^*$-s^*g open.

Theorem: 1.4.28

A set A is $(\tau_1, \tau_2)^*$-s^*g closed in X if and only if $\tau_1\tau_2$-cl(A) − A is $(\tau_1, \tau_2)^*$-s^*g open.

Proof:

Assume that A is $(\tau_1, \tau_2)^*$-s^*g closed in X.

To prove: $\tau_1\tau_2$-cl(A) − A is $(\tau_1, \tau_2)^*$-s^*g open.

Let F be $(\tau_1, \tau_2)^*$-semi closed and $F \subseteq \tau_1\tau_2$-cl(A) − A.

Since A is $(\tau_1, \tau_2)^*$-s^*g closed in X, $\tau_1\tau_2$-cl(A) − A contains no nonempty $(\tau_1, \tau_2)^*$-semi closed set.

Since $F \subseteq \tau_1\tau_2$-cl(A) − A, $F = \varphi \subset \tau_1\tau_2$-int$[\tau_1\tau_2$-cl(A) − A].

Therefore, $\tau_1\tau_2$-cl(A) − A is $(\tau_1, \tau_2)^*$-s^*g open.

Conversely,

Assume that $\tau_1\tau_2$-cl(A) − A is $(\tau_1, \tau_2)^*$-s^*g open.

Suppose that U is $(\tau_1, \tau_2)^*$-semi open and $A \subseteq U$.

Since $A \subseteq U$, $U^C \subseteq A^C$.

Therefore, $\tau_1\tau_2$-cl(A) $\cap U^C \subseteq \tau_1\tau_2$-cl(A) $\cap A^C = \tau_1\tau_2$-cl(A) − A.

Since U is $(\tau_1, \tau_2)^*$-semi open in X, U^C is $(\tau_1, \tau_2)^*$-semi closed in X.

Since $\tau_1\tau_2$-cl(A) is $(\tau_1, \tau_2)^*$-semi closed in X and U^C is $(\tau_1, \tau_2)^*$-semi closed in X,

$[(\tau_1\tau_2$-cl(A)$] \cap U^C$ is $(\tau_1, \tau_2)^*$-semi closed in X.

$\tau_1\tau_2$-cl(A) $\cap U^C \subseteq \tau_1\tau_2$-int$[\tau_1\tau_2$-cl(A) − A]

$= \tau_1\tau_2$-int$[\tau_1\tau_2$-cl(A) $\cap A^C$]

$= \varphi$. { Since $\tau_1\tau_2$-cl(A)−A is $(\tau_1, \tau_2)^*$-s^*g open }. . .

Hence $\tau_1 \tau_2$ -cl(A) $\subseteq$ U .

Therefore, A is $(\tau_1 , \tau_2)^*$-s^*g closed .

SECTION-1.5

$(1,2)^*$ -SEMI STAR GENERALIZED CONTINUOUS FUNCTIONS AND PAIRWISE SEMI STAR GENERALIZED T_S –SPACES

Definition: 1.5.1

A map f : X → Y is called **$(1, 2)^*$ -s^*g continuous** if the inverse image of $\sigma_1\sigma_2$ -closed set in Y is $(\tau_1 , \tau_2)^*$ -s^*g closed in X.

Definition: 1.5.2

A map f : X → Y is called **$(1, 2)^*$ -s^*gc irresolute** if the inverse image of $(\sigma_1 ,\sigma_2)^*$ -s^*g closed set in Y is $(\tau_1 , \tau_2)^*$ -s^*g closed in X.

Definition:1.5.3

A map f : X → Y is called **$(1, 2)^*$ -s^*g closed** if the image of $\tau_1 \tau_2$ -closed set in X is $(\sigma_1 , \sigma_2)^*$ -s^*g closed in Y.

Theorem: 1.5.4

(a) The composition of two $(1, 2)^*$ -s^*gc irresolute functions is $(1, 2)^*$ -s^*gc irresolute,i.e., if f, g are $(1, 2)^*$ -s^*gc irresolute, then $g o f$ is also $(1, 2)^*$ -s^*gc irresolute

(b) If f is $(1, 2)^*$ -s^*gc irresolute and g is $(1, 2)^*$ -s^*g continuous, then $g o f$ is also $(1, 2)^*$ -s^*g continuous.

Proof:

(a) Let f : $(X, \tau_1 , \tau_2) \to (Y, \sigma_1 , \sigma_2)$ and g : $(Y, \sigma_1 , \sigma_2) \to (Z, \mu_1 , \mu_2)$ be two $(1, 2)^*$ -s^*gc irresolute functions.

To prove: $g o f$ is $(1, 2)^*$ -s^*g continuous.

Let V be a $(\mu_1 , \mu_2)^*$ -s^*g closed in Z.

Since g is $(1, 2)^*$ -s^*gc irresolute, g^{-1} (V) is $(\sigma_1 , \sigma_2)^*$ -s^*g closed in Y

. . .

Since f is $(1, 2)^*$ -s^*gc irresolute, $f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V)$ is $(\tau_1, \tau_2)^*$ -s^*g closed in X.

Therefore, $g \circ f$ is $(1, 2)^*$ -s^*gc irresolute.

(b) Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1, 2)^*$ -s^*gc irresolute $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ be $(1, 2)^*$ -s^*g continuous

To prove: $g \circ f$ is $(1, 2)^*$ -s^*gc irresolute

Let V be a $\mu_1 \mu_2$ - closed in Z.

Since g is $(1, 2)^*$ -s^*g continuous, $g^{-1}(V)$ is $(\sigma_1, \sigma_2)^*$ -s^*g closed in Y Since f is $(1, 2)^*$ -s^*gc irresolute, $f^{-1}[g^{-1}(V)] = (g \circ f)^{-1}(V)$ is $(\tau_1, \tau_2)^*$ -s^* g closed in X.

Therefore, $g \circ f$ is $(1, 2)^*$ -s^*g continuous.

(since $\sigma_1 \sigma_2$ closed is $(\sigma_1, \sigma_2)^*$ -s^*g closed.)

Remark: 1.5.5

The composition of two $(1, 2)^*$ -s^*g continuous functions need not be $(1, 2)^*$ -s^*g continuous

Example: 1.5.6

Let X = {a, b, c}, τ_1 = { φ , X, {a}, {b, c}}, τ_2 ={ φ , X, {a}, {a, b}}. Let Y = {a, b, c}, σ_1 = { φ , Y, {a}}, σ_2 ={ φ , Y, {b, c}}.Let Z = {a, b, c}, μ_1 = { φ, Z, {c}}, μ_2 ={ φ, Z}. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \rightarrow (Z, \mu_1, \mu_2)$ be the identity maps. Then f and g are $(1, 2)^*$ -s^*g continuous maps but $g \circ f$ is not $(1, 2)^*$ -s^*g continuous.

Theorem: 1.5.7

Every $(1, 2)^*$ -continuous function is $(1, 2)^*$ -s^*g continuous.

Proof:

Let f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1, 2)^*$ -continuous function and let V be $\sigma_1 \sigma_2$ -closed set in Y.

To prove: f is $(1, 2)^*$- s^*g continuous

Since f is a $(1, 2)^*$ -continuous function, $f^{-1}(V)$ is $\tau_1 \tau_2$ -closed in X.

...

Since every $\tau_1 \tau_2$ -closed set is $(\tau_1 , \tau_2)^*$ -s^*g closed, $f^{-1}(V)$ is $(\tau_1 , \tau_2)^*$ -s^*g closed in X.

Therefore, f is $(1, 2)^*$ -s^*g continuous.

Remark: 1.5.8

The converse of the above theorem need not be true.

Example: 1.5.9

Let X = {a, b, c}, τ_1 = { φ , X, {a, b}}, τ_2 ={ φ, X}. Let Y = {a, b, c}, σ_1 = { φ, Y, {a}}, σ_2 ={ φ, Y }. Let f : $(X, \tau_1 , \tau_2) \rightarrow (Y, \sigma_1 , \sigma_2)$ be the identity map. Then the function f is $(1, 2)^*$ -s^*g continuous,but not $(1, 2)^*$ -continuous.

Theorem: 1.5.10

(a) Every $(1, 2)^*$ -s^*g continuous function is $(1, 2)^*$ -g continuous.

(b) Every $(1, 2)^*$ -s^*g continuous function is $(1, 2)^*$ -sg continuous.

(c) Every $(1, 2)^*$ -s^*g continuous function is $(1, 2)^*$-gs continuous.

Proof:

(a) Let f : $(X, \tau_1 , \tau_2) \rightarrow (Y, \sigma_1 , \sigma_2)$ be a $(1, 2)^*$ -s^*g continuous function.

To prove: f is $(1, 2)^*$ -g continuous,

Let V be a $\sigma_1 \sigma_2$ -closed set in Y.

Since f is a $(1, 2)^*$ -s^*g continuous function, $f^{-1}(V)$ is $(\tau_1 , \tau_2)^*$ -s^*g closed in X.

Since every $(\tau_1 , \tau_2)^*$ -s^*g closed set is $(\tau_1 , \tau_2)^*$ -g closed, $f^{-1}(V)$ is $(\tau_1 , \tau_2)^*$ -g closed in X.

Therefore, f is $(1, 2)^*$ -g continuous.

(b) Let f : $(X, \tau_1 , \tau_2) \rightarrow (Y, \sigma_1 , \sigma_2)$ be a $(1, 2)^*$ -s^*g continuous function.

To prove: f is $(1, 2)^*$ -sg continuous,

Let V be a $\sigma_1 \sigma_2$ -closed set in Y .

Since f is a $(1, 2)^*$ -s^*g continuous function, $f^{-1}(V)$ is $(\tau_1 , \tau_2)^*$ -s^*g closed in X.

Since every $(\tau_1 , \tau_2)^*$ -s^*g closed set is $(\tau_1 , \tau_2)^*$ -sg closed, $f^{-1}(V)$ is $(\tau_1 , \tau_2)^*$ -sg closed in X.

Therefore, f is $(1, 2)^*$ -sg continuous. ...

(c) Let $f : (X, \tau_1 , \tau_2) \rightarrow (Y, \sigma_1 , \sigma_2)$ be a $(1, 2)^*$ -s^*g continuous function.

To prove: f is $(1, 2)^*$ -gs continuous,

Let V be a $\sigma_1 \sigma_2$ -closed set in Y.

Since f is a $(1, 2)^*$ -s^*g continuous function, $f^{-1}(V)$ is $(\tau_1 , \tau_2)^*$ -s^*g closed in X.

Since every $(\tau_1 , \tau_2)^*$ -s^*g closed set is $(\tau_1 , \tau_2)^*$ -g closed, and also every $(\tau_1 , \tau_2)^*$ -s^*g closed set is $(\tau_1 , \tau_2)^*$ -gs closed,$f^{-1}(V)$ is $(\tau_1 , \tau_2)^*$ -gs closed in X.

Therefore, f is $(1, 2)^*$ -gs continuous.

Remark: 1.5.11

The converse of the above theorem need not be true.

Example: 1.5.12

Let X = {a, b, c}, τ_1 = { φ , X, {a}}, τ_2 ={ φ, X, {b}}. Let Y = {a, b, c}, σ_1 = { φ, Y, {c}}, σ_2 ={ φ, Y }. Let $f : (X, \tau_1 , \tau_2) \rightarrow (Y, \sigma_1 , \sigma_2)$ be the identity map. Then f is $(1, 2)^*$ -g continuous and $(1, 2)^*$ -gs continuous, but not $(1, 2)^*$ -s^*g continuous.

Example: 1.5.13

Let X = {a, b, c}, τ_1 = { φ, X, {a}, {b}, {a, b}}, τ_2 ={ φ, X}. Let Y = {a, b, c}, σ_1 = { φ , Y, {a}}, σ_2 ={ φ, Y, {b, c}}. Let $f : (X, \tau_1 , \tau_2) \rightarrow (Y, \sigma_1 , \sigma_2)$ be the identity map.Then f is $(1, 2)^*$ -sg continuous, but not $(1, 2)^*$ -s^*g continuous.

Theorem: 1.5.14

Every $(1, 2)^*$ -s^*gc irresolute map is $(1, 2)^*$ -s^*g continuous map.

Remark: 1.5.15

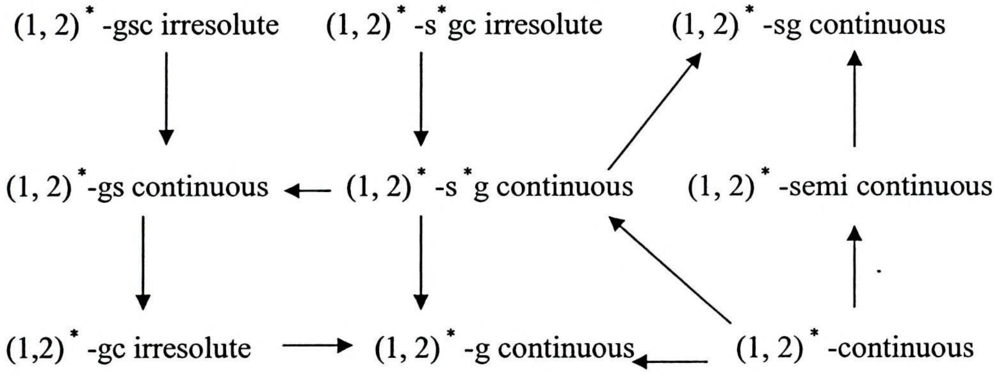
The converse of the above theorem need not be true.

Example: 1.5.16

Let X = {a, b, c}, τ_1 = { φ , X, {a}, {b, c}}, τ_2 ={ φ, X, {a}, {a, b}}. Let Y = {a, b, c}, σ_1 = { φ, Y, {a}}, σ_2={ φ, Y, {b, c}}. Let $f : (X, \tau_1 , \tau_2) \rightarrow (Y, \sigma_1 , \sigma_2)$ be the identity map. Then the function f is $(1, 2)^*$ -s^*g continuous, but not $(1, 2)^*$ -s^*gc ...

irresolute.

Remark: 1.5.17



Theorem: 1. 5. 18

(a) Every $(1, 2)^*$ -s^*g closed function is $(1, 2)^*$ -g closed,

(b) Every $(1, 2)^*$ -s^*g closed function is $(1, 2)^*$ -sg closed,

(c) Every $(1, 2)^*$ -s^*g closed function is $(1, 2)^*$ -gs closed.

Proof:

(a) Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a $(1, 2)^*$ -s^*g closed function.

To prove: f is $(1, 2)^*$ -g closed

Let V be a $\tau_1 \tau_2$ -closed set in X.

Since f is a $(1, 2)^*$ -s^*g closed function, f (V) is $(\sigma_1, \sigma_2)^*$ -s^*g closed in Y .

Since every $(\sigma_1, \sigma_2)^*$ -s^*g closed set is $(\sigma_1, \sigma_2)^*$ -g closed, f (V) is $(\sigma_1, \sigma_2)^*$ -g closed in Y .

Therefore, f is $(1, 2)^*$ -g closed.

(b) Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a $(1, 2)^*$ -s^*g closed function.

To prove: f is $(1, 2)^*$ -sg closed

Let V be a $\tau_1 \tau_2$ -closed set in X.

Since f is a $(1, 2)^*$ -s^*g closed function, f (V) is $(\sigma_1, \sigma_2)^*$ -s^*g closed in Y .

Since every $(\sigma_1, \sigma_2)^*$ -s^*g closed set is $(\sigma_1, \sigma_2)^*$ -sg closed, f (V) is $(\sigma_1, \sigma_2)^*$ -sg closed in Y .

Therefore f is $(1, 2)^*$ -g closed.

(c) Let $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a $(1, 2)^*$ -s^*g closed function.

To prove: f is $(1, 2)^*$ -gs closed

Let V be a $\tau_1 \tau_2$ -closed set in X.

Since f is a $(1, 2)^*$ -s^*g closed function, f (V) is $(\sigma_1 , \sigma_2)^*$ -s^*g closed in Y .

Since every $(\sigma_1 , \sigma_2)^*$ -s^*g closed set is $(\sigma_1 , \sigma_2)^*$ -gs closed, f (V) is $(\sigma_1 , \sigma_2)^*$ -gs closed in Y .

Therefore, f is $(1, 2)^*$ -gs closed.

Remark: 1.5.19

The converse of the above theorem need not be true.

Example: 1.5.20

Let X = {a, b, c}, τ_1 = { φ , X, {a, b}}, τ_2 ={ φ, X}. Let Y = {a, b, c}, σ_1 = { φ, Y, {a}}, σ_2 ={ φ, Y }. Let f : (X, τ_1 , τ_2) → (Y, σ_1 , σ_2) be the identity map. Then the function f is $(1, 2)^*$ -g closed, $(1, 2)^*$ -sg closed and $(1, 2)^*$ -gs closed, but not $(1, 2)^*$ -s^*g closed.

Definition: 1.5.21

A space (X, τ_1, τ_2) is a **pairwise semi star generalized T_S -space** (simply, pairwise s^*g-T_S space) if every $(\tau_1, \tau_2)^*$ –s^*g closed set in (X, τ_1, τ_2) is $\tau_1\tau_2$-closed.

Theorem: 1.5.22

If X is pairwise s^*g-T_S space, every singletone is either $(\tau_1 , \tau_2)^*$ -semi closed or $\tau_1 \tau_2$ -open.

Proof:

Let X be pairwise s^*g-T_S space.

To prove: Every singletone is either $(\tau_1 , \tau_2)^*$ -semi closed or $\tau_1 \tau_2$ -open.

Suppose {x} is not a $(\tau_1 , \tau_2)^*$ -semi closed subset for some $x \in X$.

Then, X −{x} is not $(\tau_1 , \tau_2)^*$ -semi open.

Hence X is the only $(\tau_1 , \tau_2)^*$ -semi open set containing X −{x}.

Hence X −{x} is $(\tau_1 , \tau_2)^*$ -s^*g closed.

Since (X, τ_1 , τ_2) is pairwise s^*g-T_S space, X − {x} is $\tau_1 \tau_2$ -closed. Thus {x} is $\tau_1 \tau_2$ open.