On $\pi\gamma$ Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces

By

Archana A

(20PMA003)

Supervisor

Dr. S. Prema

Thesis Submitted to

Avinashilingam Institute for Home Science and Higher Education for Women

Coimbatore-641 043

In Partial Fulfilment of the Requirement for the Degree of

Master of Science in Mathematics

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N. Balamani 18.05.2022
Signature of the Head of the Department

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Signature of the Supervisor

DECLARATION

I do hereby declare that the thesis entitled " **On $\pi\gamma$ Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces"** submitted to the Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore for the award of the degree of **Master of Science** in Mathematics is a record of original work done by me under the guidance and supervision of **Dr. S. PREMA,** Assistant Professor, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore and it has not previously formed the basis for the award of any degree, diploma, fellowship or associate ship or any other similar title of any candidate of any university.

A.Archana
Signature of the Candidate

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CHAPTER 1

1.1 INTRODUCTION

Topology

The attitude towards uncertainty, prevalent centuries ago, was expressed by the ancient Greek philosopher, Aristotle (384–322 BC), "It is the mark of an instructed mind to rest satisfied with that degree of precision which the nature of the subject admits, and not to seek exactness where only an approximation of the truth is possible".

Topology is a branch of mathematics which studies properties of the spaces that are invariant under transformation. The word topology derived from a Greek word TOPOS meaning "location" and LOGOS meaning "study". Topology is an extension of geometry. General topology known as point set topology is the branch of topology dealing with the basic set theoretic definitions and constructions used in topology. It is the foundation of most other branches of topology, including differential topology, geometric topology, algebraic topology and network topology. The fundamental concepts in point set topology are continuity, compactness and connectedness. The topological structures are modeled suitably in the fields of computer graphics, pattern recognition, artificial intelligence, data mining, information systems, rough set theory, quantum physics etc.

Fuzzy Topology

The notion of fuzzy set theory was introduced by Zadeh in the year 1965.After the introduction, it has caused great interest among both pure and applied mathematics. Several researches were conducted on the generalizations of the notion of fuzzy set. In fuzzy set theory the membership of an element to a fuzzy set is a single value between zero and one. However in reality it may not be always true that the degree of non membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some hesitation degree. A fuzzy set is created to describe linguistic variables in more detail. The linguistic variable 'PERFORMANCE' for instance, may have categories (members) of very small, small, medium, large and very large. Chang (1968) gave the topological structure to these fuzzy sets called fuzzy topological spaces and studied the topological properties. Most basic concepts like open set, closed set, neighbourhood, interior of a set, continuity and compactness etc. Fuzzy sets are designed to handle a particular kind of uncertainty, namely degree-vagueness, which results a property that can be possessed by objects to varying degrees.

Intutionistic Fuzzy Topology

Intuitionisitic fuzzy set, an extension of standard fuzzy set, has been introduced by Atanassov (1986). Intuitionisitic fuzzy set has been found to be more efficient in dealing with vagueness and ambiguity. It is characterized by a membership function (μ_A(x)) and a non membership function (ν_A(x)) with their sum being less than or equal to one ($0 \leq \mu_A(x) + \nu_A(x) \leq 1$). This relaxes the enforced duality $\nu_A(x) = 1- \mu_A(x)$ from fuzzy set theory. Intuitionistic fuzzy set allows one to address the positive and negative side of an imprecise concept separately. Intuitionistic fuzzy sets can be more precisely expressed. For example, the fact that the performance of a student changes, and other characteristics are not quite clear. There is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object.

Intuitionistic fuzzy set is useful in decision making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. Recently various applications of intuitionistic fuzzy set like artificial intelligence, intuitionistic fuzzy expert systems, intuitionistic fuzzy neural networks, intuitionistic fuzzy machine learning, intuitionistic fuzzy semantic representations etc., have appeared. Using the notion of intuitionistic fuzzy sets, Coker (1997) has constructed the basic concepts of intuitionistic fuzzy topological spaces. After giving the fundamental definitions and the necessary examples he introduced the definitions of intuitionistic fuzzy continuity, intuitionistic fuzzy compactness, intuitionistic fuzzy connectedness and obtained several preservation properties and some characterizations concerning intuitionistic fuzzy connectedness**.** IFSs are used in medical decision making in medicine and medical diagnosis.

Szmidt and Kacprzyk described a geometrical representation of an intuitionistic fuzzy set is a point of departure for our proposal of distances between intuitionistic fuzzy sets. New definitions are introduced and compared with the approach used for fuzzy sets. It is shown that all three parameters describing intuitionistic fuzzy sets should be taken into account while calculating those distances.

In this thesis, a new generalization of closed set called intuitionistic fuzzy $\pi\gamma$ generalized closed set in intuitionistic fuzzy topological spaces is introduced. Further the corresponding open sets and continuous mapping are being introduced and studied. Some interesting properties, results and their respective properties, preservation theorems, interrelations are discussed with necessary counter examples. For this, the thesis is organized in to three chapters, as given below:

In Chapter 1, the first section begins with the introduction for topology, fuzzy topology and intuitionistic fuzzy topology with examples. Second section deals with review of literature for topology, fuzzy topology and intutionistic fuzzy topology. In third section, the preliminary definitions, results, and lemma required for intuitionistic fuzzy $\pi\gamma$ closed sets are given in detail.

In Chapter 2, the first section gives a short introduction to intuitionistic fuzzy closed sets. In the second section we have introduced and studied a new class of sets called intuitionistic fuzzy $\pi\gamma$ generalized closed sets in intuitionistic fuzzy topological spaces. This section deals with the interrelation of our newly introduced sets with other type of intuitionistic fuzzy closed sets such as intuitionistic fuzzy regular closed sets. intuitionistic fuzzy semi closed sets, intuitionistic fuzzy α closed sets, intuitionistic fuzzy pre closed sets, , intuitionistic fuzzy π closed sets, , intuitionistic fuzzy γ closed sets, intuitionistic fuzzy generalized closed sets, intuitionistic fuzzy pre closed sets. Also it is shown that the converses are not true in general and they are proved with necessary counter examples. In section three, intuitionistic fuzzy $\pi\gamma$ generalized open sets are introduced and their interrelations with other intuitionistic fuzzy open sets are established. Also some of their properties are studied. In section four, we have introduced some theoretical application of intuitionistic fuzzy $\pi\gamma$ generalized closed sets and we have proved some interesting propositions.

In Chapter 3, a new type of intuitionistic fuzzy continuous mapping called intuitionistic fuzzy $\pi\gamma$ generalized continuous mapping has been introduced. This chapter is divided into two sections. In the first section, we give a short introduction to intuitionistic fuzzy continuous mappings. In the second section, we have introduced intuitionistic fuzzy $\pi\gamma$ generalized continuous mappings and analyzed the interrelations between intuitionistic fuzzy $\pi\gamma$ generalized continuous mapping with other existing continuous mappings. Some fascinating theorems concerning intuitionistic fuzzy $\pi\gamma$ generalized continuous mappings are discussed.

Throughout the thesis, the following notations are used.

- (X, τ) and (Y, σ) denote non empty intuitionistic fuzzy topological spaces on which no separation axioms are mentioned unless it is stated specifically.
- The closure and interior of a subset A of an intuitionistic fuzzy topological space is denoted by cl(A) and int(A) respectively.
- The extension of the notation of union, intersection, contained in and contain, in intutionistic fuzzy topological spaces are denoted as $\cup, \cap, \leq, \geq$.
- In all the diagrams A⟶B represents A implies B but not conversely.

1.2 REVIEW OF LITERATURE

A review of literature of recent developments on generalized notions of closed and open sets and continuous mappings in topological spaces, fuzzy topological spaces and intuitionistic fuzzy topological spaces is given below:

Topological Spaces

Closed sets are fundamental objects in topological spaces. In the study of topological spaces many concepts of topology have been generalized by introducing the concept of semi open sets (Levine (1963)) instead of open sets. The concept of generalized closed sets introduced by Levine (1970) plays a significant role in general topology. Using this concept and Levine's idea, many researchers have introduced and studied various types of generalized closed sets. Mashour, Hasanein and El. Deep (1983) have introduced α continuous and α open mapping in topological spaces. Bhattacharya and Lahari (1987) have introduced semi generalized closed sets in topology. Palaniappan and Rao (1993) have introduced regular generalized closed sets. Maki et.al (1993, 1994) have introduced generalized α closed sets and α generalized closed sets in topological spaces. b-open sets was introduced by Andrijevic (1996). Fukutake, Nasef and El-Maghrabi (2003) introduced γ generalized closed sets in topological spaces. Ganster and Steiner (2007) investigated many relationships between b generalized closed sets with generalized notions of closed sets. Ahmad Al-Omari et.al (2009) introduced generalized b-closed sets. Narmatha, Nagaveni and Noiri (2013) have introduced regular b-open sets in topological spaces.

The notion of continuous maps is one of the most important concepts in topological spaces. Mashhour, Abd El- Monsef and El-deep (1982) have investigated pre-continuous and weak pre-continuous mappings in topology. Balachandran, Sundaram, and Maki (1991) have introduced generalized continuous maps in topological spaces. Fukutake, Nasef and EL-Maghrabi (2003) have introduced γ generalized continuous maps in topological spaces. Ekici and Caldas (2004) introduced slightly γ continuous functions.

Fuzzy topological spaces

The concept of fuzzy set which was first introduced by Zadeh (1965) is a mathematical means of describing vagueness in linguistics. Subsequently, several researchers have worked on various basic concepts from classical topology using fuzzy sets and developed fuzzy topological spaces. The notion of fuzzy sets plays a very significant role in the study of fuzzy topology introduced by Chang (1968), who studied a number of basic concepts, including fuzzy continuous maps and compactness. Lowen (1982) has introduced fuzzy neighbourhood spaces. Ganguly and Saha (1986) has introduced fuzzy semi open sets in fuzzy topological spaces. Singal and Niti Prakash (1991) have introduced fuzzy pre open sets. Fuzzy α open sets and fuzzy pre open sets were introduced by Bin Shahna (1991). Thakur and Malviya (1995) have introduced generalized closed sets in fuzzy topology. Maki et.al (1998) introduced generalized closed sets in fuzzy topological spaces. Warren (1978) gave the characterizations of fuzzy continuous functions characterized by the closure of fuzzy sets, a sub basis of a fuzzy topology and fuzzy neighbourhoods. Bayaz Daraby and Nimse (2007) have discussed fuzzy generalized alpha closed set and its applications. Benchalli and Jenifer Karnel (2010) have introduced fuzzy b-open sets in fuzzy topological spaces.

Munir Abdul-Khalik, Al-Khafaji and Jaafer Jabbar Cassem (2014) have introduced fuzzy generalized b- closed set in fuzzy topological spaces. Balasubramanian and Sundaram (1997) have investigated some generalizations of fuzzy continuous functions. Abd EI-Hakeim (1999) has introduced generalized semi continuous mappings in fuzzy topological spaces. Hanafy (2009) has introduced fuzzy γ continuity and contributed some beautiful results in fuzzy topological spaces.

Intuitionistic fuzzy topological spaces

Atanassov (1986, 1988, 1989, 1994) has introduced intuitionistic fuzzy sets and also gave new results in intuitionistic fuzzy sets and operations. Intuitionistic fuzzy points are introduced by Coker and Demirci (1995). Intuitionistic fuzzy open sets and intuitionistic fuzzy closed sets are introduced by Coker (1997). Intuitionistic fuzzy semi open sets, intuitionistic fuzzy pre open sets, intuitionistic fuzzy α open sets are introduced by Joung Kon Jeon et.al (2005). Thakur and Rekha Chaturvedi (2008) have introduced generalized closed sets in intuitionistic fuzzy topology. Hanafy

(2009) studied the properties of intuitionistic fuzzy γ closed sets and intuitionistic fuzzy γ open sets. Santhi and Arun Prakash (2010) have introduced intuitionistic fuzzy semi-generalized closed sets and their applications. Rajarajeswari and Senthil Kumar (2011) have introduced generalized pre-closed sets in intuitionistic fuzzy topological spaces. The concept nowhere dense in intuitionistic fuzzy topological space is introduced by Thakur and Dhavaseelan (2015). Kanimozhi and Jayanthi (2016) have introduced and studied the concepts of intuitionistic fuzzy generalized γ closed sets, intuitionistic fuzzy generalized γ open sets and investigated their properties.

Hur and Jun (2003) have studied intuitionistic fuzzy alpha-continuous mappings. Thakur and Rekha Chaturvedi (2006) have introduced generalized continuity in intuitionistic fuzzy topology. Hanafy (2009) has introduced intuitionistic fuzzy γ continuity in intuitionistic fuzzy topological spaces. Santhi and Sakthivel (2009a) have introduced intuitionistic fuzzy generalized semi continuous mappings.

Some of the research articles which I refer for the thesis are given below.

1. FUZZY SETS

[Zadeh, 1965]

In this article, the author has introduced a new class of sets namely fuzzy sets which are characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Further the author has provided the notions of inclusion, union, intersection, complement etc., with respect to the fuzzy sets.

2. GENERAL TOPOLOGY

[Bourbaki, N, 1966]

In this book, important classes of topological spaces are studied, uniform structures are introduced and applied to topological groups. Real numbers are constructed and their properties established.

3. FUZZY TOPOLOGICAL SPACES

[Chang, 1968]

In this article, the author has introduced fuzzy topological spaces. This concept is considered to be the generalization of general topological spaces. In brief, the basic concepts such as fuzzy open set, fuzzy closed set, fuzzy neighbourhood, fuzzy continuity etc., are discussed in depth.

4. MORE ON γ-GENERALIZED CLOSED SETS IN TOPOLOGY

[Maghrabi, 2013]

In this article, the author has introduced and studied a new class of sets called γ generalized regular weakly closed set. It has been proved that the new classes of sets lie between the class of regular weakly closed sets and the class of γ generalized closed sets.

5. FUZZY GENERALIZED π CLOSED SET IN FUZZY TOPOLOGICAL SPACES

[Andal. M, Thiripurasundari. V, 2019]

In this paper, the author has introduced the concept of fuzzy π closed and fuzzy generalized π closed sets in a fuzzy topological space. Some characterizations, examples, and properties are discussed by the author.

6. FUZZY GENERALIZED ALPHA CLOSED SET AND ITS APPLICATIONS

[Bayaz Daraby and Nimse, 2007]

In this article they have defined and studied fuzzy generalized alpha closed sets, r open sets, fuzzy alpha continuous functions and their applications.

7. FUZZY GENERALIZED γ CLOSED SET IN FUZZY TOPOLOGICAL SPACE

[Dipankar, 2014]

In this article, the author has introduced and studied the concepts of fuzzy generalized γ closed sets and their basic properties in fuzzy topological spaces. Moreover, he has defined fuzzy γ T1/2 space in which every fuzzy generalized γ continuous is fuzzy γ continuous.

8. INTUITIONISTIC FUZZY SETS

[Atanassov, 1986]

In this article, the author has provided the notion of intuitionistic fuzzy sets. This is considered to be the generalization on fuzzy sets. The highlight of this particular article is that some relations and operations concerning classical sets are extended to intuitionistic fuzzy sets.

9. AN INTRODUCTION TO INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Coker, 1997]

In this article, the author has introduced intuitionistic fuzzy topological spaces. The notions of intuitionistic fuzzy interior and intuitionistic fuzzy closure are being provided and this is followed by the discussion of some important properties concerning them. Furthermore, the notion of intuitionistic fuzzy continuity is provided.

10. ON FUZZY CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Gurcay, Coker and Haydar, 1997]

This article consists of the notions of intuitionistic fuzzy semi open sets, intuitionistic fuzzy pre open sets, intuitionistic fuzzy α-open sets and their corresponding closed sets. Further the relationship between these sets are established.

11. MORE ON INTUITIONISTIC FUZZY SETS

[Atanassov, 1989]

In this article, the author has analyzed some new results on intuitionistic fuzzy sets. Two new operators on intuitionistic fuzzy sets are defined and their basic properties are studied.

12. INTUITIONISTIC FUZZY ALPHA CONTINUITY AND INTUITIONISTIC FUZZY PRECONTINUITY

[Joung Kon Jeon, Young Bae Jun and Jin Han Park, 2005]

In this article, the authors have introduced intuitionistic fuzzy alpha contuinuity and intuitionistic fuzzy pre contuinuity and discussed the relationship between the newly introduced continuous mappings with some of the previously defined intuitionistic fuzzy continuous mappings.

13. GENERALIZED CONTINUITY IN INTUITIONISTIC FUZZY TOPOLOGY

[Thakur and Rekha Chaturvedi, 2006]

In this paper, the authors have discussed and studied the concept of intuitionistic fuzzy generalized continuous mappings in intuitionistic fuzzy topological spaces. They have analyzed some of their properties and obtained some interesting theorems.

14. NOWHERE DENSE SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Thakur and Dhavaseelan, 2015]

In this article the authors have introduced the concept of nowhere dense sets and investigated the characterizations of intuitionistic fuzzy nowhere dense sets.

15. THE CATEGORY OF INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Seok Jong Lee and Eun Pyo Lee, 2000]

In this paper, the authors have introduced the concept of intuitionistic fuzzy neighbourhoods. They have investigated the properties of intuitionistic fuzzy continuous mappings, intuitionistic fuzzy open mappings and intuitionistic fuzzy closed mappings in intuitionistic fuzzy topological spaces.

16. INTUITIONISTIC FUZZY γ CONTINUITY

[Hanafy, I, M., 2009]

In this paper, the author has introduced fuzzy γ open sets and fuzzy γ continuity in intuitionistic fuzzy topological spaces and discussed some of its properties.

17. ON INTUTIONISTIC FUZZY GENERALIZED γ CLOSED SETS

[Kanimozhi. R, Jayanthi. D., 2016]

In this paper, the author have introduced intuitionistic fuzzy generalized γ closed sets, and investigated some of their properties. Some characteristics of the intuitionistic fuzzy generalized γ closed sets are also studied.

18. $\alpha\pi$g- CLOSED SETS IN INTUITIONISTC FUZZY TOPOLOGICAL SPACES

[Venkatachalam. M, Kannan, Ramesh. K., 2018]

In this paper the author has introduced $\alpha\pi$g intuitionistic fuzzy closed topological spaces and discussed about some of its properties.

19. $\pi g\gamma*$ CLOSED SETS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

[Sakthivel. K, Manikandan.M, 2019]

This article introduced the concept of $\pi g\gamma*$ closed sets and $\pi g\gamma*$ open sets in intutionistic fuzzy topological spaces. And also the author has analyses some of the properties of $\pi g\gamma*$ closed and $\pi g\gamma*$ open sets in intuitonistic fuzzy topological spaces with suitable examples.

20. ON INTUITIONISTIC FUZZY γ GENERALIZED CLOSED SETS

[Prema. S, Jayanthi. D., 2017]

In this paper, the author have introduced the notion of intutionistic fuzzy γ generalized closed sets, and investigated some of their properties and produced some characterization theorems with some examples.

21. ON INTUTITIONISTIC FUZZY $\gamma*$ GENERALIZED CLOSED SETS

[Riya V.M, Jayanthi. D, 2017]

In this article, the author has introduced the a new class of closed sets namely intutionistic fuzzy $\gamma*$ generalized closed sets intuitionistic fuzzy topological spaces, studied the relationship between the new class of set with the other existing intuitionistic fuzzy closed sets. Some properties and several characterizations are investigated. Also the author have obtained some interesting theorems.

22. INTUITIONISTIC FUZZY γ GENERALIZED CONTINUOUS MAPPINGS

[Prema. S, Jayanthi.D., 2017]

In this paper, the authors have introduced intutionistic fuzzy γ generalized continuous mappings and investigated some of their properties. Also the author has provided some characterization of intutionistic fuzzy γ generalized continuous mappings.

Notations

IFS - Intuitionistic fuzzy set

IFSs - Intuitionistic fuzzy sets

IFT - Intuitionistic fuzzy topology

IFTS - Intuitionistic fuzzy topological space

A^c - The complement of A

Int(A) - Interior of A

cl(A) - Closure of A

IFC(X) - The family of all intuitionistic fuzzy closed sets of X

IFSC(X) - The family of all intuitionistic fuzzy semi closed sets of X

IFPC(X) - The family of all intuitionistic fuzzy pre closed sets of X

IFαC(X) - The family of all intuitionistic fuzzy α closed sets of X

IFRC(X) - The family of all intuitionistic fuzzy regular closed sets of X

IFγC(X) - The family of all intuitionistic fuzzy γ closed sets of X

IFγGC(X) - The family of all intuitionistic fuzzy γ generalized closed sets of X

IFO(X) - The family of all intuitionistic fuzzy open sets of X

IFSO(X) - The family of all intuitionistic fuzzy semi open sets of X

IFPO(X) - The family of all intuitionistic fuzzy pre open sets of X

IFαO(X) - The family of all intuitionistic fuzzy α open sets of X

IFRO(X) - The family of all intuitionistic fuzzy regular open sets of X

IFγO(X) - The family of all intuitionistic fuzzy γ open sets of X

IFγGO(X) - The family of all γ generalized open sets of X

IFγGCSs - Intuitionistic fuzzy γ generalized closed sets

IFγGOSs - Intuitionistic fuzzy γ generalized open sets

IF$\pi\gamma$GCSs - Intuitionistic fuzzy $\pi\gamma$generalized closed sets

IF$\pi\gamma$GOSs - Intuitionistic fuzzy $\pi\gamma$ generalized open sets

1.3 PRELIMINARIES

In this section, the basic definitions of intuitionistic fuzzy sets, intuitionistic fuzzy continuous mappings and some results in intuitionistic fuzzy topological spaces that are used to accomplish the present study are given in detail.

Intuitionistic Fuzzy Sets

Definition 1.3.1:

Let X be a non empty fixed set. An ***intuitionistic fuzzy set*** (IFS in short) A in X is an object having the form A = $\{\langle x, \mu_A(x), \nu_A(x)\rangle / x \epsilon X\}$ where the functions $\mu_A(x)$: X → [0,1] and $\nu_A(x)$: X → [0,1] denotes the degree of membership (namely $\mu_A(x)$) and the degree of non – membership (namely $\nu_A(x)$) of each element x ϵX to the set A, respectively, and 0 ≤ $\mu_A(x) + \nu_A(x) \leq 1$ for each x ϵX. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 1.3.2:

Let A and B be IFSs of the form A= $\{< x, \mu_A(x), \nu_A(x) > / x \epsilon X\}$ and B= $\{< x, \mu_B(x), \nu_B(x) > / x \epsilon X\}$. Then

a) A ⊆ B if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all x ϵX
b) A=B if and only if A ⊆ B and B ⊆ A
c) $A^c = \{< x, \nu_A(x), \mu_A(x) > / x \epsilon X\}$
d) A ∩ B = $\{< x, \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x) > / x \epsilon X\}$
e) A ∪ B = $\{< x, \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x) > / x \epsilon X\}$

For the sake of simplicity, we shall use the notation A= $< x, (\mu_A, \mu_B), (\nu_A, \nu_B) >$ instead of A= <x, $(A/\mu_A, B/\mu_B)$, $(A/\nu_A, B/\nu_B)$>.

The intuitionistic fuzzy sets 0~ = $\{< x, 0,1) > / x \epsilon X\}$ and 1~ = $\{< x, 0,1) > / x \epsilon X\}$ are respectively the empty set and the whole set of X.

Definition 1.3.3:

An ***intuitionistic fuzzy topology*** (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

i. $0_\sim, 1_\sim \in \tau$
ii. $G1 \cap G2 \in \tau$, for any $G1, G2 \in \tau$
iii. $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an ***intuitionistic fuzzy topological space*** (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 1.3.4:

Let A, B and C be intuitionistic fuzzy sets in X. Then

i. $(A \subseteq B)$ and $(C \subseteq D) \Rightarrow (A \cup C) \subseteq (B \cup D)$ and $(A \cap C) \subseteq (B \cap D)$
ii. $A \subseteq B$ and $A \subseteq C \Rightarrow A \subseteq (B \cap C)$
iii. $A \subseteq C$ and $B \subseteq C \Rightarrow (A \cup B) \subseteq C$
iv. $A \subseteq B$ and $B \subseteq C \Rightarrow A \subseteq C$
v. $(A \cup B)^c = A^c \cap B^c$
vi. $(A \cap B)^c = A^c \cup B^c$
vii. $A \subseteq B \Rightarrow B^c \subseteq A^c$
viii. $(A^c)^c = A$
ix. $(0_\sim)^c = 1_\sim$
x. $(1_\sim) = 0_\sim$

Definition 1.3.5:

Let (X, τ) be an IFTS and $A = < x,, \mu_A, \nu_A >$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 1.3.6:

Let (X,τ) be an IFTS and A, B be IFSs in X. Then the following properties hold

i. $int(A) \subseteq A$
ii. $A \subseteq cl(A)$
iii. $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$
iv. $A \subseteq B \Rightarrow int(A) \subseteq cl(B)$
v. $int(int(A)) = int(A)$
vi. $cl(cl(A)) = cl(A)$
vii. $int(A \cap B) = int(A) \cap int(B)$
viii. $cl(A \cup B) = cl(A) \cup cl(B)$
ix. $int(1\sim) = 1\sim$
x. $cl(0\sim) = 0\sim$

Definition 1.3.7:

An IFS A of an IFTS (X, τ) is an

(i) ***intuitionistic fuzzy regular open set*** (IFROS in short) if $A = int(cl(A))$,

(ii) ***intuitionistic fuzzy regular closed set*** (IFRCS in short) if $cl(int(A))=A$.

Definition 1.3.8:

An IFS A of an IFTS (X, τ) is an

(i) ***intuitionistic fuzzy semi open set*** (IFSOS in short) if $A \subseteq cl(int(A)$.

(ii) ***intuitionistic fuzzy semi closed set*** (IFSCS in short) if $int(cl(A)) \subseteq A$.

Definition 1.3.9:

An IFS A of an IFTS (X, τ) is an

(i) ***intuitionistic fuzzy α-open set*** (IFαOS in short) if $A \subseteq int(cl(int(A)))$,

(ii) ***intuitionistic fuzzy α-closed set*** (IFαCS in short) if $cl(int(cl(A)) \subseteq A$.

Definition 1.3.10:

An IFS A of an IFTS (X, τ) is an

(i) ***intuitionistic fuzzy pre open set*** (IFPOS in short) if A ⊆ int(cl(A)).

(ii) ***intuitionistic fuzzy pre closed set*** (IFPCS in short) if cl(int(A)) ⊆ A,

Definition 1.3.11:

The union of IFROSs is called ***intuitionistic fuzzy π-open set*** (IFπOS in short) of an IFTS (X, τ). The complement of IFπOS is called ***intuitionistic fuzzy π -closed set*** (IFπCS in short).

Definition 1.3.12:

An IFS A of an IFTS (X, τ) is an

(i) ***intuitionistic fuzzy γ-open set*** (IFγOS in short) if A ⊆ int(cl(A)) ∪ cl(int(A)),

(ii) ***intuitionistic fuzzy γ-closed set*** (IFγCS in short) if cl(int(A)) ∩ int(cl(A)) ⊆ A.

Definition 1.3.13:

Let A be an IFS in an IFTS in (X,τ). Then the intuitionistic fuzzy γ- interior and intuitionistic fuzzy γ-closure of A are defined by

i. γint(A)= ∪ {G/G is an IFγOS in X and G⊆ A},

ii. γcl(A) = ∩{K/K is an IFγCS in X and A⊆K}.

Note that for any IFS A in (X,τ), we have $\gamma cl(A^c) = (\gamma int(A))^c$ and $\gamma int(A^c) = (\gamma cl(A))^c$.

Definition 1.3.14:

Let A be an IFS in (X,τ), then

i. γint(A)⊆ A∩((cl(int(A)) ∩int(cl(A)))

ii. γcl(A)⊇ A ∪((cl(int(A)) ∩int(cl(A)))

Definition 1.3.15:

An IFS A of an IFTS (X, τ) is an ***intuitionistic fuzzy generalized closed set*** (IFGCS in short) if cl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X. The complement of IFGCS is called ***intuitionistic fuzzy generalized open set*** (IFGOS in short).

Definition 1.3.16:

An IFS A is said to be an ***intuitionistic fuzzy generalized pre-closed set*** (IFGPCS in short) in (X,τ) if pcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X. The family of all IFGPCSs of an IFTS (X,τ) is denoted by IFGPC(X).

Definition 1.3.17:

An IFS A is said to be an ***intuitionistic fuzzy generalized semi-closed set*** (IFGSCS in short) in (X, τ) if scl(A)⊆ U whenever A ⊆ U and U is an IFOS in X. The family of all IFGSCSs of an IFTS (X, τ) is denoted by IFGSC(X).

Definition 1.3.18:

An IFS A in an IFTS (X,τ) is said to be ***intuitionistic fuzzy generalized γ closed sets***(IFGγCS for short) if γcl(A)⊆U and U is an IFOS in (X,τ). The family of all IFGγCSs of an IFTS (X,τ) is denoted by IFGγC(X).

Definition 1.3.19:

An IFS A is an IFTS (X,τ) is said to be an ***intuitionistic fuzzy γ generalized closed set*** (IFγGCS for short) if γcl(A) ⊆ U whenever A ⊆ U and U is an IFγOS in (X,τ).

The complement A^c of an IFγGCS A is an IFTS (X, τ) is ***called intuitionistic fuzzy γ generalized open set*** (IFγGOS in short) in X.

Definition 1.3.20:

An A in (X,τ) is called an ***intuitionitic fuzzy nowhere dense set*** if there exist no IFOS U such that U ⊆ $cl(A)$. That is int(cl(A))=$0_\sim$

Definition 1.3.21:

An intuitonistic fuzzy point (IFP in short) written as $p_{(\alpha,\beta)}$ is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}= \begin{cases} (\alpha,\beta) & \text{if } x = p \\ (0,1) & otherwsie \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 1.3.22:

Two IFSs are said to be q-coincident (A_qB in short) if and only if there exists an element x ϵ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 1.3.23:

For any two IFSs A and B are said to be not q-coincident, ($A_{\bar{q}}$ B) if and only if $A \subseteq B^c$.

Definition 1.3.24:

Let A, A_i(i $\in$ J) be intuitionistic fuzzy sets in X and B, B_j(j $\in$ K) be intuitionistic fuzzy sets in Y and f: X → Y be a mapping. Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]
d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]
e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
g) $f^{-1}(0_\sim) = 0_\sim$
h) $f^{-1}(1_\sim) = 1_\sim$
i) $f^{-1}(B^c) = (f^{-1}(B))^c$.

Definition 1.3.25:

Let X and Y be two non empty sets and f: X → Y be a mapping. If $A = \{\langle x, (\mu_A(x), \nu_A(x) / x \in X\rangle\}$ is an IFS in X, then the ***image*** of A under f, denoted by f(A), is the IFS in Y defined by

$$f(A) = \{\langle y, f(\mu_A)(y), f\text{-}(\nu_A)(y) / y \in Y\rangle\},$$

where $f\text{-}(\nu_A) = 1 - f(1 - \nu_A)$.

Definition 1.3.26:

Let X and Y be two non empty sets and f: X → Y be a mapping. If $B = \{\langle y, (\mu_B(y), \nu_B(y) / y \in Y\rangle\}$ is an IFS in Y, then the ***preimage*** of B under f is denoted and defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X\rangle\}$$

where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$

Intuitionistic Fuzzy Continuous Mappings

Definition 1.3.27:

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an **intuitionistic fuzzy continuous** (IF continuous) mapping if $f^{-1}(V)$ is an IFCS in(X,τ) for every IFCS V of (Y,σ)

Definition 1.3.28:

Let f be a mapping from an IFTS (X,τ) into an IFTS (Y,σ). Then f is said to be an

i. ***intuitionistic fuzzy semi continuous*** (IFS continuous) ***mapping*** if $f^{-1}(V)$ is an IFSCS in (X, τ) for every IFCS V of (Y, σ),

ii. ***intuitionistic fuzzy α*** **continuous** (IFα continuous) ***mapping*** if $f^{-1}(V)$ is an IFαCS in (X, τ) for every IFCS V of (Y, σ),

iii. ***intuitionistic fuzzy pre continuous*** (IFP continuous) ***mapping*** if $f^{-1}(V)$ is an IFPCS in (X, τ) for every IFCS V of (Y, σ).

Definition 1.3.29:

Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an ***intuitionistic fuzzy γ continuous*** (IFγ continuous) mapping if $f^{-1}(V)$ is an IFγCS in (X, τ) for every IFCS V of (Y, σ).

Definition 1.3.30:

A mapping f : (x, τ) ⟶ (y,σ) is called an ***intuitionistic fuzzy γ generalized continuous*** (IFγG continuous) mapping if f^{-1} (V) is an IFγGCS in (X, τ) for every IFCS V of (Y, σ).

CHAPTER 2

2.1 $\pi\gamma$ Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces

In this section $\pi\gamma$ generalized closed sets in intuitionistic fuzzy topological spaces is introduced. Also we have established the relationship between basic intutionistic fuzzy sets and intuitionistic fuzzy $\pi\gamma$ generalized closed sets. Also we have analyzed some properties of $\pi\gamma$ generalized closed sets in intuitionistic fuzzy topological spaces.

Definition 2.1.1:

An IFS A in (X, τ) is said to be an *intuitionistic fuzzy $\pi\gamma$ generalized closed sets* (IF$\pi\gamma$GCS in short) if γcl(A)⊆ U whenever A ⊆ U and U is an $IF\pi OS$ in (X, τ).

Example 2.1.2:

Let X = {a,b} and let τ $=\{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on X, where $G_1 = <x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)>$ and $G_2 = <x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)>$. Then the IFS A $=< x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)>$ is an IF$\pi\gamma$GCS in (X,τ).

Theorem 2.1.3:

Every *intuitionistic fuzzy closed set* (IFCS in short) in (X, τ) is an IF$\pi\gamma$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\gamma cl(A) \subseteq cl(A) =$ A ⊆ U. We have $\gamma cl(A) \subseteq U$. Therefore A is an IF$\pi\gamma$GCS in (X, τ).

Example 2.1.4:

Let X ={a, b} and let τ $=\{0_\sim, G_1, G_2, 1_\sim\}$ is an IFT on X, where $G_1 = <x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)>$ and $G_2 = <x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)>$. Then the IFS A $=< x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)>$ is an IF$\pi\gamma$GCS in (X, τ) but not an IFCS in X as cl(A)= $G_1^c \neq$ A.

Theorem 2.1.5:

Every *intuitionistic fuzzy regular closed set* (IFRCS in short) in (X, τ) is an IF$\pi\gamma$GCS in (X, τ) but not conversely.

Proof:

Since every IFRCS is an IFCS in (X, τ). Therefore A is an IF$\pi\gamma$GCS in (X, τ).

Example 2.1.6:

Let X ={a,b} and let τ={$0_\sim, G_1, G_2, 1_\sim$} is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi\gamma$GCS in (X,$\tau$) but not an IFRCS in X as cl(int(A)) = cl(G_1)=$G_1^c \neq$A.

Theorem 2.1.7:

Every *intuitionistic fuzzy semi closed set* (IFSCS in short) in (X, τ) is an IF$\pi\gamma$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFSCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\gamma cl(A) \subseteq Scl(A)$ = A ⊆ U. By hypothesis A⊆ U . Hence $\gamma cl(A) \subseteq U$. Therefore A is an IF$\pi\gamma$GCS in (X, τ).

Example 2.1.8:

Let X ={a,b} and let τ ={$0_\sim, G_1, G_2, 1_\sim$}is an IFT on X, where G_1 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> and G_2 =<$x, (0.2_a, 0.2_b), (0.8_a, 0.8_b)$> . Then the IFS A =< $x, (0.4_a, 0.6_b), (0.6_a, 0.4_b)$> is an IF$\pi\gamma$GCS in (X, τ) but not an IFSCS in (X,τ) as int(cl(A))=int(G_2^c) =$G_1 \not\subseteq$ A.

Theorem 2.1.9:

Every *intuitionistic fuzzy* α *closed set* (IFαCS in short) in (X, τ) is an IF$\pi\gamma$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFαCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\gamma cl(A) \subseteq \alpha cl(A)$ = A⊆ U. By hypothesis $\gamma cl(A) \subseteq U$. Therefore A is an IFπγGCS in (X, τ).

Example 2.1.10:

Let X= {a,b} and let τ ={$0_\sim, G_1, G_2, 1_\sim$} is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi$γGCS in (X, τ) but not an IFαCS in (X, τ) as cl(int(cl(A))) = cl(int(G_1^c))=cl(G_1) = $G_1^c \nsubseteq$ A.

Theorem 2.1.11:

Every *intuitionistic fuzzy pre closed set* (IFPCS in short) in (X, τ) is an IFπγGCS in (X, τ) but not conversely.

Proof:

Let A be an IFPCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\gamma cl(A) \subseteq pcl(A)$ = A⊆ U. By hypothesis $\gamma cl(A) \subseteq U$. Therefore A is an IFπγGCS in (X, τ).

Example 2.1.12:

Let X ={a,b} and let τ ={$0_\sim, G_1, G_2, 1_\sim$} is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi$γGCS in (X, τ) but not an IFRCS in (X, τ) as cl(int(A))=cl(G_1)= $G_1^c \nsubseteq$ A.

Theorem 2.1.13:

Every *intuitionistic fuzzy π closed set* (IFπCS in short) in (X, τ) is an IFπγGCS in (X, τ) but not conversely.

Proof:

Let A be an IFπCS in (X,τ) and let A ⊆ U.Since every IFπCS is an IFCS. Therefore A is an IFπγGCS in (X, τ).

Example 2.1.14:

Let X ={a,b} and let τ ={$0_{\sim}, G_1, G_2, 1_{\sim}$}is an IFT on X, where G_1 =<x, $(0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<x, $(0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< x, $(0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi\gamma$GCS in (X, τ) but not an IFπCS in (X, τ).

Theorem 2.1.15:

Every *intuitionistic fuzzy γ closed set* (IFγCS in short) in (X, τ) is an IF$\pi\gamma$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFγCS in X. Then $\gamma cl(A) = A$. Let A ⊆ U and U be an IFπOS in (X, τ). By hypothesis $\gamma cl(A) \subseteq U$. Therefore A is an IF$\pi\gamma$GCS in (X, τ).

Example 2.1.16:

Let X ={a,b} and let τ ={$0_{\sim}, G_1, G_2, 1_{\sim}$} is an IFT on X, where G_1 =<x, $(0.5_a, 0.6_b), (0.5_a, 0.4_b)$> and G_2 =<x, $(0.2_a, 0.2_b), (0.8_a, 0.8_b)$> . Then the IFS A =< x, $(0.4_a, 0.6_b), (0.6_a, 0.4_b)$> is an IF$\pi\gamma$GCS in (X, τ) but not an IFγCS in (X,τ) as cl(int(A)) ∩ int(cl(A)) = cl(G_2) ∩ int(G_2^c) = G_1^c ∩ G_1= G_1^c ⊈ A .

Theorem 2.1.17:

Every *intuitionistic fuzzy generalized closed set* (IFGCS in short) in (X, τ) is an IF$\pi\gamma$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFGCS and let A ⊆ U and U be an IFπOS in (X, τ). As $\gamma cl(A) \subseteq cl(A)$ = A⊆ U. We have $\gamma cl(A) \subseteq U$. Therefore A is an IF$\pi\gamma$GCS in (X, τ).

Example 2.1.18:

Let X={a,b} and let τ ={$0_{\sim}, G_1, G_2, 1_{\sim}$}is an IFT on X, where G_1 =<x, $(0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<x, $(0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< x, $(0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi\gamma$GCS in (X, τ) but not an IFGCS in (X, τ) as cl(A) = G_1^c ⊈ G_1 where A ⊆ G_1.

Theorem 2.1.19:

Every *intuitionistic fuzzy generalized pre closed set* (IFGPCS in short) in (X, τ) is an IF$\pi\gamma$GCS in (X, τ) but not conversely.

Proof:

Let A be an IFGPCS and let A ⊆ U and U be an IFπOS in (X, τ). That is U is an IFOS in X. By hypothesis $pcl(A) \subseteq U$, which implies γcl(A) ⊆ pcl(A)⊆ U. That is $\gamma cl(A) \subseteq U$. Therefore A is an IF$\pi\gamma$GCS in (X, τ).

Example 2.1.20:

Let X ={a,b} and let τ ={$0_\sim, G_1, G_2, 1_\sim$}is an IFT on X, where G_1 =<$x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> and G_2 =<$x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)$> . Then the IFS A =< $x, (0.4_a, 0.5_b), (0.6_a, 0.5_b)$> is an IF$\pi\gamma$GCS in (X,$\tau$) but not an IFGPCS in (X, τ).

Theorem 2.1.21:

Every *intuitionistc fuzzy generalized semi closed set* (IFGSCS in short) in (X, τ) is an IF$\pi\gamma$GCS in (X, τ) but not conversely.

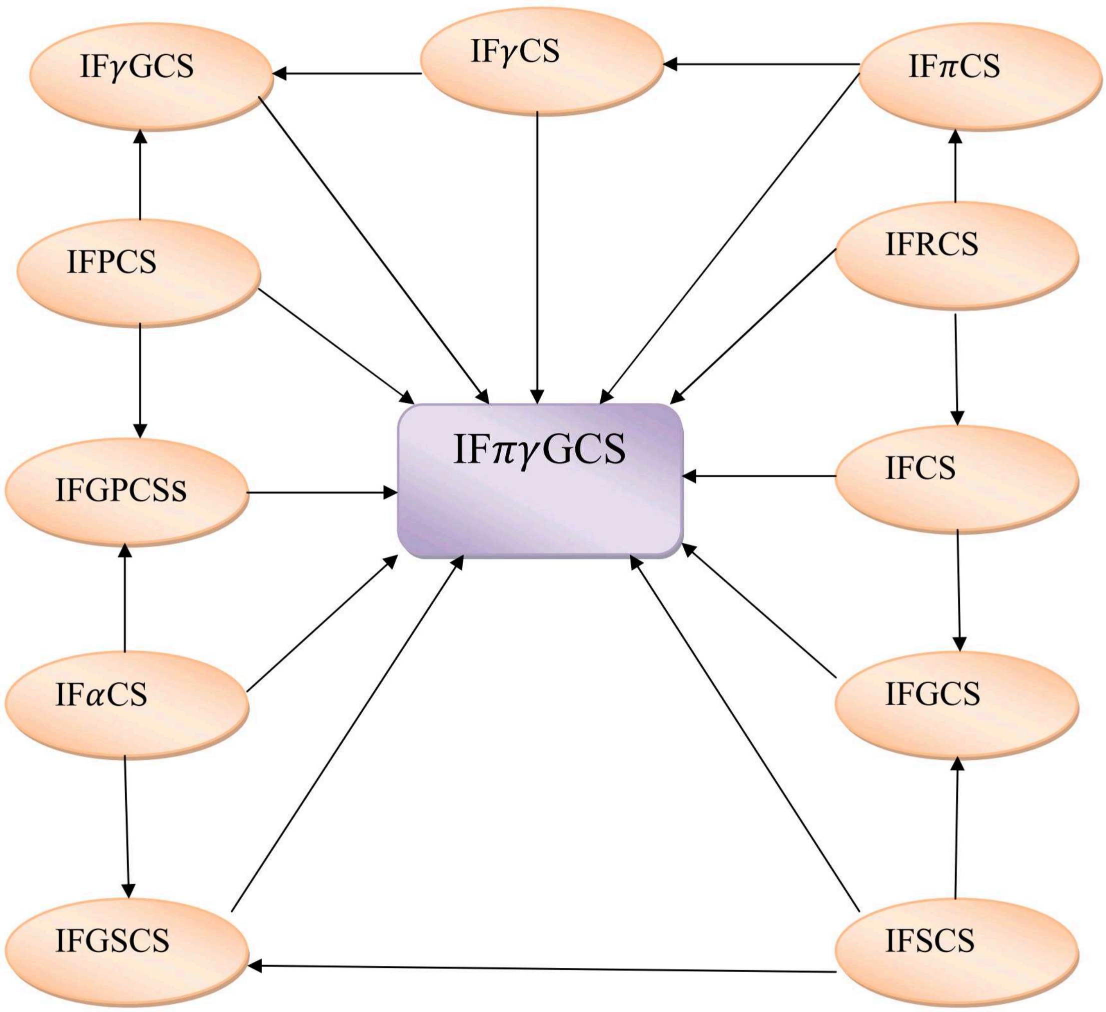
Proof:

Let A be an IFGSCS in X. Let A ⊆ U and U be an IFπOS in (X, τ). Therefore scl(A) = A ∪ int(cl(A)) ⊆ U, by hypothesis. This implies int(cl(A)) ⊆ U. Now cl(int(A)) ∩ int(cl(A)) ⊆ cl(int(A)) ∩ U ⊆ cl(A) ∩ U ⊆ cl(U) ∩ U ⊆ U. Hence A is an IF$\pi\gamma$GCS in (X, τ).

Example 2.1.22:

Let X ={a,b} and let τ ={$0_\sim, G_1, G_2, 1_\sim$}is an IFT on X, where G_1 =<$x, (0.5_a, 0.3_b), (0.5_a, 0.7_b)$> and G_2 =<$x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)$> . Then the IFS A =< $x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)$> is an IF$\pi\gamma$GCS in (X,$\tau$) but not an IFGSCS in (X, τ) as scl(A)= A∪ int(cl(A)) =A∪ G_1= $G_1 \nsubseteq G_2$, but A ⊆ G_2.

In the following diagram, we have provided relationship between various types of intuitionistic fuzzy closed sets.



Remark 2.1.23:

The intersection of any two IF$\pi\gamma$GCS need not be an IF$\pi\gamma$GCS in (X,τ) in general.

Example 2.1.24:

Let X={a,b}, G_1=<x,(0.4_a,0.2_b),(0.6_a,0.8_b)> and G_2=<x,(0.4_a,0.4_b),(0.5_a,0.5_b)>. Then τ = {$0_\sim$,G_1, G_2, $1_\sim$} is an IFT on X. Here the

IFSs A=<x,$(0.4_a,0.5_b),(0.5_a,0.4_b)$> and B=<x, $(0.5_a,0.2_b),(0.4_a,0.6_b)$> are IF$\pi\gamma$GCS in (X,$\tau$) but A ∩ B =<x,$(0.4_a,0.2_b),(0.5_a,0.6_b)$> is not an IF$\pi\gamma$GCS in (X,$\tau$).

Theorem 2.1.25:

Let (X, τ) be an IFTS. Then for every A ϵ IFπγGC(X) and for every B ϵ IFS(X), A ⊆B ⊆ $\gamma cl(A)$ implies B ϵ IFπγGC(X).

Proof:

Let B ⊆ U and U be an IFπOS. Since A ⊆ B, A ⊆ U, by hypothesis B⊆ $\gamma cl(A)$. Therefore $\gamma cl(B) \subseteq \gamma cl(\gamma cl(A)) = \gamma cl(A) \subseteq$ U. since A is an IFπγGCS. Hence B ϵ IFπγGC(X).

Theorem 2.1.26:

If A is both an IFOS and IFπγGCS in (X, τ) then A is an IFγCS in (X, τ).

Proof:

Since A be an IFπOS and A ⊆ A, by hypothesis $\gamma cl(A)$ ⊆ A. Hence A is an IFγCS in (X, τ).

Theorem 2.1.27:

If A is both an IFπOS and an IF$\pi\gamma$GCS in (X, τ) then A is an IFγCS in (X, τ).

Proof:

Let A be an IFπOS and an IF$\pi\gamma$GCS in (X,τ). Then γcl(A) ⊆ A, as A⊆A. But A ⊆ γcl(A). Therefore γcl(A) = A. Hence A is an IFγCS in (X, τ).

Theorem 2.1.28:

If A is an IFπOS and an IF$\pi\gamma$GCS in (X,τ), then A is an IFSCS in (X, τ).

Proof:

Let A be an IFπOS and an IF$\pi\gamma$GCS in (X,τ). That is A is an IFOS in X. Then γcl(A)⊆ A, as A ⊆ A. Clearly int(cl(A)) = cl(A) ∩ int(cl(A)) = cl(int(A)) ∩ int(cl(A)) ⊆ A. This implies int(cl(A)) ⊆A. Hence A is an IFSCS in (X, τ).

Theorem 2.1.29:

For an IFS A in (X, τ), A is both an IFπOS and an IF$\pi\gamma$GCS in X, then A is an IFROS in (X, τ).

Proof:

Let A be an IFπOS and an IF$\pi\gamma$GCS in X. That A is a IFOS in X. Then int(cl(A)) = int(cl(A)) ∩ cl(A) = int(cl(A)) ∩ cl(int(A)) ⊆ γcl(A) ⊆ A. Since A is an IFOS, it is an IFPOS and A ⊆ int(cl(A)). Therefore A = int(cl(A)) and A is an IFROS in (X, τ).

Theorem 2.1.30:

If an IFS A of an IFTS (X, τ) is an intuitionistic fuzzy nowhere dense then A is an IF$\pi\gamma$GCS in (X, τ).

Proof:

If A is an intuitionistic fuzzy nowhere dense, then by definition int(cl(A))= $0_\sim$. Let A ⊆ U where U is an IFπOS in X. Now $\gamma cl(A)$ ⊆ scl(A)= A ∪ int(cl(A)= A ∪ $0_\sim$ = A⊆U. This implies γcl(A)⊆ U . Hence A is an IF$\pi\gamma$GCS in (X, τ).

Theorem 2.1.31:

Let A ⊆ Y ⊆ X and suppose that A is an IF$\pi\gamma$GCS in X then A is an IF$\pi\gamma$GCS relative to Y.

Proof:

Given that A ⊆Y ⊆ X and A is an IF$\pi\gamma$GCS in X. Now let A ⊆Y ∩ U where U is an IFπOS in X. Since A is an IF$\pi\gamma$GCS in X, A ⊆ U implies $\gamma_y$cl(A) ⊆ U. It follows that Y ∩ γcl(A)= γcl(A) ⊆ Y ∩ U = U. Thus A is an IF$\pi\gamma$GCS relative to Y.

Theorem 2.1.32:

Let F ⊆ A ⊆ X where A is an IFπOS and an IF$\pi\gamma$GCS in X. Then F is an IF$\pi\gamma$GCS in A if and only if F is an IF$\pi\gamma$GCS in (X,τ).

Proof:

Necessity:

Let U be an IFπOS in X and F ⊆ U. Also let F be an IF$\pi\gamma$GCS in A. Then clearly F ⊆ A ∩ U and A ∩ U is an IFπOS in A. Hence γ_A(cl(A)) ⊆ A ∩ U and by theorem 2.1.27, A is an IFγCS. Therefore γ(cl(A)) ⊆ A. Now γcl(F)⊆ γcl(F)∩ A = $\gamma_A$cl(F)⊆ A ∩ U ⊆ U. That is γcl(F) ⊆ U, whenever F ⊆ U. Hence F is an IF$\pi\gamma$GCS in (X,τ).

Sufficiency:

Let V be an IFπOS in A such that F⊆V. Since A is an IFπOS in X, V is an IFπOS in X. Therefore γcl(F) ⊆V as F is an IF$\pi\gamma$GCS in (X,τ). Thus, $\gamma_A$cl(F) = γcl(F)∩ A ⊆ V ∩ A ⊆ V. Hence F is an IF$\pi\gamma$GCS in A.

Theorem 2.1.33:

An IFS A of an IFTS (X, τ) is an IF$\pi\gamma$GCS if and only if $A_{\bar{q}}F$ implies $\gamma cl(A)_{\bar{q}}F$ for every IFπCS F of X.

Proof:

Necessity:

Let F be an IFπCS in X and $A_{\bar{q}}\,F$,then A⊆ F^c, by definition, F^c is an IFπOS, Then $\gamma cl(A)$ ⊆ F^c, by hypothesis. Hence by definition, $\gamma cl(A)_{\bar{q}}\,F$.

Sufficiency:

Let U be an IFπOS such that A⊆ U. Then U^c is an IFπCS and A ⊆ $(U^c)^c$. Therefore, by hypothesis $A_{\bar{q}}U^c$ implies $\gamma cl(A)_{\bar{q}}\, U^c$. Hence $\gamma cl(A)$ ⊆ $U^c)^c$ =0. Therefore $\gamma cl(A) \subseteq U$ and A is an IF$\pi\gamma$GCS in X.

2.2 $\pi\gamma$ Generalized Open Sets in Intuitionistic Fuzzy Topological Spaces

In this section, we have introduced a new type of intutionistic fuzzy $\pi\gamma$ generalized open sets which is a complement of intutionistic fuzzy $\pi\gamma$ generalized closed sets. Also, we have established the relationship between basic intutionistic fuzzy sets and newly intuitionistic fuzzy $\pi\gamma$ generalized closed sets. Also, we have analyzed some properties of $\pi\gamma$ generalized closed sets in intuitionistic fuzzy topological spaces.

Definition 2.2.1:

An IFS A is said to be an intuitionistic fuzzy $\pi\gamma$ generalized open sets (IF$\pi\gamma$GOS in short) in (X,τ) if the complement A^c is an IF$\pi\gamma$GOS in X.

The family of all IF$\pi\gamma$GOSs of an IFTS (X,τ) is denoted by IF$\pi\gamma$GO(X).

Example 2.2.2:

Let X={a,b} and let τ= {$0_\sim$,G_1, G_2, $1_\sim$} where G_1=<x,(0.4,0.2),(0.6,0.8) >, G_2 =<x. (0.7,0.8),(0.3,0.2)>. Then IFS A=<x, (0.5,0.5),(0.5,0.5)> is an IF$\pi\gamma$GOS in (X, τ) .

Theorem 2.2.3:

For any IFTS (X,τ), we have the following:

- Every IFOS is an IF$\pi\gamma$GOS in (X,τ).
- Every IFαOS is an IF$\pi\gamma$GOS in (X,τ).
- Every IFROS is an IF$\pi\gamma$GOS in (X,τ).
- Every IFPOS is an IF$\pi\gamma$GOS in (X,τ).
- Every IFγOS is an IF$\pi\gamma$GOS in (X,τ).
- Every IFπOS is an IF$\pi\gamma$GOS in (X,τ). But the converse are not true in general.

Proof: Straight forward.

Example 2.2.4:

Let X={a,b} and let τ= {$0_{\sim}$,G_1, G_2, $1_{\sim}$} where G_1=<x,(0.4_a,0.2_b),(0.6_a,0.8_b) >, G_2 =<x. (0.7_a,0.8_b),(0.3_a,0.2_b)>. Then IFS A=<x, (0.5_a,0.5_b),(0.5_a,0.5_b)> is an IF$\pi\gamma$GOS but not an IFOS in (X, τ) as int(A) = $G_1 \neq$ A.

Example 2.2.5:

Let X={a,b} and let τ= {$0_{\sim}$,G_1, G_2, $1_{\sim}$} where G_1=<x,(0.4_a,0.2_b),(0.6_a,0.8_b) >, G_2 =<x. (0.7_a,0.8_b),(0.3_a,0.2_b)>. Then IFS A=<x, (0.5_a,0.5_b),(0.5_a,0.5_b)> is an IF$\pi\gamma$GOS but not an IFαOS in (X, τ) as int(cl(in(A))) = int(cl(G_1))= int(G_1^c) = G_1, A $\nsubseteq G_1$.

Example 2.2.6:

Let X={a,b} and let τ= {$0_{\sim}$,G_1, G_2, $1_{\sim}$} where G_1=<x,(0.4_a,0.2_b),(0.6_a,0.8_b) >, G_2 =<x. (0.7_a,0.8_b),(0.3_a,0.2_b)>. Then IFS A=<x, (0.5_a,0.5_b),(0.5_a,0.5_b)> is an IF$\pi\gamma$GOS but not an IFROS in (X, τ) as int(cl(A))=int(G_1^c)= G_1, A $\neq G_1$.

Example 2.2.7:

Let X={a,b} and let τ= {$0_{\sim}$,G_1, G_2, $1_{\sim}$} where G_1=<x,(0.4_a,0.2_b),(0.6_a,0.8_b) >, G_2 =<x. (0.7_a,0.8_b),(0.3_a,0.2_b)>. Then IFS A=<x, (0.5_a,0.5_b),(0.5_a,0.5_b)> is an IF$\pi\gamma$GOS but not an IFPOS in (X, τ) as int(cl(A))= int(G_1^c) = G_1, A $\nsubseteq G_1$.

Example 2.2.8:

Let X={a,b} and let τ= {$0_{\sim}$,G_1, G_2, $1_{\sim}$} where G_1=<x,(0.4_a,0.2_b),(0.6_a,0.8_b) >, G_2 =<x. (0.7_a,0.8_b),(0.3_a,0.2_b)>. Then IFS A=<x, (0.5_a,0.5_b),(0.5_a,0.5_b)> is an IF$\pi\gamma$GOS but not an IFγOS in (X, τ) as int(cl(A)) $\cap$ cl(int(A))= int(G_1^c) $\cap$ cl(G_1) = $G_1 \cap G_1^c$ = G_1, A $\nsubseteq G_1$.

Example 2.2.9:

Let X={a,b} and let τ= {$0_{\sim}$,G_1, G_2, $1_{\sim}$} where G_1=<x,(0.4_a,0.2_b),(0.6_a,0.8_b) >, G_2 =<x. (0.7_a,0.8_b),(0.3_a,0.2_b)>. Then IFS A=<x, (0.5_a,0.5_b),(0.5_a,0.5_b)> is an IF$\pi\gamma$GOS but not an IFπOS in (X, τ) as int(cl(A))= int(G_1^c) =G_1, A$\neq G_1$.

Theorem 2.2.10:

Let (X, τ) be an IFTS. Then for every IFS A and for every B ϵ IFRC(X), B ⊆ A ⊆cl(int(B))∩int(cl(B)) implies A is an IF$\pi\gamma$GOS in X.

Proof: Let B be an IFRCS in X. Then B=cl(int(B)). By hypothesis A ⊆ cl(int(A)) ∩ int(cl(A)) = B ∩ int(cl(A)) ⊆ int(cl(B)) ⊆ int(cl(A)) as B ⊆ A. Therefore A is an IFPOS. Therefore A is an IF$\pi\gamma$GOS in X.

Theorem 2.2.11:

Let (X,τ) be an IFTS. Then for every A ϵ IF$\pi\gamma$GO(X) and for every B ϵ IFS(X), $\gamma\ int\ (A)$ ⊆ B ⊆ A ⇒ B ϵ IF$\pi\gamma$GO(X).

Proof:

Let A be an IF$\pi\gamma$GOS of X and B be any IFS on X. Let $\gamma\ int\ (A)$ ⊆ B ⊆ A. Then A^c is an IF$\pi\gamma$GCS and $A^c \subseteq B^c \subseteq\ \gamma\ cl(A^c)$. Therefore B^c is an IF$\pi\gamma$GCS which implies B is an IF$\pi\gamma$GOS in X. Hence B ϵ IF$\pi\gamma$GO(X).

Theorem 2.2.12:

If A is an IFRCS and B is an IFαOS, then A∪B is an IF$\pi\gamma$GOS in (X, τ).

Proof:

Let B be an IFαOS and A be an IFRCS. Then B⊆ int(cl(int(B))) and cl(int(A)) = A. Therefore A∪B ⊆ A ∪ int(cl(int(B))) = cl(int(A)) ∪ int(cl(int(B))) ⊆ cl(int(A)) ∪cl(int(B)) ⊆cl(int(A∪B)). Therefore we have A∪B ⊆cl(int(A∪B)). Therefore A∪B is an IFSOS and hence by theorem 2.2.2,A∪B is an IF$\pi\gamma$GOS in X.

Theorem 2.2.13:

If an IFS A of an IFTS in both an IFCS and an IFGOS, then A is an IF$\pi\gamma$GOS in (X, τ).

Proof:

Suppose A is both an IFCS and IFGOS. Then as $A \subseteq A$, by hypothesis $A \subseteq int(A)$. But $int(A) \subseteq A$. Therefore $int(A) = A$. We have A is an IFπOS, since every IFπOS is an IF$\pi\gamma$GOS. Hence A is an IF$\pi\gamma$GOS in X.

Theorem 2.2.14:

Let (X,τ) be an IFTS. Then for every $A \epsilon$ IFS(X) and for every $B \epsilon$ IFγO(X), $B \subseteq A \subseteq int(cl(int(B)\,)\,) \Longrightarrow A \epsilon \text{IF}\pi\gamma\text{GO}(X)$.

Proof:

Let B be an IFγOS. Then $B \subseteq cl(int(cl(B)))$. By hypothesis $A \subseteq int(cl(int(B))) \subseteq int(cl(int(cl(int(cl(B))))) \subseteq int(cl(cl(int(cl(B))))) = int(cl(int(cl(B)))) \subseteq int(cl(cl(A)) \subseteq int(cl(A))$ as $B \subseteq A$. Therefore A is an IFPOS. By theorem 2.2.3, A is an IF$\pi\gamma$GOS. Hence $A \epsilon \text{IF}\pi\gamma\text{GO}(X)$.

CHAPTER 3

3.1 $\pi\gamma$ Generalized Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

In this section, we have introduced a new type of intutionistic fuzzy $\pi\gamma$ generalized continuous mappings. Also, we have established the relationship between basic intutionistic fuzzy continuous mappings and newly introduced intuitionistic fuzzy $\pi\gamma$ generalized continuous mappings. Also, we have analyzed some properties of $\pi\gamma$ generalized continuous mappings in intuitionistic fuzzy topological spaces.

Definition 3.1.1:

A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is called an intuitionistic fuzzy $\pi\,\gamma$ generalized continuous (IF$\pi\gamma$G continuous for short) mappings if $f^{-1}(V)$ is an IF$\pi\gamma$GCS in (X,τ) for every IFCS V of (Y,σ).

For the sake of simplicity, we shall use the notation A= $< x, (\mu_A, \mu_B), (\nu_A, \nu_B) >$ instead of A= $<x, (a/\mu_A, b/\mu_B), (a/\nu_A, b/\nu_B)>$ in the following examples. Similarly we shall use the notation B= $< y, (\mu_u, \mu_v), (\nu_u, \nu_v) >$ instead of B= $<y, (a/\mu_u, b/\mu_v), (a/\nu_u, b/\nu_v)>$ in the following examples.

The intuitionistic fuzzy sets $0_\sim = \{< x, 0,1) >/x \in X\}$ and $1_\sim = \{< x, 0,1) >/x \in X\}$ are respectively the empty set and the whole set of X.

Example 3.1.2

Let X={a,b} , Y={u,v} and G_1 = $<x, (0.4_a,0.2_b),(0.6_a,0.8_b)>$, $G_2=<x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$, $G_3=<y,(0.5_u,0.6_v),(0.5_u,0.4_v)>$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS $G_3^c = <y,(0.5_u,0.4_v),(0.5_u,0.6_v)>$ is an IFCS in Y. Then $f^{-1}(G_3^c) = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>$ is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping.

Theorem 3.1.3:

Every IF continuous mapping is an IF$\pi\gamma$G continuous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IF continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFCS in X. Since every IFCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.4:

Let X={a,b} , Y={u,v} and G_1 = <x, $(0.4_a,0.2_b),(0.6_a,0.8_b)$>, G_2 =<x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$>,$G_3$=<y,$(0.5_u,0.6_v),(0.5_u,0.4_v)$>.Then $\tau= \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS G_3^c = <y,$(0.5_u,0.4_v),(0.5_u,0.6_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFCS in X, as $cl(f^{-1}(G_3^c)) = G_2^c \neq f^{-1}(G_3^c)$, f is not an IF continuous mapping.

Theorem 3.1.5:

Every IFS continuous mapping is an IF$\pi\gamma$G continuous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSCS in X. Since every IFSCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.6:

Let X={a,b}, Y={u,v} and G_1 = <x,$(0.5_a,0.6_b),(0.5_a,0.4_b)$>, G_2=<x,$(0.4_a,0.3_b),(0.6_a,0.7_b)$>,$G_3$=<y,$(0.7_u,0.8_v),(0.3_u,0.2_v)$>. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS G_3^c = <y, $(0.3_u,0.2_v),(0.7_u,0.8_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x, $(0.3_a,0.2_b),(0.7_a,0.8_b)$> is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but since

$f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) = int(G_1^c) = G_1 \nsubseteq f^{-1}(G_3^c)$, f is not an IFS continuous mapping.

Theorem 3.1.7:

Every IFP continuous mapping is an IF$\pi\gamma$G continuous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFP continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFPCS in X. Since every IFPCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.8:

Let X={a,b} , Y={u,v} and $G_1 = \langle x, (0.4_a,0.2_b),(0.6_a,0.8_b)\rangle$, $G_2=\langle x,(0.5_a,0.4_b),(0.5_a,0.6_b)\rangle$, $G_3=\langle y,(0.5_u,0.6_v),(0.5_u,0.4_v)\rangle$. Then $\tau=\{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS $G_3^c = \langle y,(0.5_u,0.4_v),(0.5_u,0.6_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x,(0.5_a,0.4_b),(0.5_a,0.6_b)\rangle$ is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFPCS in X, as $cl(int(f^{-1}(G_3^c))) = cl(G_2)=G_2^c \nsubseteq f^{-1}(G_3^c)$, f is not an IFP continuous mapping.

Theorem 3.1.9:

Every IFR continuous mapping is an IF$\pi\gamma$G continuous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFR continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFRCS in X. Since every IFRCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.10:

Let X={a,b} , Y={u,v} and G_1 = <x, (0.4_a,0.2_b),(0.6_a,0.8_b)>, G_2=<x,(0.5_a,0.4_b),(0.5_a,0.6_b)>,G_3=<y,(0.5_u,0.6_v),(0.5_u,0.4_v)>. Then τ ={$0_\sim, G_1, G_2, 1_\sim$} and σ={$0_\sim, G_3, 1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y,(0.5_u,0.4_v),(0.5_u,0.6_v)> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFRCS in X, as cl(int($f^{-1}(G_3^c)$)) = cl(G_2)=$G_2^c \neq f^{-1}(G_3^c)$, f is not an IFR continuous mapping.

Theorem 3.1.11:

Every IFα continuous mapping is an IF$\pi\gamma$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: (X,τ) → (Y,σ) be an IFα continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFαCS in X. Since every IFαCS is an IF$\pi\gamma$GCS, f^{-1}(V) is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.12:

Let X={a,b} , Y={u,v} and G_1 = <x, (0.4_a,0.2_b),(0.6_a,0.8_b)>, G_2=<x,(0.5_a,0.4_b),(0.5_a,0.6_b)>,G_3=<y,(0.5_u,0.6_v),(0.5_u,0.4_v)>. Then τ ={$0_\sim, G_1, G_2, 1_\sim$} and σ={$0_\sim, G_3, 1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y,(0.5_u,0.4_v),(0.5_u,0.6_v)> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but not an IFα continuous mapping, since $f^{-1}(G_3^c)$ is not an IFαCS in X, as cl(int(cl($f^{-1}(G_3^c)$))= cl(int(G_2^c))= cl(G_2)= $G_2^c \nsubseteq f^{-1}(G_3^c)$.

Theorem 3.1.13:

Every IFπ continuous mapping is an IF$\pi\gamma$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFπ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFπCS in X. Since every IFπCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.14:

Let X={a,b} , Y={u,v} and G_1 = <x, $(0.5_a,0.3_b),(0.5_a,0.7_b)$>, G_2=<x,$(0.4_a,0.3_b),(0.6_a,0.7_b)$>,$G_3$=<y,$(0.7_u,0.8_v),(0.3_u,0.2_v)$>. Then $\tau =\{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma=\{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : $(X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS G_3^c = <y,$(0.3_u,0.2_v),(0.7_u,0.8_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,$(0.3_a,0.2_b),(0.7_a,0.8_b)$> is an IFS in X.

Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but not an IFπ continuous mapping, since $f^{-1}(G_3^c)$ is not an IFπCS in X, as $cl(int(f^{-1}(G_3^c))= 0_\sim \neq f^{-1}(G_3^c)$.

Theorem 3.1.15:

Every IFγ continuous mapping is an IF$\pi\gamma$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFγ continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFγCS in X. Since every IFγCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.16:

Let X={a,b} , Y={u,v} and G_1 = <x,$(0.5_a,0.4_b),(0.5_a,0.6_b)$>, G_2=<x,$(0.4_a,0.3_b),(0.6_a,0.7_b)$>,$G_3$=<y,$(0.5_u,0.6_v),(0.4_u,0.4_v)$>.Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ by f(a)= u and f(b) = v. The IFS G_3^c = <y, $(0.4_u,0.4_v),(0.5_u,0.6_v)$> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,$(0.4_a,0.4_b),(0.5_a,0.6_b)$> is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but since

$f^{-1}(G_3^c)$ is not an IFSCS in X, as $int(cl(f^{-1}(G_3^c))) \cap cl(int(f^{-1}(G_3^c))) = int(G_1^c) \cap cl(G_2) = G_1 \cap G_1^c \nsubseteq f^{-1}(G_3^c)$, $f^{-1}(G_3^c)$ is not an IFS continuous mapping.

Theorem 3.1.17:

Every IFG continuous mapping is an IF$\pi\gamma$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFG continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGCS in X. Since every IFGCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.18:

Let $X=\{a,b\}$, $Y=\{u,v\}$ and $G_1 = \langle x,(0.5_a,0.4_b),(0.5_a,0.6_b)\rangle$, $G_2=\langle x,(0.4_a,0.3_b),(0.6_a,0.7_b)\rangle$, $G_3=\langle y,(0.5_u,0.6_v),(0.4_u,0.4_v)\rangle$. Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X,\tau) \rightarrow (Y, \sigma)$ by f(a)= u and f(b) = v. The IFS $G_3^c = \langle y, (0.4_u,0.4_v),(0.5_u,0.6_v)\rangle$ is an IFCS in Y. Then $f^{-1}(G_3^c) = \langle x,(0.4_a,0.4_b),(0.5_a,0.6_b)\rangle$ is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGCS in X, as $cl(f^{-1}(G_3^c)) = G_1^c \nsubseteq G$, $f^{-1}(G_3^c)$ is not an IFG continuous mapping.

Theorem 3.1.19:

Every IFGS continuous mapping is an IF$\pi\gamma$G continous mapping in (X,τ) but not conversely in general.

Proof:

Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFGS continuous mapping. Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFGSCS in X. Since every IFGSCS is an IF$\pi\gamma$GCS, $f^{-1}(V)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.20:

Let X={a,b} , Y={u,v} and G_1 = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>, G_2=<x,(0.4_a,0.3_b),(0.6_a,0.7_b)>,G_3=<y,(0.5_u,0.6_v),(0.4_u,0.4_v)>.Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y,(0.4_u,0.4_v),(0.5_u,0.6_v)> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,(0.4_a,0.4_b),(0.5_a,0.6_b)> is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGSCS in X, as $f^{-1}(G_3^c) \cup int(cl(f^{-1}(G_3^c))) = f^{-1}(G_3^c) \cup int(G_1^c) = f^{-1}(G_3^c) \cup G_1 = G_1 \nsubseteq G_2$, $f^{-1}(G_3^c)$ is not an IFGS continuous mapping.

Theorem 3.1.21:

Every IFGP continuous mapping is an IF$\pi\gamma$G continous mapping in (X,τ) but not conversely in general.

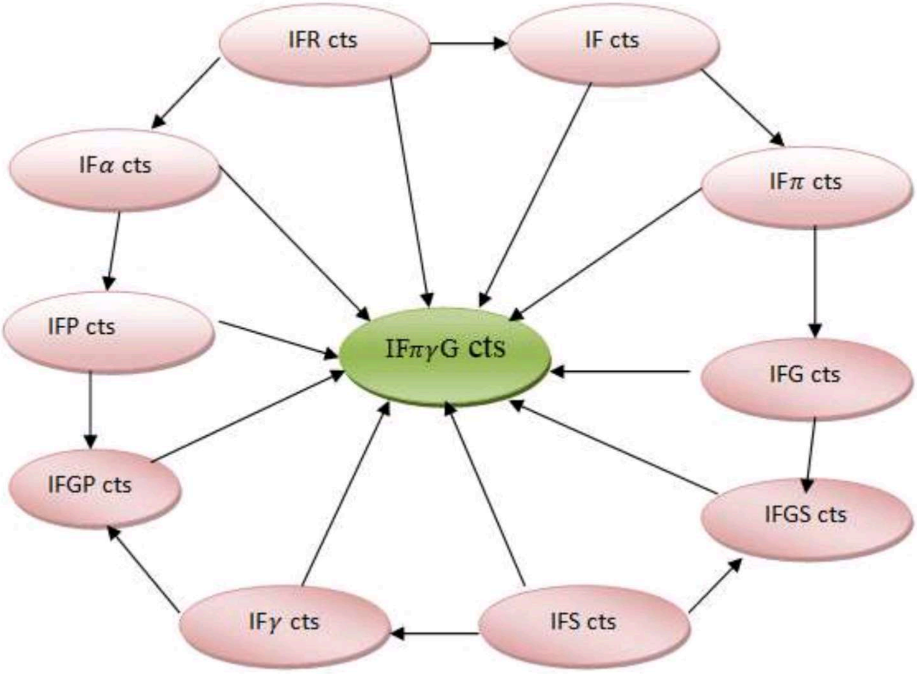
Proof:

Let f: (X,τ) → (Y,σ) be an IFGS continuous mapping. Let V be an IFCS in Y. Then f^{-1}(V) is an IFGPCS in X. Since every IFGPCS is an IF$\pi\gamma$GCS, f^{-1}(V) is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Example 3.1.22:

Let X={a,b} , Y={u,v} and G_1 = <x,(0.5_a,0.4_b),(0.5_a,0.6_b)>, G_2=<x,(0.4_a,0.3_b),(0.6_a,0.7_b)>,G_3=<y,(0.5_u,0.6_v),(0.4_u,0.4_v)>.Then $\tau = \{0_\sim, G_1, G_2, 1_\sim\}$ and $\sigma = \{0_\sim, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f : (X,τ) → (Y, σ) by f(a)= u and f(b) = v. The IFS G_3^c = <y ,(0.4_u,0.4_v),(0.5_u,0.6_v)> is an IFCS in Y. Then $f^{-1}(G_3^c)$ = <x,(0.4_a,0.4_b),(0.5_a,0.6_b)> is an IFS in X. Hence $f^{-1}(G_3^c)$ is an IF$\pi\gamma$GCS in (X,τ). Therefore f is an IF$\pi\gamma$G continuous mapping but since $f^{-1}(G_3^c)$ is not an IFGPCS in X, $f^{-1}(G_3^c)$ is not an IFGP continuous mapping.

The relationship between various types of intutionistic fuzzy continuity is given in the following figure. In this figure 'cts' means continuous.



Theorem 3.1.23

A mapping f: (X,τ) → (Y,σ) is an IF$\pi\gamma$G continuous mapping if and only if the inverse image of each IFπOS in Y is an IF$\pi\gamma$GOS in (X,τ).

Proof:

Necessity:

Let A be an IFπOS in Y. This implies A^c is an IFπCS in Y. Then $f^{-1}(A^c)$ is an IF$\pi\gamma$GCS in X, by hypothesis. Since $f^{-1}(A^c) = (f^{-1}(A)^c)$, $f^{-1}(A)$ is an IF$\pi\gamma$GOS in X.

Sufficiency:

Let A be an IFπCS in Y. Then A^c is an IFπOS in Y. By hypothesis $f^{-1}(A^c)$ is an IF$\pi\gamma$GOS in X. Since $f^{-1}(A^c) = (f^{-1}(A)^c)$, $(f^{-1}(A)^c)$ is an IF$\pi\gamma$GOS in X. Therefore $f^{-1}(A)$ is an IF$\pi\gamma$GCS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

Theorem 3.1.24:

If $f : (X,\tau) \rightarrow (Y,\sigma)$ is an IF$\pi\gamma$G continous mapping then for each IFP $p_{(\alpha,\beta)}$of X and each $A \epsilon \sigma$ such that $f(p_{(\alpha,\beta)})\ _q A$, there exists an IF$\pi\gamma$GOS B of X such that $p_{(\alpha,\beta)}\ _q B$ and $f(B) \subseteq A$.

Proof:

Let $p_{(\alpha,\beta)}$ be an IFP of X and $A \epsilon \sigma$ such that $f(p_{(\alpha,\beta)})_q A$. Put $B= f^{-1}(A)$. Then by hypothesis, B is an IF$\pi\gamma$GOS in X such that $p_{(\alpha,\beta)}X_q B$ and $f(B) =f(f^{-1}(A)) \subseteq A$.

Theorem 3.1.25:

A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is an IF$\pi\gamma$G continous mapping if $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof:

Let A be an IFπOS in Y then A^c is an IFπCS in Y. By hypothesis, $cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(cl(A^c))= f^{-1}(A^c)$. Now $(int(cl(int(f^{-1}(A)))))^c = cl(int(cl(f^{-1}(A^c)))) \subseteq f^{-1}(A^c) = f^{-1}(cl(A))^c$. This implies that $f^{-1}(A) \subseteq (int(cl(int(f^{-1}(A)))))$. Hence $f^{-1}(A)$ is an IFαOS and hence it is an IF$\pi\gamma$GOS. Therefore f is an IF$\pi\gamma$G continuous mapping.

Theorem 3.1.26:

Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be an IF$\pi\gamma$G continuous mapping and $g: (Y,\sigma) \rightarrow (Z,\delta)$ is an IF continuous mapping then $g \circ f : (X,\tau) \rightarrow (Z,\delta)$ is an IF$\pi\gamma$ continuous mapping.

Proof:

Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFCS in Y, by hypothesis. Since f is an IF$\pi\gamma$G continuous mapping, $f^{-1}(g^{-1}(V))$ is an IF$\pi\gamma$GCS in X. Hence $g \circ f$ is an IF$\pi\gamma$G continuous mapping.

Theorem 3.1.27:

Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be an mapping from an IFTS X into an IFTS Y that satisfies $f^{-1}(int(B)) = int(cl(f^{-1}(B)))$ for every IFS B in Y. Then f is an IF$\pi\gamma$G continuous mapping.

Proof:

Let B be an IFπOS in Y. Then int(cl(B))= B, by hypothesis $f^{-1}(B) =$ int(cl($f^{-1}(B)$)). This implies $f^{-1}(B)$ is an IFROS in X. Therefore it is an IF$\pi\gamma$GOS in X. Hence f is an IF$\pi\gamma$G continuous mapping.

SUMMARY AND CONCLUSION

Summary and Conclusion

The basic concept of fuzzy set was introduced by L.A. Zadeh in the year 1965. The fuzzy set theory was developed by Zadeh and others have found many applications in the domain mathematics and elsewhere. C.L. Chang in the year 1968 first introduced the concept of fuzzy topological spaces and he used the fuzzy set theory for defining and introducing fuzzy topological spaces. The notion of intutionistic fuzzy sets was introduced by Atanassaov as a generalization of fuzzy sets in 1997, Coker introduced the concept of intutionistic fuzzy topological spaces.

In this thesis, we have introduced intutionistic fuzzy $\pi\gamma$ generalized closed sets(intutionistic fuzzy $\pi\gamma$ generalized open sets) and compared intutionistic fuzzy $\pi\gamma$ generalized closed sets and intutionistc fuzzy $\pi\gamma$ generalized open sets with some of the basic intutionistic fuzzy sets such as intuitionistic fuzzy closed sets(intuitionistic fuzzy open sets), intuitionistic fuzzy regular closed sets(intuitionistic fuzzy regular open sets), intuitionistic fuzzy semi closed sets(intuitionistic fuzzy semi open sets), intuitionistic fuzzy α closed sets(intuitionistic fuzzy α open sets), intuitionistic fuzzy pre closed sets(intuitionistic fuzzy pre open sets), intuitionistic fuzzy π closed sets(intuitionistic fuzzy π open sets), intuitionistic fuzzy γ closed sets(intuitionistic fuzzy γ open sets), intuitionistic fuzzy pre closed sets(intuitionistic fuzzy pre open sets), intuitionistic fuzzy generalized closed sets, intuitionistic fuzzy generalized semi closed sets and intuitionistic fuzzy generalized pre closed sets. Also, the intersection properties of intutionistic fuzzy $\pi\gamma$ generalized closed sets, few properties of intutionistic fuzzy $\pi\gamma$ generalized closed sets and intutionistic fuzzy $\pi\gamma$ generalized open sets are analyzed and discussed with suitable examples.

Also, we have introduced a new concept of intutionistic fuzzy $\pi\gamma$ generalized continuous mappings and made an attempt to compare with basic intutionistic fuzzy continuous mappings. And analyzed some of the properties of intuitionistic fuzzy $\pi\gamma$ generalized continuous mappings.

The future research directions based on this research work may be extended as follows:

1. The notion of intuitionistic fuzzy $\pi\gamma$ generalized closed sets can be extended to bitopological spaces, cubic and neutrosophic topological spaces.
2. The theoretical developments studied in this thesis may be focused on the applications.

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