CHAPTER - VI

CHAPTER VI

EXTENT ANALYSIS METHOD ON FUZZY ANALYTIC HIERARCHY PROCESS

The Extent Analysis Method is used to consider the extent to which an object can satisfy the goal, i.e., satisfaction extent. In this method the "extent" is quantified using triangular fuzzy number. On the basis of fuzzy values for the extent analysis of each object, a fuzzy synthetic degree values can be obtained, which is defined as follows:

Let $X = \{x_1, x_2, \dots, x_n\}$ be an object set and $U = \{u_1, u_2, \dots, u_m\}$ be a goal set. Each object is taken and extent analysis for each goal g_i is performed, respectively. Therefore, m extent analysis values for each object can be obtained, with the following signs:

$$M_{g_i}^1, M_{g_i}^2, \dots, M_{g_i}^m \qquad i = 1,2, \dots n$$

where all $M_{g_i}^j$ $(j = 1,2, \dots m)$ are triangular fuzzy numbers.

The steps for Extent Analysis Method on Fuzzy AHP can be given as in the following.

Step 1: Construction of the Fuzzy AHP Comparison Matrix

The first task of the Fuzzy AHP method is to decide on the relative importance of each pair of factors in the same hierarchy. The pairwise comparisons are described by values taken from a pre-defined set of ratio scale values as presented in Table 7.

Linguistic scale	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Equally important	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly more important	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly more important	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strongly more important	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolutely more important	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

Table 7. Triangular Fuzzy Conversion Scale

By using triangular fuzzy numbers, via pairwise comparison, the fuzzy evaluation matrix $A = (\widetilde{a_{ij}})_{n\times m}$ is constructed; where $\widetilde{a_{ij}} = (l_{ij}, m_{ij}, u_{ij})$ is the relative importance of i^{th} element over j^{th} element in pair wise comparison and l_{ij}, m_{ij}, u_{ij} are the lower bound, model, upper bound values of $\widetilde{a_{ij}}$ respectively. Also $\widetilde{a_{ij}}$ are satisfied with

$$l_{ij} = \frac{1}{l_{ji}},\quad m_{ij} = \frac{1}{m_{ji}},\quad u_{ij} = \frac{1}{u_{ji}}$$

The ratio of comparison between the relative preferences of elements indexed i and j on a criterion can be modeled through a fuzzy scale value associated with a degree of fuzziness. For example, essential or strong importance of element i over element j under a certain criterion: then $\widetilde{a_{ij}} = (l, 5, u)$, where l and u represent a fuzzy degree of judgment. The greater $u - l$, the fuzzier the degree; when $u - l = 0$, the judgment is a non fuzzy number. This stays the same to scale 5 under general meaning. If strong importance of element j over element i holds, then the pairwise comparison scale can be represented by the fuzzy number

$$\widetilde{a_{ij}}^{-1} = \left(\frac{1}{u}, \frac{1}{m}, \frac{1}{l}\right)$$

***Step 2:* Calculation of the Value of Fuzzy Synthetic Extent**

The value of Fuzzy Synthetic Extent with respect to the i^{th} object is defined as

$$S_i = \sum_{j=1}^{m} M_{g_i}^{j} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^{j}\right]^{-1} \tag{24}$$

Here S_i is defined as the fuzzy synthetic extent and $\otimes$ defined as the multiplication of Triangular Fuzzy Numbers.

To obtain $\sum_{j=1}^{m} M_{g_i}^{j}$, perform the fuzzy addition operation of m extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{j=1}^{m} l_{ij}, \sum_{j=1}^{m} m_{ij}, \sum_{j=1}^{m} u_{ij} \right)$$

and to obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j\right]^{-1}$, perform the fuzzy addition operation of $M_{g_i}^j$ $(j = 1, \dots m,\ i = 1, \dots n)$ values such that

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{i=1}^{n}\sum_{j=1}^{m} l_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{m} m_{ij}, \sum_{i=1}^{n}\sum_{j=1}^{m} u_{ij} \right) \tag{25}$$

and then compute the inverse of the vector in Equation (25) such that

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} u_{ij}}, \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} m_{ij}}, \frac{1}{\sum_{i=1}^{n}\sum_{j=1}^{m} l_{ij}} \right)$$

Step 3: **Calculation of the Sets of Weight Values of the Fuzzy AHP**

To obtain the estimates for the sets of weight values under each criterion, it is necessary to consider a principle of comparison for fuzzy numbers.

Definition 6.1 [16]

The degree of possibility of

$$M_1 = (l_1, m_1, u_1) \geq M_2 = (l_2, m_2, u_2)$$

is defined as

$$V(M_1 \geq M_2) = \sup_{x \geq y}\left[min\left(\mu_{M_1}(x), \mu_{M_2}(x)\right)\right]$$

When a pair (x, y) exists such that $x \geq y$ and $\mu_{M_1}(x) = \mu_{M_2}(x)$, then we have $V(M_1 \geq M_2) = 1$. Since M_1 and M_2 are convex fuzzy numbers we have that

$$V(M_1 \geq M_2) = 1 \qquad iff \quad m_1 \geq m_2$$

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_1}(d)$$

where d is the ordinate of the highest intersection point D between $\mu_{M_1}(d)$ and $\mu_{M_2}(d)$. Also the above equation can be equivalently expressed as follows:

$$V(M_2 \geq M_1) = hgt(M_1 \cap M_2) = \mu_{M_1}(d)$$

$$= \begin{cases} 1 & if \ m_2 \geq m_1 \\ 0 & if \ \ l_1 \geq u_2 \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} & otherwise \end{cases} \qquad (26)$$

The following figure illustrates Equation (26)

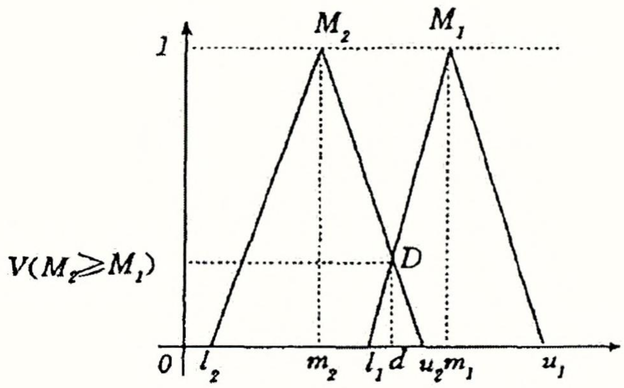

Fig.6 The intersection between M_1 and M_2

To compare M_1 and M_2 , we need both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Definition 6.2 [15]

The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbersM_i $(i = 1, \cdots k)$ can be defined by

$$V(M \geq M_1, \cdots, M_k) = V[(M \geq M_1) \ and \ (M \geq M_2) \ and \ \cdots and \ (M \geq M_k)]$$

$$= \min V(M \geq M_i) \qquad i = 1, \cdots k$$

Assume that

$$d'(A_i) = \min \ V(S_i \geq S_k)$$

For $k = \ 1, 2, ..., n \ ; \ \ k \neq i$.

Then the weight vector is given by

$$W' = \left(d'(A_1), d'(A_2), \cdots, d'(A_n)\right)^T$$

where A_i $(i = 1, \cdots, n)$ are n elements.

Via normalization, the normalized weight vectors are

$$W = (d(A_1), d(A_2), \cdots, d(A_n))^T$$

where W is a non fuzzy number.

Step 4: **Aggregate the Relative Weights of Decision Elements to Obtain an Overall Rating for the Alternatives.**

Finally the alternative with highest weight is chosen as the best alternative.

After comparison is made, it is necessary to check the consistency ratio of the comparison. To do so, the graded mean integration approach is utilized for defuzzyfying the matrix. According to the graded mean integration approach, a fuzzy number $\widetilde{M} = (l, m, u)$ can be transformed into a crisp number by employing the below equation:

$$P(\widetilde{M}) = \frac{l + 4m + u}{6}$$

After the defuzzification of each value in the matrix, 'Consistency Ratio' (CR) [38] of the matrix can easily be calculated and checked whether CR is smaller than 0.10 or not.