A STUDY ON SPHERICAL FUZZY NEUTROSOPHIC CUBIC GRAPH

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Degree of Master of Science (M.Sc.)

By

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DECLARATION

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I declare that the thesis entitled **"A Study On Spherical Fuzzy Neutrosophic Cubic Graph"** submitted by me for the degree of **Master of Science (M. Sc.)** is the record of work carried out by me during the period from December 2022 to May 2023 under the guidance of **Dr.K. Akalyadevi, M.Sc., M.Phil., Ph.D.,** Assistant Professor and Head, Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore, and has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship, Titles in this institute or any other University or other similar institution of Higher Learning.

Signature of the Candidate

Signature of the Supervisor

CERTIFICATE

CERTIFICATE

I certify that the thesis entitled **"A Study On Spherical Fuzzy Neutrosophic Cubic Graph"** submitted for the degree of **Master of Science (M. Sc.)** by Ms. Abirami.R, is the record of research work carried out by her during the period from December 2022 to May 2023 under my guidance and supervision, and that this work has not formed the basis for the award of any Degree, Diploma, Associateship, Fellowship or other Titles in this institute or any other University or institution of Higher Learning.

Signature of the
Head of the Department

Signature of the Supervisor

Signature of the Director

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Chapter-III deals with the new ideas of Spherical fuzzy neutrosophic cubic graph. We also defined some of the properties such as Cartesian product, composition, N-join, M-join, N-union and M-union with certain illustrations.

CHAPTER 1

CHAPTER 1

1.1 INTRODUCTION

Fuzzy sets were introduced by Zadeh (1965). The notion of fuzzy set theory has caused great interest among both pure and applied mathematics. This day's fuzzy set hypothesis has arisen as a likely space of interdisciplinary exploration. It has fruitful applications in different fields as a phenomenal apparatus for addressing human information and discernment.

In 1983, Atanassov presented the idea of intuitionistic fuzzy set as a speculation of Zadeh's thought of fluffy set. Atanassov added another segment that decides the level of non-membership to the meaning of fuzzy set alongside level of membership, which is pretty much autonomous of one another. The as it where necessity is that the amount of these two degrees isn't greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields, including computer science, engineering, mathematics, medicine, chemistry, and economics.

Yager, proposed a brand-new extension of fuzzy set called Pythagorean fuzzy set (PFS), which has been successfully applied in many fields for decision making procedures. PFS is characterized by a membership and non-membership function satisfies the condition that the square sum of membership and non-membership is less than or equal to one.

Spherical fuzzy set is a generalization of picture fuzzy set and Pythagorean fuzzy set. There is a need of spherical fuzzy set to tackle an interesting scenario emerge when picture fuzzy sets and Pythagorean fuzzy sets both failed to handle. We can study the neutral degree in spherical fuzzy set where as in Pythagorean fuzzy sets and picture fuzzy sets it doesn't. In spherical fuzzy set, membership degrees are gratifying the condition $0 \leq P^2(x) + I^2(x) + N^2(x) \leq 1$.

The idea of neutrosophic set is introduced by Smarandache, which is a generalization of the fuzzy set, intuitionistic fuzzy set. The neutrosophic sets are characterized by a truth function (T), an indeterminate function (I) and a false function (F) independently. Smarandache [63] introduced the new concepts of neutrosophic perspectives: Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic, Hedge Algebras and Applications.

The concept of cubic set is characterized by fuzzy set and interval valued fuzzy set, which is an important tool to deal with uncertainty and vagueness. The hybrid platform of cubic set contains more information than a fuzzy set. Neutrosophic set combined with cubic sets gave the new concept of neutrosophic cubic set introduced by Jun et.al

Zhang (1994) initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are extension of fuzzy sets whose range of membership degree is $[-1, 1]$. In a bipolar fuzzy sets, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $(0, 1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1, 0)$ of an element indicates that the element somewhat satisfies the implicit counter property.

In Mathematics, Graph theory is the investigation of graphs, which are numerical designs used to display the relationship between two or more objects or with set of vertices and edges. The graph theory origin can be traced back to Euler's work on the Konigsberg bridges problem in 1735. The concepts of graph theory have applications in many areas of computer science such as data mining, image segmentation, clustering, image capturing, networking, etc.

The kaufmann (1973) gave first definition of fuzzy graph based on Zadeh's fuzzy relations. Generalization of fundamental ideas of graph theory like paths, cycles, trees, connectedness, and their properties to fuzzy graph theory have been done by Rosenfeld.

Atanassov et al. (2006) discussed a new generalization of the intuitionistic fuzzy graphs, using as a basis the concepts of intuitionistic fuzzy sets, intuitionistic fuzzy relations and index matrices. Parvathi et al. (2006) gave a new definition for min-max intuitionistic fuzzy graph.

Recently, Naz et al. initially presented the idea of Pythagorean fuzzy graph (PFG) as a as a change of the possibility of IFG by Atanassov and investigated a few properties of procedure on PFGs alongside its applications in dynamic. Akram et al. characterized some more fundamental procedure on PFGs and furthermore investigated their significant properties.

Chapter-I deals with the Introduction, Review of literature and studied some of the basic definitions like Fuzzy Set, Intuitionistic Fuzzy Set, Pythagorean Fuzzy Set, Neutrosophic Fuzzy Set, Spherical Fuzzy Set, Bipolar Fuzzy Set, Fuzzy Graph, Intuitionistic Fuzzy Graph, Pythagorean Fuzzy Graph, Neutrosophic Fuzzy Graph, Spherical Fuzzy Graph.

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Chapter-III deals with the new ideas of Spherical fuzzy neutrosophic cubic graph. We also defined some of the properties such as Cartesian product, composition, N-join, M-join, N-union and M-union with certain illustrations.

1.2 REVIEW OF LITERATURE

In 1965, Zadeh was introduced a fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterised by a membership (characteristic) function that assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established. In particular, a separation theorem for convex fuzzy sets is proved without requiring that the fuzzy sets be disjoint.

In 1983, Atanassov defined the concept Intuitionistic Fuzzy Set (IFS) as a generalization of the concept fuzzy set. Various properties are proved, which are connected to the operations and relations over sets, and with modal and topological operators, defined over the set of IFS's.

In 1989, Atanassov defined new results on Intuitionistic fuzzy sets. Two new operators on intuitionistic fuzzy sets are defined and their basic properties are studied.

In 1994, Zhang first introduced a bipolar fuzzy set theory, which is presented for cognitive modelling and multiagent decision analysis. Firstly, notions of bipolar fuzziness are introduced. Secondly, an interval-based bipolar fuzzy logic is defined, which generalises the real-valued bipolar fuzzy logic by allowing interval-based linguistic variables.

Neutrosophic set theory firstly proposed in 1998 by Florentin Smarandache, who also developed the concept of single valued neutrosophic set, oriented towards real world scientific and engineering applications. Since then, the single valued neutrosophic set theory has been extensively studied in books and monographs introducing neutrosophic sets and its applications, by many authors around the world.

In 2005, Young et al. using the notion of intuitionistic fuzzy sets, the concepts of intuitionistic fuzzy semi-preopen sets and intuitionistic fuzzy semi-precontinuous mappings were introduced. The relation between an intuitionistic fuzzy precontinuous mapping and an intuitionistic semi-precontinuous mapping is given. Characterizations of intuitionistic fuzzy semi-preopen sets and intuitionist fuzzy semi-precontinuous mappings are given.

Yager et al. (2013) has proposed the Pythagorean Fuzzy Set (PFS) as an effective tool for handling the uncertainty or vague information more adequately in real-world situations. In PFSs, the sum of squares of the degrees of membership and non-membership is less than or equal to 1. For example, if a decision maker provides the membership degree 0.6 and non-membership degree 0.7 in his evaluation, then this situation cannot handle by intuitionistic fuzzy set theory because of 0.6+0.7 > 1. However, it is easily observed that $(0.6)^2 + (0.7)^2 < 1$, that is to say, the Pythagorean Fuzzy Set (PFS) is capable to represent this evaluation information. In other words, the PFSs are more powerful to handle problems in uncertain situations. Under PFS environment, many researchers have started work in different directions and obtained various significant results.

In 2014, Yager introduced a class of nonstandard Pythagorean fuzzy subsets whose membership grades are pairs (a,b) satisfying the requirement $a^2 + b^2 \leq 1$. They also introduce a variety of aggregation operations for these Pythagorean fuzzy subsets. Then look at multicriteria decision making in the case where the criteria satisfaction is expressed using Pythagorean membership grades. The issue of having to choose the best alternative in multicriteria decision making leads us to consider the problem of comparing Pythagorean membership grades.

Graph is a diagrammatic way of representing the relationship between two or more things. Fuzzy graph was found by Rosenfeld (1975), he described that fuzzy analogues of different basic graph-theoretic ideas like bridges, paths, cycles, trees, connectedness and some properties. Krassimir T. Atanassov (1999), were introduced the concept intuitionistic fuzzy graph. Parvathi.R and Karunambigai.M.G (2006) gives a new definition to the intuitionistic fuzzy graph and describe its properties. Muhammad Akram et al. (2019) extended the definition of Pythagorean fuzzy sets (PFS) to Pythagorean fuzzy graphs (PFG) and its properties, and studied the regularity of Pythagorean fuzzy graph (PFG) product. The Pythagorean fuzzy graph is an extension of intuitionistic fuzzy graph and it provides an accurate value for the problem which is vague.

In 2006, Atanassov et al. discuss a new generalization of the Intuitionistic Fuzzy Graphs (IFGs), using as a basis the concepts of the Intuitionistic Fuzzy Sets (IFSs), Intuitionistic Fuzzy Relations (IFRs), and Index Matrices (IMs) and their basic concepts.

In 2006, Parvathi et al. introduced a new definition for intuitionistic fuzzy graphs and some properties of intuitionistic fuzzy graphs are considered and the authors introduced the notions of various concepts. These concepts are analysed through suitable illustrations. In 2007, Karunambigai et al. present a model based on dynamic programming to find the shortest paths in intuitionistic fuzzy graphs.

In 2011, Akram introduced the notion of bipolar fuzzy graphs, described various methods of their construction, discussed the concept of isomorphism's of these graphs, and investigated some of their important properties. They then introduced the notion of strong bipolar fuzzy graphs and studied some of their properties, and also discussed some propositions of self-complementary and self-weakly complementary strong bipolar fuzzy graphs.

In 2019, Xindong Peng et al. present an overview of the Pythagorean fuzzy set with the aim of offering a clear perspective on the different concepts, tools, and trends related to their extension. In particular, we provide two novel algorithms for decision-making problems in a Pythagorean fuzzy environment. It may serve as a foundation for developing more algorithms in decision-making.

In 2020, Akalyadevi et al. introduced spherical fuzzy graph in bipolar environment and discussed the operation on bipolar spherical fuzzy graphs namely, symmetric difference and rejection with brief description on degree and total degree of bipolar spherical fuzzy graphs.

In 2020, Akalyadevi et al. Compared to fuzzy set and all other versions of fuzzy set, neutrosophic sets can handle imprecise information in a more effective way. A Neutrosophic cubic set, which is the generalization of neutrosophic set, are more flexibile as well as compatible to the system compared to other existing fuzzy models. On other hand, graph is a very easy way to understand and handle a problem physically in the form of diagrams. We introduce spherical fuzzy neutrosophic cubic graph and single-valued neutrosophic spherical cubic graphs in bipolar setting and discuss some of their properties such as Cartesian product, composition, m-join, n-join, m-union, n-union. We also present a numerical example of the defined model which depicts the advantage of the same. Finally, we define a score function and minimum spanning tree algorithm of an undirected bipolar single-valued neutrosophic spherical cubic graph with a numerical example.

In 2020, Akalyadevi et al. proposed an algorithm for finding minimum spanning tree of an undirected bipolar graph where the edge lengths are represented by bipolar spherical fuzzy number. To construct the minimum spanning tree of undirected bipolar spherical fuzzy graph, a new algorithm and score function based on matrix approach has been introduced. The proposed method compare with some existing method are also discussed.

NOTATIONS

- FS = Fuzzy set
- FR = Fuzzy Relation
- IFS = Intuitionistic Fuzzy Set
- IFR = Intuitionistic Fuzzy Relation
- PFS = Pythagorean Fuzzy Set
- PFR = Pythagorean Fuzzy Relation
- CS = Cubic Set
- NS = Neutrosophic Set
- SFS = Spherical Fuzzy Set
- BFS = Bipolar Fuzzy Set
- SFR = Spherical Fuzzy Graph
- FG = Fuzzy Graph
- IFG = Intuitionistic Fuzzy Graph
- PFG = Pythagorean Fuzzy Graph
- CG = Cubic Graph
- SFG = Spherical Fuzzy Graph
- NG = Neutrosophic Graph
- NCG = Neutrosophic Cubic Graph

1.3 PRELIMINIARIES

Definition 1.3.1

A **Fuzzy Set** (FS) on a universe Y is an object of the form represents the membership

$$m=\{\langle\eta,\alpha_m(\eta)\rangle\mid\eta\in Y\}$$

Where $\alpha_m:Y\to[0,1]$ represents the membership functions of m.

Definition 1.3.2

A Fuzzy set on $Y\times Y$ is said to be a **Fuzzy Relation** (FR) on Y, denoted by

$$n=\{\eta\lambda,\alpha_n(\eta\lambda)\mid uv\in Y\times Y\}$$

Where $\alpha_n:Y\times Y\to[0,1]$ represents the membership function of m.

Definition 1.3.3

An **Intuitionistic Fuzzy Set** (IFS) on a universe X is an object of the form

$$m=\{\langle\eta,\alpha_m(\eta),\beta_m(\eta)\rangle\mid\eta\in Y\},$$

Where $\alpha_m:Y\to[0,1]$ and $\beta_m:Y\to[0,1]$ represents the membership and non-membership functions of m, and α_m,β_m satisfies the condition $0\le\alpha_m(\eta)+\beta_m(\eta)\le 1$ for all $\eta\in Y$.

Definition 1.3.4

An **Intuitionistic Fuzzy Relation** (IFR) $n=(\alpha_n(\eta,\lambda),\beta_n(\eta,\lambda))$ in an Universe $Y\times Y(m(Y\to Y))$ is an intutionistic fuzzy set of the form

$$n=\{\langle(\eta,\lambda),\alpha_n(\eta,\lambda),\beta_n(\eta,\lambda)\rangle\mid(u,v)\in Y\times Y\}$$

Where $\alpha_n:Y\times Y\to[0,1]$ and $\beta_n:Y\times Y\to[0,1]$. The Intuitionistic fuzzy relation m satisfies $0\le\alpha_n(\eta,\lambda)+\beta_n(\eta,\lambda)\le 1$ for all $\eta,\lambda\in Y$.

Definition 1.3.5

A **Pythagorean Fuzzy Set** (PFS) on a universe Y is an object of the form

$$m=\{\langle\eta,\ \alpha_m(\eta),\beta_m(\eta)\rangle \mid \eta\in Y\}$$

Where $\alpha_m : Y\to[0,1]$ and $\beta_m : Y\to[0,1]$ represents the membership and non-membership functions of m, and α_m,β_m satisfies the condition $0\le\alpha_m^2(\eta)+\beta_m^2(\eta)\le 1$ for all $\eta\in Y$.

Definition 1.3.6

A Pythagorean Fuzzy Set n on $Y\times Y$ is said to be a **Pythagorean Fuzzy Relation (**PFR) on X, denoted by

$$n=\{\langle\eta\lambda,\alpha_n(\eta\lambda),\beta_n(\eta\lambda)\rangle \mid \eta\lambda\in Y\times Y\}$$

Where $\alpha_n : Y\times Y\to[0,1]$ and $\beta_n : Y\times Y\to[0,1]$ represents the membership function and non- membership functions of n, α_n,β_n satisfies the condition $0\le\alpha_m^2(\eta\lambda)+\beta_m^2(\eta\lambda)\le 1$ for all $\eta\ \lambda\in Y\times Y$.

Definition 1.3.7

Let Y be a Universe, then a **Cubic Set** (CS) has the following form

$$m=\{\langle y,\alpha_m(y),\beta_m(y)\rangle \mid y\in Y\}$$

In which α is an interval value fuzzy set and β is a fuzzy set.

Definition 1.3.8

Let Y be a Universe. A **Neutrosophic Set** (NS) over Y is defined by

$$m=\{\langle y,(T_P(y),I_P(y),F_P(y))\rangle \mid y\in Y\}$$

Where $T_P(y), I_P(y)$ and $F_P(y)$ are called truth-membership function, Indeterminancy membership function and falsity-membership function, respectively. They are, defined by $T_P(y): Y\to[^-0,1^+]$, $I_P(y): Y\to[^-0,1^+]$, $F_P(y): Y\to[^-0,1^+]$ such that $^-0\le T_P(y)+I_P(y)+F_P(y)\le 3^+$.

Definition 1.3.9

A **Spherical Fuzzy Set** (SFS) on a universe Y is an object of the form

$$m=\{\langle\eta,\ \alpha_m(\eta),\ \beta_m(\eta),\tau_m(\eta)\rangle\,|\,\eta\in Y\}$$

Where $\alpha_m:Y\to[0,1]$, $\beta_m:Y\to[0,1]$ and $\tau_m:Y\to[0,1]$ represents the membership, non-membership and Indeterminancy functions of m, and α_m,β_m and τ_m satisfies the condition $0\le\alpha_m^2(\eta)+\beta_m^2(\eta)+\tau_m^2(\eta)\le1$ for all $\eta\in Y$.

Definition 1.3.10

A Spherical Fuzzy Set n on $Y\times Y$ is said to be **Spherical Fuzzy Relation** (SPR) on Y, denoted by

$$n=\{\langle\eta\lambda,\alpha_n(\eta\lambda),\beta_n(\eta\lambda),\tau_n(\eta\lambda)\rangle\,|\,\eta\lambda\in Y\times Y\}$$

$$\alpha_n:Y\times Y\to[0,1],\ \beta_n:Y\times Y\to[0,1],\ \tau_n:Y\times Y\to[0,1]$$

represents the membership ,non-membership and Indeterminancy of n and α_n,β_n and τ_n satisfies the condition $0\le\alpha_n^2(\eta\lambda)+\beta_n^2(\eta\lambda)+\tau_n^2(\eta\lambda)\le1$ for all $\eta\lambda\in Y\times Y$.

Definition 1.3.11

Let Y is a nonempty set. A **Bipolar Fuzzy Set** (BFS) B in Y is an object having the form

$$B=\{y,(\alpha_B^P(y),\alpha_B^N(y))\,|\,y\in Y\},$$

Where, $\alpha_B^P:Y\to[0,1]$ and $\alpha_B^N:Y\to[0,1]$ are mappings.

We use the positive membership degree $\alpha_B^P(y)$ to denote the satisfaction degree of an element y to the property corresponding to a bipolar fuzzy set B and the negative membership degree $\alpha_B^N(y)$ to denote the satisfaction degree of an element y to some implicit counter property corresponding to a bipolar fuzzy set B. If and it is the situation that y is regarded as having only positive satisfaction for B. If $\alpha_B^P(y)=0$ and $\alpha_B^N(y)\neq0$ it is situation that y does not satisfy the property of B but somewhat satisfies the counter property of B. It is possible for an element y to be such that $\alpha_B^P(y) \neq 0$ and $\alpha_B^N(y) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of For the sake of simplicity, we will use the symbol $B = (\alpha_B^P, \alpha_B^N)$, for the bipolar fuzzy set.

Definition 1.3.12

A **Graph** G is finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G, called edges. The vertex set and the edge set of G are respectively denoted by V(G) and E(G) is denoted by G=(V, E).

Definition 1.3.18

A **Fuzzy Graph** (FG) on a non-empty set Y is a pair $g=(m,n)$ with m a FS on Y and n a fuzzy relation on Y such that

$$\alpha_n(\eta\lambda) \leq \alpha_m(\eta) \wedge \alpha_m(\lambda)$$

For all $\eta, \lambda \in Y$, where $m: Y \rightarrow [0,1]$ and $n: Y \times Y \rightarrow [0,1]$.

Definition 1.3.19

An **Intuitionistic Fuzzy Graph** (IFG) on a non-empty set Y is a pair $g=(m,n)$ with m an IFS on Y and n an IFR on Y such that

$$\alpha_n(\eta\lambda) \leq \alpha_m(\eta) \wedge \alpha_m(\lambda), \ \beta_n(\eta\lambda) \geq \beta_m(\eta) \vee \beta_m(\lambda)$$

And $0 \leq \alpha_n(\eta\lambda) + \beta_n(\eta\lambda) \leq 1$ for all $\eta, \lambda \in Y$, where $\alpha_n: Y \times Y \rightarrow [0,1]$ and $\beta_n: Y \times Y \rightarrow [0,1]$ represents the membership function and non- membership functions of n, respectively.

Definition 1.3.20

A **Pythagorean Fuzzy Graph** (PFG) on a non-empty set Y is a pair $g=(m,n)$ with m a PFS on Y and n a PFR on Y such that

$$\alpha_n(\eta\lambda) \leq \alpha_m(\eta) \wedge \alpha_m(\lambda), \ \beta_n(\eta\lambda) \geq \beta_m(\eta) \vee \beta_m(\lambda)$$

and $0 \leq \alpha_n^2(\eta\lambda) + \beta_n^2(\eta\lambda) \leq 1$ for all $\eta, \lambda \in Y$, where $\alpha_n: Y \times Y \rightarrow [0,1]$ and $\beta_n: Y \times Y \rightarrow [0,1]$ represents the membership function and non- membership functions of n, respectively.

Definition 1.3.21

A **Cubic Graph** is a triplet $G=(G^{*},P,Q)$ where $G^{*}=(V,E)$ is a graph, $P=(\alpha_P,\beta_P)$ is a cubic set on V and $Q=(\alpha_Q,\beta_Q)$ is a cubic set on $V\times V$ such that $\alpha_Q(\eta\lambda)\le \min\{\alpha_P(\eta),\alpha_P(\lambda)\}$, $\beta_Q(\eta\lambda)\ge \max\{\beta_P(\eta),\beta_P(\lambda)\}$

Definition 1.3.22

A **Spherical Fuzzy Graph** (SFG) on a non-empty set Y is a pair $g=(m,n)$ with m an SFS on Y and n an SFR on Y such that

$$\alpha_n(\eta\lambda)\le\alpha_m(\eta)\wedge\alpha_m(\lambda)\ ,\ \beta_n(\eta\lambda)\le\beta_m(\eta)\wedge\beta_m(\lambda)\ ,\tau_n(\eta\lambda)\ge\tau_m(\eta\lambda)\vee\tau_m(\eta\lambda)$$

and $0\le\alpha_n^2(\eta\lambda)+\beta_n^2(\eta\lambda)+\tau_n^2(\eta\lambda)\le 1$ for all $\eta,\lambda\in Y$, where, $\alpha_n : Y\times Y\to[0,1]$ $\beta_n : Y\times Y\to[0,1]$ and $\tau_n : Y\times Y\to[0,1]$ represents the membership, non-membership and indeterminacy functions of n, respectively.

CHAPTER 2

CHAPTER-2

Bipolar Spherical Fuzzy Neutrosophic Cubic Graph

Definition 2.1

Let X be a non-empty set. A Bipolar Spherical Fuzzy Neutrosophic Cubic Set (BSFNCS)

$$A = \{x, (T_A^{P+}, I_A^{P+}, F_A^{P+}), (T_A^{P-}, I_A^{P-}, F_A^{P-}), \lambda_A \mid x \in X\}$$

Where $T_A^{P+}, I_A^{P+}, F_A^{P+} : X \to [0,1]$, $T_A^{P-}, I_A^{P-}, F_A^{P-} : X \to [0,1]$, $\lambda_A : X \to [0,1]$ are the mappings such that $0 \le \left((T_A^{P+})^2 + (I_A^{P+})^2 + (F_A^{P+})^2\right) \le \sqrt{3}$ and $0 \le \left((T_A^{P-})^2 + (I_A^{P-})^2 + (F_A^{P-})^2\right) \le \sqrt{3}$ and T_A^{P+} denote the positive truth membership function, I_A^{P+} denote the positive indeterminancy membership function, F_A^{P+} denote the positive falsity membership function, T_A^{P-} denote the negative truth membership function, I_A^{P-} denote the negative indeterminancy membership function, F_A^{P-} denote the negative falsity membership function and λ_A denote the fuzzy membership function.

Definition 2.2

Let $G^* = (V, E)$ be a graph and G(P,Q) is a Bipolar Spherical Fuzzy Neutrosophic Cubic Graph (BSFNCG) of G^*, if

$$P = (A, \lambda) = \{V, (T_A^{P+}, I_A^{P+}, F_A^{P+}), (T_A^{P-}, I_A^{P-}, F_A^{P-}), \lambda_A\}$$

is the BSFNCS representation of vertex set V and

$$Q = (B, \mu) = \{V, (T_A^{P+}, I_A^{P+}, F_A^{P+}), (T_A^{P-}, I_A^{P-}, F_A^{P-}), \mu_B\}$$

is the BSFNCS representation of edge set E such that

1. $T_B^{P+}(u_i v_i) \le r\min\{T_A^{P+}(u_i), T_A^{P+}(v_i)\}, T_\mu^{P+}(u_i, v_i) \ge r\max\{T_\mu^{P+}(u_i), T_\mu^{P+}(v_i)\}$
   $T_B^{P-}(u_i v_i) \ge r\max\{T_A^{P-}(u_i), T_A^{P-}(v_i)\}, T_\mu^{P-}(u_i, v_i) \ge r\min\{T_\mu^{P-}(u_i), T_\mu^{P-}(v_i)\}$

2. $I_B^{P+}(u_i v_i) \le r\min\{I_A^{P+}(u_i), I_A^{P+}(v_i)\}, I_\mu^{P+}(u_i, v_i) \ge r\max\{I_\mu^{P+}(u_i), I_\mu^{P+}(v_i)\}$
   $I_B^{P-}(u_i v_i) \ge r\max\{I_A^{P-}(u_i), I_A^{P-}(v_i)\}, I_\mu^{P-}(u_i, v_i) \ge r\min\{I_\mu^{P-}(u_i), I_\mu^{P-}(v_i)\}$

3. $F_B^{P+}(u_i v_i) \le r\max\{F_A^{P+}(u_i), F_A^{P+}(v_i)\}, F_\mu^{P+}(u_i, v_i) \ge r\min\{F_\mu^{P+}(u_i), F_\mu^{P+}(v_i)\}$
   $F_B^{P-}(u_i v_i) \ge r\min\{F_A^{P-}(u_i), F_A^{P-}(v_i)\}, F_\mu^{P-}(u_i, v_i) \ge r\max\{F_\mu^{P-}(u_i), F_\mu^{P-}(v_i)\}$

Let $G^* = (V, E)$ be a graph and G(P,Q) is a Bipolar Spherical Fuzzy Neutrosophic Cubic Graph (BSFNCG) of G^*, if

$P = (A, \lambda) = \{V, (T_A^{P+}, I_A^{P+}, F_A^{P+}), (T_A^{P-}, I_A^{P-}, F_A^{P-}), \lambda_A\}$

is the BSFNCS representation of vertex set V and

$$Q = (B, \mu) = \{V, (T_A^{P+}, I_A^{P+}, F_A^{P+}), (T_A^{P-}, I_A^{P-}, F_A^{P-}), \mu_B\}$$

the BSFNCS representation of edge set E and λ and μ are a Bipolar Spherical Fuzzy Neutrosophic Cubic Sets.

Example 2.3

Let $G^* = (V, E)$ be a graph where V={a,b,c,d} and E={ab,ac,ad,bc,bd,cd} where P and Q are as follows:

$$P = \left\langle \begin{array}{l} \left\{ \begin{array}{l} a, ([0.3,0.5], 0.2), ([0.8,0.9], 0.5), ([0.2,0.4], 0.5) \\ ([-0.4,-0.3], -0.2), ([-0.6,-0.5], -0.8), ([-0.9,-0.8], -0.6) \end{array} \right\} \\ \left\{ \begin{array}{l} b, ([0.9,0.1], 0.7), ([0.6,0.8], 0.1), ([0.4,0.7], 0.1) \\ ([-0.8,-0.7], -0.5), ([-0.5,-0.2], -0.1), ([-0.3,-0.2], -0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} c, ([0.3,0.6], 0.8), ([0.4,0.6], 0.7), ([0.5,0.6], 0.4), \\ ([-0.5,-0.4], -0.1), ([-0.6,-0.3], -0.1), ([-0.7,-0.6], -0.3) \end{array} \right\} \\ \left\{ \begin{array}{l} d, ([0.1,0.3], 0.5), ([0.2,0.3], 0.6), ([0.6,0.7], 0.8) \\ ([-0.8,-0.6], -0.2), ([-0.7,-0.3], -0.2), ([-0.9,-0.6], -0.4) \end{array} \right\} \end{array} \right\rangle$$

$$Q=\left\langle \begin{array}{l} \left\{ \begin{array}{l} ab,([0.3,0.1],0.7),([0.6,0.8],0.5),([0.4,0.7],0.1) \\ ([-0.4,-0.7],-0.1),([-0.4,-0.3],-0.5),([-0.5,-0.2],-0.8) \end{array} \right\} \\ \left\{ \begin{array}{l} ac,([0.3,0.5,],0.8),([0.4,0.6],0.7),(0.5,0.6),0.4 \\ ([-0.4,-0.3],-0.2),([-0.6,-0.3],-0.8),([-0.9,-0.8],-0.3) \end{array} \right\} \\ \left\{ \begin{array}{l} ad,([0.1,0.3],0.5),([0.2,0.3],0.6),([0.6,0.7],0.5) \\ ([-0.4,-0.3],-0.2),([-0.6,0.3],0.8),([-0.9,-0.8],-0.4) \end{array} \right\} \\ \left\{ \begin{array}{l} bc,([0.3,0.1],0.8),([0.4,0.6],0.7),([0.5,0.7],0.1) \\ ([-o.5,-0.4],-0.5),([-0.5,-0.2],-0.1),([-0.7,-0.6],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} bd,([0.1,0.1],0.7),([0.2,0.3],0.6),([0.6,0.7],0.1) \\ ([-0.8,-0.6],-0.5),([-0.5,-0.2],-0.2),([-0.9,-0.6],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} cd([0.1,0.3],0.8),([0.2,0.3],0.7),([0.6,0.7],0.4) \\ ([--0.5,-0.4],-0.2),([-0.6,-0.3],-0.2),([-0.9,-0.6],-0.3) \end{array} \right\} \end{array} \right\rangle$$

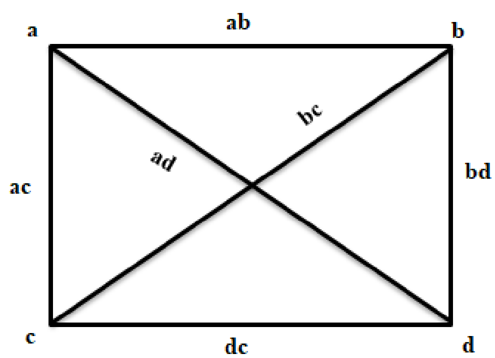

Figure 2.1

The vertex set in P and the edge set in Q are represented for the graph $G^* = (V, E)$

Remark 2.4

1. If $n \geq 3$ in the vertex set and $n \geq 3$ in the set of edges then the graphs in a bipolar neutrosophic cubic polygon only when we join each vertex to the corresponding vertex through an edge.

2. If we have infinite elements in the vertex set and by joining the edge and every edge with each other we get a bipolar neutrosophic cubic curve.

Definition 2.5

Let G = (P, Q) be a bipolar spherical fuzzy neutrosophic cubic graph. The order of bipolar spherical fuzzy neutrosophic cubic graph is defined by

$$O(G) = \sum_{u \in V} \left\{ \begin{array}{l} \left(T_A^{P+}, T_\lambda^{P+}\right)(u), \left(I_A^{P+}, I_\lambda^{P+}\right)(u), \left(F_A^{P+}, F_A^{P+}\right)(u), \\ (T_A^{P-}, T_\lambda^{P-})(u), \left(I_A^{P-}, I_A^{P-}\right)(u), \left(F_A^{P-}, F_A^{P-}\right)(u) \end{array} \right\}$$

and the degree of a vertex u and G is defined by

$$\deg(u) = \sum_{uv \in V} \left\{ \begin{array}{l} \left(T_B^{P+}, T_\mu^{P+}\right)(uv), \left(I_B^{P+}, I_\mu^{P+}\right)(uv), \left(F_B^{P+}, F_B^{P+}\right)(uv), \\ (T_B^{P-}, T_\mu^{P-})(uv), \left(I_B^{P-}, I_\mu^{P-}\right)(uv), \left(F_B^{P-}, F_\mu^{P-}\right)(uv) \end{array} \right\}$$

Example 2.6

In the above example, the order of a bipolar spherical fuzzy neutrosophic cubic graph

$$\deg(a) = \left\{ \begin{array}{l} ([0.9, 0.2], 2), ([1.2, 1.7], 1.8), ([1.5, 2], 1), \\ ([-1.2, -0.9], -0.9), ([-1.7, -0.8], -2.4), ([-2.7, -2.4], -0.8) \end{array} \right\}$$

$$\deg(b) = \left\{ \begin{array}{l} ([0.7, 0.3], 2.2), ([1.2, 1.7], 1.8), ([1.5, 2.1], 0.3), \\ ([-1.7, -1.3], -1.5), ([-1.5, -0.6], -1.1), ([-2.5, -2], -0.3) \end{array} \right\}$$

$$\deg(c) = \left\{ \begin{array}{l} ([0.7, 0.9], 2.4), ([1, 1.5], 2.1), ([1.6, 2], 0.9), \\ ([-1.4, -1.1], -0.9), ([-1.7, -1.8], -1.1), ([-2.5, -2], -0.7) \end{array} \right\}$$

$$\deg(d) = \left\{ \begin{array}{l} ([0.3, 0.7], 2), ([0.6, 0.9], 1.9), ([1.8, 2.1], 1), \\ ([-1.7, -1.3], -0.9), ([-1.7, -0.8], -1.2), ([-2.7, -2], -0.8) \end{array} \right\}$$

Definition 2.7

Let $G_1=(P_1,Q_1)$ be a bipolar spherical fuzzy neutrosophic cubic graph of $G_1^*=(V_1,E_1)$ and $G_2=(P_2,Q_2)$ be a bipolar spherical fuzzy neutrosophic cubic graph of $G_2^*=(V_2,E_2)$. The Cartesian product of G_1 and G_2 is denoted by

$$G_1\times G_2=(P_1\times P_2),Q_1\times Q_2$$

$$=\begin{pmatrix}(A_1^{P+},\lambda_1^{P+})\times(A_2^{P+},\lambda_2^{P+}),(A_1^{P-},\lambda_1^{P-})\times(A_2^{P-},\lambda_2^{P-}),\\(B_1^{P+},\mu_1^{P+})\times(B_2^{P+},\mu_2^{P+}),(B_1^{P-},\mu_1^{P-})\times(B_2^{P-},\mu_2^{P-})\end{pmatrix}$$

$$=\begin{pmatrix}(A_1^{P+}\times A_2^{P+},\lambda_1^{P+}\times\lambda_2^{P+}),(A_1^{P-}\times A_2^{P-},\lambda_1^{P-}\times\lambda_2^{P-})\\(B_1^{P+}\times B_2^{P+},\mu_1^{P+}\times\mu_2^{P+}),(B_1^{P-}\times B_2^{P-},\mu_1^{P-}\times\mu_1^{P-}\times\mu_2^{P-})\end{pmatrix}$$

$$G_1\times G_2=\left\langle\begin{matrix}(T^{P+}_{A_1\times A_2},T^{P+}_{\lambda_1\times\lambda_2}),(I^{P+}_{A_1\times A_2},I^{P+}_{\lambda_1\times\lambda_2}),(F^{P+}_{A_1\times A_2},F^{P+}_{\lambda_1\times\lambda_2})\\(T^{P-}_{A_1\times A_2},T^{P-}_{\lambda_1\times\lambda_2}),(I^{P-}_{A_1\times A_2},I^{P-}_{\lambda_1\times\lambda_2}),(F^{P-}_{A_1\times A_1},F^{P-}_{\lambda_1\times\lambda_2})\\(T^{P+}_{B_1\times B_2},T^{P+}_{\mu_1\times\mu_2}),(I^{P+}_{B_1\times B_2},I^{P+}_{\mu_1\times\mu_2}),(F^{P+}_{B_1\times B_2},F^{P+}_{\mu_1\times\mu_2})\\(T^{P-}_{B_1\times B_2},T^{P-}_{\mu_1\times\mu_1}),(I^{P-}_{B_1\times B_2},I^{P-}_{\mu_1\times\mu_2}),(F^{P-}_{B_1\times B_2},F^{P-}_{\mu_1\times\mu_2})\end{matrix}\right\rangle$$

and is defined as follows:

1. $$\begin{pmatrix}T^{P+}_{A_1\times A_2}(u,v)=r\min\{T^{P+}_{A_1}(u),T^{P+}_{A_2}(v)\},T^{P+}_{\lambda_1\times\lambda_2}(u,v)=r\max\{T^{P+}_{\lambda_1}(u),T^{P+}_{\lambda_2}(v)\},\\T^{P-}_{A_1\times A_2}(u,v)=r\max\{T^{P-}_{A_1}(u),T^{P-}_{A_2}(v)\},T^{P-}_{\lambda_1\times\lambda_2}(u,v)=r\min\{T^{P-}_{\lambda_1}(u),T^{P-}_{\lambda_2}(v)\}\end{pmatrix}$$

2. $$\begin{pmatrix}I^{P+}_{A_1\times A_2}(u,v)=r\min\{I^{P+}_{A_1}(u),I^{P+}_{A_2}(v)\},I^{P+}_{\lambda_1\times\lambda_2}(u,v)=r\max\{I^{P+}_{\lambda_1}(u),I^{P+}_{\lambda_2}(v)\},\\I^{P-}_{A_1\times A_2}(u,v)=r\max\{I^{P-}_{A_1}(u),I^{P-}_{A_2}(v)\},I^{P-}_{\lambda_1\times\lambda_2}(u,v)=r\min\{I^{P-}_{\lambda_1}(u),I^{P-}_{\lambda_2}(v)\}\end{pmatrix}$$

3. $$\begin{pmatrix}F^{P+}_{A_1\times A_2}(u,v)=r\max\{F^{P+}_{A_1}(u),F^{P+}_{A_2}(v)\},F^{P+}_{\lambda_1\times\lambda_2}(u,v)=r\min\{F^{P+}_{\lambda_1}(u),F^{P+}_{\lambda_2}(v)\},\\F^{P-}_{A_1\times A_2}(u,v)=r\min\{F^{P-}_{A_1}(u),F^{P-}_{A_2}(v)\},F^{P-}_{\lambda_1\times\lambda_2}(u,v)=r\max\{F^{P-}_{\lambda_1}(u),F^{P-}_{\lambda_2}(v)\}\end{pmatrix}$$

$$4.\begin{pmatrix} T^{P+}_{B_1\times B_2}((u,v_1)(u,v_2)) = r\min\{T^{P+}_{A_1}(u), T^{P+}_{B_2}(v_1 v_2)\}, \\ T^{P-}_{B_1\times B_2}((u,v_1)(u,v_2) = r\max\{T^{P-}_{A_1}(u), T^{P+}_{B_2}(v_1 v_2)\}, \\ T^{P+}_{\mu_1\times\mu_2}((u,v_1)(u,v_2)) = r\max\{T^{P+}_{\lambda_1}(u), T^{P+}_{\mu_2}(v_1 v_2)\}, \\ T^{P-}_{\mu_1\times\mu_2}((u,v_1)(u,v_2)) = r\min\{T^{P-}_{\lambda_1}(u), T^{P+}_{\mu_2}(v_1 v_2)\} \end{pmatrix}$$

$$5.\begin{pmatrix} I^{P+}_{B_1\times B_2}((u,v_1)(u,v_2)) = r\min\{I^{P+}_{A_1}(u), I^{P+}_{B_2}(v_1 v_2)\}, \\ I^{P-}_{B_1\times B_2}((u,v_1)(u,v_2) = r\max\{I^{P-}_{A_1}(u), I^{P+}_{B_2}(v_1 v_2)\}, \\ I^{P+}_{\mu_1\times\mu_2}((u,v_1)(u,v_2)) = r\max\{I^{P+}_{\lambda_1}(u), I^{P+}_{\mu_2}(v_1 v_2)\}, \\ I^{P-}_{\mu_1\times\mu_2}((u,v_1)(u,v_2)) = r\min\{I^{P-}_{\lambda_1}(u), I^{P+}_{\mu_2}(v_1 v_2)\} \end{pmatrix}$$

$$6.\begin{pmatrix} F^{P+}_{B_1\times B_2}((u,v_1)(u,v_2)) = r\max\{F^{P+}_{A_1}(u), F^{P+}_{B_2}(v_1 v_2)\}, \\ F^{P-}_{B_1\times B_2}((u,v_1)(u,v_2) = r\min\{F^{P-}_{A_1}(u), F^{P+}_{B_2}(v_1 v_2)\}, \\ F^{P+}_{\mu_1\times\mu_2}((u,v_1)(u,v_2)) = r\min\{F^{P+}_{\lambda_1}(u), F^{P+}_{\mu_2}(v_1 v_2)\}, \\ F^{P-}_{\mu_1\times\mu_2}((u,v_1)(u,v_2)) = r\max\{F^{P-}_{\lambda_1}(u), F^{P+}_{\mu_2}(v_1 v_2)\} \end{pmatrix}$$

$$7.\begin{pmatrix} T^{P+}_{B_1\times B_2}((u_1,v)(u_2,v)) = r\min\{T^{P+}_{B_1}(u_1,u_2), T^{P+}_{\lambda_2}(v)\}, \\ T^{P-}_{B_1\times B_2}((u_1,v)(u_2,v) = r\max\{T^{P-}_{B_1}(u_1,u_2), T^{P+}_{\lambda_2}(v)\}, \\ T^{P+}_{\mu_1\times\mu_2}((u_1,v)(u_2,v)) = r\max\{T^{P+}_{\mu_1}(u_1,u_2), T^{P+}_{\lambda_2}(v)\}, \\ T^{P-}_{\mu_1\times\mu_2}((u_1,v)(u_2,v)) = r\min\{T^{P-}_{\mu_1}(u_1,u_2), T^{P+}_{\lambda_2}(v)\} \end{pmatrix}$$

$$8.\begin{pmatrix} I^{P+}_{B_1\times B_2}((u_1,v)(u_2,v)) = r\min\{I^{P+}_{B_1}(u_1,u_2), I^{P+}_{\lambda_2}(v)\}, \\ I^{P-}_{B_1\times B_2}((u_1,v)(u_2,v) = r\max\{I^{P-}_{B_1}(u_1,u_2), I^{P+}_{\lambda_2}(v)\}, \\ I^{P+}_{\mu_1\times\mu_2}((u_1,v)(u_2,v)) = r\max\{I^{P+}_{\mu_1}(u_1,u_2), I^{P+}_{\lambda_2}(v)\}, \\ I^{P-}_{\mu_1\times\mu_2}((u_1,v)(u_2,v)) = r\min\{I^{P-}_{\mu_1}(u_1,u_2), I^{P+}_{\lambda_2}(v)\} \end{pmatrix}$$

$$9.\begin{pmatrix} F^{P+}_{B_1\times B_2}((u_1,v)(u_2,v)) = r\max\{F^{P+}_{B_1}(u_1,u_2), F^{P+}_{\lambda_2}(v)\}, \\ F^{P-}_{B_1\times B_2}((u_1,v)(u_2,v) = r\min\{F^{P-}_{B_1}(u_1,u_2), F^{P+}_{\lambda_2}(v)\}, \\ F^{P+}_{\mu_1\times\mu_2}((u_1,v)(u_2,v)) = r\min\{F^{P+}_{\mu_1}(u_1,u_2), F^{P+}_{\lambda_2}(v)\}, \\ F^{P-}_{\mu_1\times\mu_2}((u_1,v)(u_2,v)) = r\max\{F^{P-}_{\mu_1}(u_1,u_2), F^{P+}_{\lambda_2}(v)\} \end{pmatrix}$$

Example 2.8

Let $G_1 = (P_1, Q_1)$ be a bipolar spherical fuzzy neutrosophic cubic graph of $G_1^* = (V_1, E_1)$ as shown in figure, where $V_1 = \{a, b, c\}$, $E_1 = \{ab, bc, ac\}$.

$$P_1 = \left\langle \begin{matrix} \left\{ \begin{matrix} \text{a}, ([0.3,0.6],0.8), ([0.4,0.6],0.7), ([0.5,0.6],0.4), \\ ([-0.8,-0.6],-0.2), ([-0.7,-0.3],-0.2), ([-0.9,-0.6],-0.4) \end{matrix} \right\} \\ \left\{ \begin{matrix} \text{b}, ([0.1,0.3],0.5), ([0.2,0.3],0.6), ([0.6,0.7],0.8) \\ ([-0.4,-0.3],-0.2), ([-0.6,-0.5],-0.8), ([-0.9,-0.8],-0.6) \end{matrix} \right\} \\ \left\{ \begin{matrix} \text{c}, ([0.3,0.5],0.2), ([0.8,0.9],0.5), ([0.2,0.4],0.5) \\ ([-0.8,-0.7],-0.5), ([-0.5,-0.2],-0.1), ([-0.3,-0.2],-0.1) \end{matrix} \right\} \end{matrix} \right\rangle$$

$$Q_1 = \left\langle \begin{matrix} \left\{ \begin{matrix} \text{ab}, ([0.1,0.3],0.8), ([0.2,0.3],0.7), ([0.6,0.7],0.4), \\ ([-0.4,-0.3],-0.2), ([-0.6,-0.3],-0.8), ([-0.9,-0.8],-0.4) \end{matrix} \right\} \\ \left\{ \begin{matrix} \text{ac}, ([0.3,0.5],0.8), ([0.4,0.6],0.7), ([0.5,0.6],0.4) \\ ([-0.8,-0.6],-0.5), ([-0.5,-0.2],-0.2), ([-0.9,-0.6],-0.1) \end{matrix} \right\} \\ \left\{ \begin{matrix} \text{bc}, ([0.1,0.3],0.5), ([0.2,0.3],0.6), ([0.6,0.7],0.4) \\ ([-0.4,-0.3],-0.5), ([-0.5,-0.2],-0.8), ([-0.9,-0.8],-0.1) \end{matrix} \right\} \end{matrix} \right\rangle$$

And $G_2 = (P_2, Q_2)$ be a bipolar spherical fuzzy neutrosophic cubic graph of $G_2^* = (V_2, E_2)$ as shown in figure, where $\text{V}_2 = \{\text{x}, \text{y}, \text{z}\}$, $\text{E}_2 = \{\text{xy}, \text{yz}, \text{xz}\}$

$$P_2 = \left\langle \begin{matrix} \left\{ \begin{matrix} \text{x}, ([0.5,0.6],0.3), ([0.4,0.7],0.1), ([0.2,0.3],0.5), \\ ([-0.7,-0.6],-0.1), ([-0.4,-0.2],-0.5), ([-0.5,-0.4],-0.3) \end{matrix} \right\} \\ \left\{ \begin{matrix} \text{y}, ([0.1,0.2],0.4), ([0.7,0.3],0.9), ([0.2,0.4],0.1) \\ ([-0.3,-0.2],-0.1), ([-0.5,-0.3],-0.2), ([-0.7,-0.5],-0.3) \end{matrix} \right\} \\ \left\{ \begin{matrix} \text{z}, ([0.3,0.5],0.2), ([0.5,0.6],0.7), ([0.2,0.6],0.8) \\ ([-0.8,-0.7],-0.5), ([-0.7,-0.4],-0.3), ([-0.9,-0.4],-0.2) \end{matrix} \right\} \end{matrix} \right\rangle$$

$$Q_2 = \left\langle \begin{matrix} \left\{ \begin{matrix} xy, ([0.1,0.2],0.4), ([0.4,0.3],0.9), ([0.2,0.4],0.1), \\ ([-0.3,-0.2],-0.1), ([-0.4,-0.2],-0.5), ([-0.7,-0.5],-0.3) \end{matrix} \right\} \\ \left\{ \begin{matrix} xz, ([0.3,0.5],0.3), ([0.4,0.6],0.7), ([0.2,0.6],0.5) \\ ([-0.7,-0.6],-0.5), ([-0.4,-0.2],-0.5), ([-0.9,-0.4],-0.2) \end{matrix} \right\} \\ \left\{ \begin{matrix} yz, ([0.1,0.2],0.4), ([0.5,0.3],0.9), ([0.2,0.6],0.1) \\ ([-0.3,-0.2],-0.5), ([-0.5,-0.3],-0.3), ([-0.9,-0.5],-0.2) \end{matrix} \right\} \end{matrix} \right\rangle$$

Then $G_1 \times G_2$ is a bipolar spherical fuzzy neutrosophic cubic graph of $G_1^* \times G_2^*$ as shown in figure, where

$V_1 \times V_2 = \{(a,x),(a,y),(a,z),\}(b,x),(b,y),(b,z),(c,x),(c,y),(c,z)\}$ and

$$p_1 \times p_2 = \left\langle \begin{matrix} \left\{ \begin{matrix} (a,x), ([0.3,0.6],0.8), ([0.4,0.6],0.7), ([0.5,0.6],0.4), \\ ([-0.7,-0.6],-0.2), ([-0.4,-0.2],-0.5), ([-0.9,-0.6],-0.3) \end{matrix} \right\} \\ \left\{ \begin{matrix} (a,y), ([0.1,0.2],0.8), ([0.4,0.3],0.9), ([0.5,0.6],0.1), \\ ([-0.3,-0.2],-0.2), ([-0.5,-0.3],-0.2), ([-0.9,-0.6],-0.3) \end{matrix} \right\} \\ \left\{ \begin{matrix} (a,z), ([0.3,0.5),0.8), ([0.4,0.6],0.7), ([0.5,0.6],0.4), \\ ([-0.8,-0.6],-0.5), ([-0.7,-0.3],-0.3), ([-0.9,-0.6],-0.2) \end{matrix} \right\} \\ \left\{ \begin{matrix} (b,x), ([0.1,0.3],0.5), ([0.2,0.3],0.6), ([0.6,0.7],0.5), \\ ([-0.4,-0.3],-0.2), ([-0.4,-0.2],-0.8), ([-0.9,-0.8],-0.3) \end{matrix} \right\} \\ \left\{ \begin{matrix} (b,y), ([0.1,0.2],0.5), ([0.2,0.3],0.9), ([0.6,0.7],0.1), \\ ([-0.3,-0.2],-0.2), ([-0.5,-0.3],-0.8), ([-0.9,-0.8],-0.3) \end{matrix} \right\} \\ \left\{ \begin{matrix} (b,z), ([0.1,0.3],0.5), ([0.2,0.3],0.7), ([0.6,0.7],0.8), \\ ([-0.4,-0.3],-0.5), ([-0.6,-0.4],-0.8), ([-0.9,-0.8],-0.2) \end{matrix} \right\} \\ \left\{ \begin{matrix} (c,x), ([0.3,0.5],0.3), ([0.4,0.7],0.5), ([0.2,0.4],0.5), \\ ([-0.7,-0.6],-0.5), ([-0.4,-0.2],-0.5), ([-0.5,-0.4],-0.1) \end{matrix} \right\} \\ \left\{ \begin{matrix} (c,y), ([0.1,0.2],0.4), ([0.7,0.3],0.9), ([0.2,0.4],0.1), \\ ([-0.3,-0.2],-0.5), ([-0.5,-0.2],-0.2), ([-0.7,-0.5],-0.1) \end{matrix} \right\} \\ \left\{ \begin{matrix} (c,z), ([0.3,0.5],0.2), ([0.5,0.6],0.7), ([0.2,0.6],0.5), \\ ([-0.8,-0.7],-0.5), ([-0.5,-0.2],-0.3), ([-0.9,-0.4],-0.1) \end{matrix} \right\} \end{matrix} \right\rangle$$

$$Q_1 \times Q_2 = \left\langle \begin{array}{l} \left\{ \begin{array}{l} ((a,x)(a,y)),([0.1,0.2],0.8),([0.4,0.3],0.9),([0.5,0.6],0.1), \\ ([-0.3,-0.2],-0.2),([-0.4,-0.2],-0.5),([-0.9,-0.6],-0.3) \end{array} \right\} \\ \left\{ \begin{array}{l} ((a,y)(a,z)),([0.1,0.2],0.8),([0.4,0.3],0.9),([0.5,0.6],0.1), \\ ([-0.3,-0.2],-0.5),([-0.5,-0.3],-0.3),([-0.9,-0.6],-0.2) \end{array} \right\} \\ \left\{ \begin{array}{l} ((a,z)(b,z)),([0.1,0.3],0.8),([0.2,0.3],0.7),([0.6,0.7],0.4), \\ ([-0.4,-0.3],-0.5),([-0.6,-0.3],-0.8),([-0.9,-0.8],-0.2) \end{array} \right\} \\ \left\{ \begin{array}{l} ((b,x)(b,y)),([0.1,0.2],0.5),([0.2,0.3],0.9),([0.6,0.7],0.1), \\ ([-0.3,-0.2],-0.2),([-0.4,-0.2],-0.8),([-0.9,-0.8],-0.3) \end{array} \right\} \\ \left\{ \begin{array}{l} ((b,x)(b,z)),([0.1,0.3],0.5),([0.2,0.3],0.7),([0.6,0.7],0.5), \\ ([-0.4,-0.3],-0.5),([-0.4,-0.2],-0.8),([-0.9,-0.8],-0.2) \end{array} \right\} \\ \left\{ \begin{array}{l} ((c,y)(c,z)),([0.1,0.2],0.4),([0.5,0.3],0.9),([0.2,0.6],0.1), \\ ([-0.3,-0.2],-0.5),([-0.5,-0.2],-0.3),([-0.9,-0.5],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} ((c,x)(c,z)),([0.3,0.5],0.3),([0.4,0.6],0.7),([0.2,0.6],0.5), \\ ([-0.7,-0.6],-0.5),([-0.4,-0.2],-0.5),([-0.9,-0.4],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} ((a,x)(c,x)),([0.3,0.5],0.8),([0.4,0.6],0.7),([0.5,0.6],0.4), \\ ([-0.7,-0.6],-0.5),([-0.4,-0.2],-0.5),([-0.9,-0.6],-0.1) \end{array} \right\} \end{array} \right\rangle$$

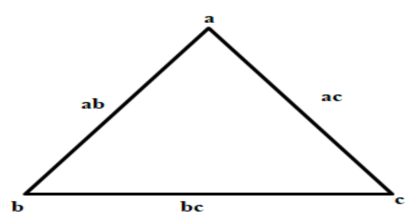

Figure 2.2

The vertex set in P_1 and the edge set in Q_1 are represented for the graph $G_1 = (P_1, Q_1)$

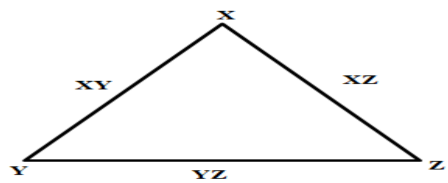

Figure 2.3

The vertex set in P_2 and the edge set in Q_2 are represented for the graph $G_2 = (P_2, Q_2)$

Proposition 2.9

The Cartesian product of two bipolar spherical neutrosophic cubic graph is again a bipolar spherical fuzzy neutrosophic cubic graph.

Proof

For $P_1 \times P_2$ the condition is obvious. Now we verify the condition only for $Q_1 \times Q_2$ where

$$Q_1 \times Q_2 = \left\{(T^{P+}_{B_1\times B_2}, T^{P+}_{\mu_1\times\mu_2}), (I^{P+}_{B_1\times B_2}, I^{P+}_{\mu_1\times\mu_2}), (F^{P+}_{B_1\times B_2}, F^{P+}_{\mu_1\times\mu_2}), (T^{P-}_{B_1\times B_2}, T^{P-}_{\mu_1\times\mu_2}), (I^{P-}_{B_1\times B_2}, I^{P-}_{\mu_1\times\mu_2}), (F^{P-}_{B_1\times B_2}, F^{P-}_{\mu_1\times\mu_2})\right\}$$

Then

$$\begin{pmatrix} T^{P+}_{B_1\times B_2}((u,u_2)(u,v_2)) = r\min\left\{T^{P+}_{A_1}(u), T^{P+}_{B_2}(u_2\, v_2)\right\}, \\ \leq r\min\left\{(T^{P+}_{A_1}(u), (r\min(\, T^{P+}_{A_2}(u_2), T^{P+}_{A_2}(v_2)))\right\} \\ = r\min\left\{r\min(\, T^{P+}_{A_1}(u), T^{P+}_{A_2}(u_2)), r\min(\, T^{P+}_{A_1}(u), T^{P+}_{A_2}(v_2))\right\} \\ = r\min\left\{((T^{P+}_{A_1} \times T^{P+}_{A_2})(uu_2)), ((T^{P+}_{A_1} \times T^{P+}_{A_2})(uv_2))\right\} \end{pmatrix}$$

$$\left(\begin{array}{l} T^{P-}_{B_1\times B_2}((u,u_2)(u,v_2)) = r\max\left\{T^{P-}_{A_1}(u), T^{P-}_{B_2}(u_2\,v_2)\right\}, \\ \geq r\min\left\{(T^{P-}_{A_1}(u), (r\max(T^{P-}_{A_2}(u_2), T^{P-}_{A_2}(v_2)))\right\} \\ = r\max\left\{r\max(T^{P-}_{A_1}(u), T^{P-}_{A_2}(u_2)), r\max(T^{P-}_{A_1}(u), T^{P-}_{A_2}(v_2))\right\} \\ = r\max\left\{((T^{P-}_{A_1}\times T^{P-}_{A_2})(uu_2)), ((T^{P-}_{A_1}\times T^{P-}_{A_2})(uv_2))\right\} \end{array}\right)$$

$$\left(\begin{array}{l} T^{P+}_{\mu_1\times \mu_2}((u,u_2)(u,v_2)) = r\max\left\{T^{P+}_{\lambda_1}(u), T^{P+}_{\mu_2}(u_2\,v_2)\right\}, \\ \leq r\max\left\{(T^{P+}_{\lambda_1}(u), (r\max(T^{P+}_{\lambda_2}(u_2), T^{P+}_{\lambda_2}(v_2)))\right\} \\ = r\max\left\{r\max(T^{P+}_{\lambda_1}(u), T^{P+}_{\lambda_2}(u_2)), r\max(T^{P+}_{\lambda_1}(u), T^{P+}_{\lambda_2}(v_2))\right\} \\ = r\max\left\{((T^{P+}_{\lambda_1}\times T^{P+}_{\lambda_2})(uu_2)), ((T^{P+}_{\lambda_1}\times T^{P+}_{\lambda_2})(uv_2))\right\} \end{array}\right)$$

$$\left(\begin{array}{l} T^{P-}_{\mu_1\times \mu_2}((u,u_2)(u,v_2)) = r\min\left\{T^{P-}_{\lambda_1}(u), T^{P-}_{\mu_2}(u_2\,v_2)\right\}, \\ \geq r\min\left\{(T^{P-}_{\lambda_1}(u), (r\min(T^{P-}_{\lambda_2}(u_2), T^{P-}_{\lambda_2}(v_2)))\right\} \\ = r\min\left\{r\min(T^{P-}_{\lambda_1}(u), T^{P-}_{\lambda_2}(u_2)), r\min(T^{P-}_{\lambda_1}(u), T^{P-}_{\lambda_2}(v_2))\right\} \\ = r\min\left\{((T^{P-}_{\lambda_1}\times T^{P-}_{\lambda_2})(uu_2)), ((T^{P-}_{\lambda_1}\times T^{P-}_{\lambda_2})(uv_2))\right\} \end{array}\right)$$

$$\left(\begin{array}{l} I^{P+}_{B_1\times B_2}((u,u_2)(u,v_2)) = r\min\left\{I^{P+}_{A_1}(u), I^{P+}_{B_2}(u_2\,v_2)\right\}, \\ \leq r\min\left\{(I^{P+}_{A_1}(u), (r\min(I^{P+}_{A_2}(u_2), I^{P+}_{A_2}(v_2)))\right\} \\ = r\min\left\{r\min(I^{P+}_{A_1}(u), I^{P+}_{A_2}(u_2)), r\min(I^{P+}_{A_1}(u), I^{P+}_{A_2}(v_2))\right\} \\ = r\min\left\{((I^{P+}_{A_1}\times I^{P+}_{A_2})(uu_2)), ((I^{P+}_{A_1}\times I^{P+}_{A_2})(uv_2))\right\} \end{array}\right)$$

$$\left(\begin{array}{l} I^{P-}_{B_1\times B_2}((u,u_2)(u,v_2)) = r\max\left\{I^{P-}_{A_1}(u), I^{P-}_{B_2}(u_2\,v_2)\right\}, \\ \geq r\max\left\{(I^{P-}_{A_1}(u), (r\max(I^{P-}_{A_2}(u_2), I^{P-}_{A_2}(v_2)))\right\} \\ = r\max\left\{r\max(I^{P-}_{A_1}(u), I^{P-}_{A_2}(u_2)), r\max(I^{P-}_{A_1}(u), I^{P-}_{A_2}(v_2))\right\} \\ = r\max\left\{((I^{P-}_{A_1}\times I^{P-}_{A_2})(uu_2)), ((I^{P-}_{A_1}\times I^{P-}_{A_2})(uv_2))\right\} \end{array}\right)$$

$$\left(\begin{array}{l} I^{P+}_{\mu_1\times \mu_2}((u,u_2)(u,v_2)) = r\max\left\{I^{P+}_{\lambda_1}(u), I^{P+}_{\mu_2}(u_2\,v_2)\right\}, \\ \leq r\max\left\{(I^{P+}_{\lambda_1}(u), (r\max(I^{P+}_{\lambda_2}(u_2), I^{P+}_{\lambda_2}(v_2)))\right\} \\ = r\max\left\{r\max(I^{P+}_{\lambda_1}(u), I^{P+}_{\lambda_2}(u_2)), r\max(I^{P+}_{\lambda_1}(u), I^{P+}_{\lambda_2}(v_2))\right\} \\ = r\max\left\{((I^{P+}_{\lambda_1}\times I^{P+}_{\lambda_2})(uu_2)), ((I^{P+}_{\lambda_1}\times I^{P+}_{\lambda_2})(uv_2))\right\} \end{array}\right)$$

$$\begin{pmatrix} I^{P-}_{\mu_1\times\mu_2}((u,u_2)(u,v_2)) = r\min\left\{I^{P-}_{\lambda_1}(u), I^{P-}_{\mu_2}(u_2\, v_2)\right\}, \\ \geq r\min\left\{(I^{P-}_{\lambda_1}(u), (r\min(\, I^{P-}_{\lambda_2}(u_2), I^{P-}_{\lambda_2}(v_2)))\right\} \\ = r\min\left\{r\min(\, I^{P-}_{\lambda_1}(u), I^{P-}_{\lambda_2}(u_2)), r\min(\, I^{P-}_{\lambda_1}(u), I^{P-}_{\lambda_2}(v_2))\right\} \\ = r\min\left\{((I^{P-}_{\lambda_1}\times I^{P-}_{\lambda_2})(uu_2)), ((I^{P-}_{\lambda_1}\times I^{P-}_{\lambda_2})(uv_2))\right\} \end{pmatrix}$$

$$\begin{pmatrix} F^{P+}_{B_1\times B_2}((u,u_2)(u,v_2)) = r\max\left\{F^{P+}_{A_1}(u), F^{P+}_{B_2}(u_2\, v_2)\right\}, \\ \leq r\max\left\{(F^{P+}_{A_1}(u), (r\max(\, F^{P+}_{A_2}(u_2), F^{P+}_{A_2}(v_2)))\right\} \\ = r\max\left\{r\max(\, F^{P+}_{A_1}(u), F^{P+}_{A_2}(u_2)), r\max(\, F^{P+}_{A_1}(u), F^{P+}_{A_2}(v_2))\right\} \\ = r\max\left\{((F^{P+}_{A_1}\times F^{P+}_{A_2})(uu_2)), ((F^{P+}_{A_1}\times F^{P+}_{A_2})(uv_2))\right\} \end{pmatrix}$$

$$\begin{pmatrix} F^{P-}_{B_1\times B_2}((u,u_2)(u,v_2)) = r\min\left\{F^{P-}_{A_1}(u), F^{P-}_{B_2}(u_2\, v_2)\right\}, \\ \geq r\min\left\{(F^{P-}_{A_1}(u), (r\min(\, F^{P-}_{A_2}(u_2), F^{P-}_{A_2}(v_2)))\right\} \\ = r\min\left\{r\min(\, F^{P-}_{A_1}(u), F^{P-}_{A_2}(u_2)), r\min(\, F^{P-}_{A_1}(u), F^{P-}_{A_2}(v_2))\right\} \\ = r\min\left\{((F^{P-}_{A_1}\times F^{P-}_{A_2})(uu_2)), ((F^{P-}_{A_1}\times F^{P-}_{A_2})(uv_2))\right\} \end{pmatrix}$$

$$\begin{pmatrix} F^{P+}_{\mu_1\times\mu_2}((u,u_2)(u,v_2)) = r\min\left\{F^{P+}_{\lambda_1}(u), F^{P+}_{\mu_2}(u_2\, v_2)\right\}, \\ \leq r\min\left\{(F^{P+}_{\lambda_1}(u), (r\min(\, F^{P+}_{\lambda_2}(u_2), F^{P+}_{\lambda_2}(v_2)))\right\} \\ = r\min\left\{r\min(\, F^{P+}_{\lambda_1}(u), F^{P+}_{\lambda_2}(u_2)), r\min(\, F^{P+}_{\lambda_1}(u), F^{P+}_{\lambda_2}(v_2))\right\} \\ = r\min\left\{((F^{P+}_{\lambda_1}\times F^{P+}_{\lambda_2})(uu_2)), ((F^{P+}_{\lambda_1}\times F^{P+}_{\lambda_2})(uv_2))\right\} \end{pmatrix}$$

$$\begin{pmatrix} F^{P-}_{\mu_1\times\mu_2}((u,u_2)(u,v_2)) = r\max\left\{F^{P-}_{\lambda_1}(u), F^{P-}_{\mu_2}(u_2\, v_2)\right\}, \\ \geq r\max\left\{(F^{P-}_{\lambda_1}(u), (r\max(\, F^{P-}_{\lambda_2}(u_2), F^{P-}_{\lambda_2}(v_2)))\right\} \\ = r\max\left\{r\max(\, F^{P-}_{\lambda_1}(u), F^{P-}_{\lambda_2}(u_2)), r\max(\, F^{P-}_{\lambda_1}(u), F^{P-}_{\lambda_2}(v_2))\right\} \\ = r\max\left\{((F^{P-}_{\lambda_1}\times F^{P-}_{\lambda_2})(uu_2)), ((F^{P-}_{\lambda_1}\times F^{P-}_{\lambda_2})(uv_2))\right\} \end{pmatrix}$$

Similarly, we can prove it for $w \in V_2$ and $u_1, u_2 \in E_2$

Definition 2.10

Let $G_1=(P_1,Q_1)$ and $G_2=(P_2,Q_2)$ be two bipolar spherical fuzzy neutrosophic cubic graphs. The degree of a vertex in $G_1\times G_2$ can be defined as follows for any $(u_1\times u_2)\in v_1\times v_2$

$$\begin{aligned}
&\deg(T_{A_1}^{P+}\times T_{A_2}^{P+})(u_1,u_2)\\
&=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\max(T_{B_1}^{P+}\times T_{B_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(T_{A_1}^{P+}(u),T_{B_2}^{P+}(u_2,v_2))\\
&+\sum_{u_1=v_2=w,u_1v_1\in E} r\max(T_{A_2}^{P+}(w),T_{B_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(T_{B_1}^{P+}(u_1,v_1),T_{B_2}^{P+}(u_2,v_2))
\end{aligned}$$

$$\begin{aligned}
&\deg(T_{A_1}^{P-}\times T_{A_2}^{P-})(u_1,u_2)\\
&=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\min(T_{B_1}^{P+}\times T_{B_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(T_{A_1}^{P+}(u),T_{B_2}^{P+}(u_2,v_2))\\
&+\sum_{u_1=v_2=w,u_1v_1\in E} r\min(T_{A_2}^{P+}(w),T_{B_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(T_{B_1}^{P+}(u_1,v_1),T_{B_2}^{P+}(u_2,v_2))
\end{aligned}$$

$$\begin{aligned}
&\deg(T_{\lambda_1}^{P+}\times T_{\lambda_2}^{P+})(u_1,u_2)\\
&=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\min(T_{\mu_1}^{P+}\times T_{\mu_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(T_{\lambda_1}^{P+}(u),T_{\lambda_2}^{P+}(u_2,v_2))\\
&+\sum_{u_1=v_2=w,u_1v_1\in E} r\min(T_{\lambda_2}^{P+}(w),T_{\mu_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(T_{\mu_1}^{P+}(u_1,v_1),T_{\mu_2}^{P+}(u_2,v_2))
\end{aligned}$$

$$\begin{aligned}
&\deg(T_{\lambda_1}^{P-}\times T_{\lambda_2}^{P-})(u_1,u_2)\\
&=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\max(T_{\mu_1}^{P-}\times T_{\mu_2}^{P-})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(T_{\lambda_1}^{P-}(u),T_{\lambda_2}^{P-}(u_2,v_2))\\
&+\sum_{u_1=v_2=w,u_1v_1\in E} r\max(T_{\lambda_2}^{P-}(w),T_{\mu_1}^{P-}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(T_{\mu_1}^{P-}(u_1,v_1),T_{\mu_2}^{P-}(u_2,v_2))
\end{aligned}$$

$$\begin{aligned}
&\deg(I_{A_1}^{P+}\times I_{A_2}^{P+})(u_1,u_2)\\
&=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\max(I_{B_1}^{P+}\times I_{B_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(I_{A_1}^{P+}(u),I_{B_2}^{P+}(u_2,v_2))\\
&+\sum_{u_1=v_2=w,u_1v_1\in E} r\max(I_{A_2}^{P+}(w),I_{B_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(I_{B_1}^{P+}(u_1,v_1),I_{B_2}^{P+}(u_2,v_2))
\end{aligned}$$

$$\deg(I_{A_1}^{P-}\times I_{A_2}^{P-})(u_1,u_2)$$
$$=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\min(I_{B_1}^{P+}\times I_{B_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(I_{A_1}^{P+}(u),I_{B_2}^{P+}(u_2,v_2))$$
$$+\sum_{u_1=v_2=w,u_1v_1\in E} r\min(I_{A_2}^{P+}(w),I_{B_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(I_{B_1}^{P+}(u_1,v_1),I_{B_2}^{P+}(u_2,v_2))$$

$$\deg(I_{\lambda_1}^{P+}\times I_{\lambda_2}^{P+})(u_1,u_2)$$
$$=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\min(I_{\mu_1}^{P+}\times I_{\mu_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(I_{\lambda_1}^{P+}(u),I_{\lambda_2}^{P+}(u_2,v_2))$$
$$+\sum_{u_1=v_2=w,u_1v_1\in E} r\min(I_{\lambda_2}^{P+}(w),I_{\mu_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(I_{\mu_1}^{P+}(u_1,v_1),I_{\mu_2}^{P+}(u_2,v_2))$$

$$\deg(I_{\lambda_1}^{P-}\times I_{\lambda_2}^{P-})(u_1,u_2)$$
$$=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\max(I_{\mu_1}^{P-}\times I_{\mu_2}^{P-})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(I_{\lambda_1}^{P-}(u),I_{\lambda_2}^{P-}(u_2,v_2))$$
$$+\sum_{u_1=v_2=w,u_1v_1\in E} r\max(I_{\lambda_2}^{P-}(w),I_{\mu_1}^{P-}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(I_{\mu_1}^{P-}(u_1,v_1),I_{\mu_2}^{P-}(u_2,v_2))$$

$$\deg(F_{A_1}^{P+}\times F_{A_2}^{P+})(u_1,u_2)$$
$$=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\min(F_{B_1}^{P+}\times F_{B_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(F_{A_1}^{P+}(u),F_{B_2}^{P+}(u_2,v_2))$$
$$+\sum_{u_1=v_2=w,u_1v_1\in E} r\min(F_{A_2}^{P+}(w),F_{B_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\min(F_{B_1}^{P+}(u_1,v_1),F_{B_2}^{P+}(u_2,v_2))$$

$$\deg(F_{A_1}^{P-}\times F_{A_2}^{P-})(u_1,u_2)$$
$$=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\max(F_{B_1}^{P+}\times F_{B_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(F_{A_1}^{P+}(u),F_{B_2}^{P+}(u_2,v_2))$$
$$+\sum_{u_1=v_2=w,u_1v_1\in E} r\max(F_{A_2}^{P+}(w),F_{B_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(F_{B_1}^{P+}(u_1,v_1),F_{B_2}^{P+}(u_2,v_2))$$

$$\deg(F_{\lambda_1}^{P+}\times F_{\lambda_2}^{P+})(u_1,u_2)$$
$$=\sum_{(u_1u_2)(v_1v_2)\in E_2} r\max(F_{\mu_1}^{P+}\times F_{\mu_2}^{P+})((u_1,u_2)(v_1,v_2))=\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(F_{\lambda_1}^{P+}(u),F_{\lambda_2}^{P+}(u_2,v_2))$$
$$+\sum_{u_1=v_2=w,u_1v_1\in E} r\max(F_{\lambda_2}^{P+}(w),I_{\mu_1}^{P+}(u_1,v_1))+\sum_{u_1=v_1=u,u_2v_2\in E_2} r\max(F_{\mu_1}^{P+}(u_1,v_1),I_{\mu_2}^{P+}(u_2,v_2))$$

$$\deg(F^{P-}_{\lambda_1} \times F^{P-}_{\lambda_2})(u_1, u_2)$$
$$= \sum_{(u_1u_2)(v_1v_2)\in E_2} r\min(F^{P-}_{\mu_1} \times F^{P-}_{\mu_2})((u_1, u_2)(v_1, v_2)) = \sum_{u_1=v_1=u, u_2 v_2 \in E_2} r\min(F^{P-}_{\lambda_1}(u), F^{P-}_{\lambda_2}(u_2, v_2))$$
$$+ \sum_{u_1=v_2=w, u_1v_1\in E} r\min(F^{P-}_{\lambda_2}(w), F^{P-}_{\mu_1}(u_1, v_1)) + \sum_{u_1=v_1=u, u_2 v_2 \in E_2} r\min(F^{P-}_{\mu_1}(u_1, v_1), F^{P-}_{\mu_2}(u_2, v_2))$$

Definition 2.11

Let $G_1 = (P_1, Q_1)$ be a bipolar spherical fuzzy neutrosophic cubic graph of $G_1^* = (V_1^*, E_1^*)$ and $G_2 = (P_2, Q_2)$ be a bipolar spherical fuzzy neutrosophic cubic graph of $G_2^* = (V_2^*, E_2^*)$. Then the composition of G_1 and G_1 is denoted by $G_1[G_2]$ and defined as follows:

$$G_1[G_2] = \begin{array}{l} (P_1, Q_1)[P_2, Q_2] \\ \{(P_1[P_2], Q_1[Q_2])\} \\ \{(A_1, \lambda_1)[A_2, \lambda_2], (B_1, \mu_1)[B_2, \mu_2]\} \\ \{(A_1[A_2], \lambda_1[\lambda_2]), (B_1[B_2], \mu_1[\mu_2])\} \end{array}$$

$$= \left\{ \begin{array}{l} \left\langle \begin{array}{l} ((T^{P+}_{A_1} \circ T^{P+}_{A_2}), (T^{P+}_{\lambda_1} \circ T^{P+}_{\lambda_2})), ((I^{P+}_{A_1} \circ I^{P+}_{A_2}), (I^{P+}_{\lambda_1} \circ I^{P+}_{\lambda_2})), ((F^{P+}_{A_1} \circ F^{P+}_{A_2}), (F^{P+}_{\lambda_1} \circ F^{P+}_{\lambda_2})), \\ ((T^{P-}_{A_1} \circ T^{P-}_{A_2}), (T^{P-}_{\lambda_1} \circ T^{P-}_{\lambda_2})), ((I^{P-}_{A_1} \circ I^{P-}_{A_2}), (I^{P-}_{\lambda_1} \circ I^{P-}_{\lambda_2})), ((F^{P-}_{A_1} \circ F^{P-}_{A_2}), (F^{P-}_{\lambda_1} \circ F^{P-}_{\lambda_2})) \end{array} \right\rangle \\ \left\langle \begin{array}{l} ((T^{P+}_{B_1} \circ T^{P+}_{B_2}), (T^{P+}_{\mu_1} \circ T^{P+}_{\mu_2})), ((I^{P+}_{B_1} \circ I^{P+}_{B_2}), (I^{P+}_{\mu_1} \circ I^{P+}_{\mu_2})), ((F^{P+}_{B_1} \circ F^{P+}_{B_2}), (F^{P+}_{\mu_1} \circ F^{P+}_{\mu_2})), \\ ((T^{P-}_{B_1} \circ T^{P-}_{B_2}), (T^{P-}_{\mu_1} \circ T^{P-}_{\mu_2})), ((I^{P-}_{B_1} \circ I^{P-}_{B_2}), (I^{P-}_{\mu_1} \circ I^{P-}_{\mu_2})), ((F^{P-}_{B_1} \circ F^{P-}_{B_2}), (F^{P-}_{\mu_1} \circ F^{P-}_{\mu_2})) \end{array} \right\rangle \end{array} \right.$$

1. $\forall (u, v) \in (v_1, v_2) = V$

$$(T^{P+}_{A_1} \circ T^{P+}_{A_2})(u, v) = r\min(T^{P+}_{A_1}(u), T^{P+}_{A_2}(v)), (T^{P+}_{\lambda_1} \circ T^{P+}_{\lambda_2})(u, v) = \max(T^{P+}_{\lambda_1}(u), T^{P+}_{\lambda_2}(v))$$
$$(T^{P-}_{A_1} \circ T^{P-}_{A_2})(u, v) = r\max(T^{P-}_{A_1}(u), T^{P-}_{A_2}(v)), (T^{P-}_{\lambda_1} \circ T^{P-}_{\lambda_2})(u, v) = \min(T^{P-}_{\lambda_1}(u), T^{P-}_{\lambda_2}(v))$$

$$(I^{P+}_{A_1} \circ I^{P+}_{A_2})(u, v) = r\min(I^{P+}_{A_1}(u), I^{P+}_{A_2}(v)), (I^{P+}_{\lambda_1} \circ I^{P+}_{\lambda_2})(u, v) = \max(I^{P+}_{\lambda_1}(u), I^{P+}_{\lambda_2}(v))$$
$$(I^{P-}_{A_1} \circ I^{P-}_{A_2})(u, v) = r\max(I^{P-}_{A_1}(u), I^{P-}_{A_2}(v)), (I^{P-}_{\lambda_1} \circ I^{P-}_{\lambda_2})(u, v) = \min(I^{P-}_{\lambda_1}(u), I^{P-}_{\lambda_2}(v))$$

$$(F^{P+}_{A_1} \circ F^{P+}_{A_2})(u, v) = r\max(F^{P+}_{A_1}(u), F^{P+}_{A_2}(v)), (F^{P+}_{\lambda_1} \circ F^{P+}_{\lambda_2})(u, v) = \min(F^{P+}_{\lambda_1}(u), F^{P+}_{\lambda_2}(v))$$
$$(F^{P-}_{A_1} \circ F^{P-}_{A_2})(u, v) = r\min(F^{P-}_{A_1}(u), F^{P-}_{A_2}(v)), (F^{P-}_{\lambda_1} \circ F^{P-}_{\lambda_2})(u, v) = \max(F^{P-}_{\lambda_1}(u), F^{P-}_{\lambda_2}(v))$$

2. $\forall u \in V_1$ and $v_1 v_2 \in E$

$$(T^{P+}_{B_1} \circ T^{P+}_{B_2})((u, v_1)(u, v_2)) = r\min(T^{P+}_{A_1}(u), T^{P+}_{B_2}(v_1 v_2)),$$
$$(T^{P+}_{\mu_1} \circ T^{P+}_{\mu_2})((u, v_1)(u, v_2)) = \max(T^{P+}_{\lambda_1}(u), T^{P+}_{\mu_2}(v_1 v_2))$$

$$(T_{B_1}^{P-} \circ T_{B_2}^{P-})((u, v_1)(u, v_2)) = r\max(T_{A_1}^{P-}(u), T_{B_2}^{P-}(v_1 v_2)),$$
$$(T_{\mu_1}^{P-} \circ T_{\mu_2}^{P-})((u, v_1)(u, v_2)) = \min(T_{\lambda_1}^{P-}(u), T_{\mu_2}^{P-}(v_1 v_2))$$

$$(I_{B_1}^{P+} \circ I_{B_2}^{P+})((u, v_1)(u, v_2)) = r\min(I_{A_1}^{P+}(u), I_{B_2}^{P+}(v_1 v_2)),$$
$$(I_{\mu_1}^{P+} \circ I_{\mu_2}^{P+})((u, v_1)(u, v_2)) = \max(I_{\lambda_1}^{P+}(u), I_{\mu_2}^{P+}(v_1 v_2))$$

$$(I_{B_1}^{P-} \circ I_{B_2}^{P-})((u, v_1)(u, v_2)) = r\max(I_{A_1}^{P-}(u), I_{B_2}^{P-}(v_1 v_2)),$$
$$(I_{\mu_1}^{P-} \circ I_{\mu_2}^{P-})((u, v_1)(u, v_2)) = \min(I_{\lambda_1}^{P-}(u), I_{\mu_2}^{P-}(v_1 v_2))$$

$$(F_{B_1}^{P+} \circ F_{B_2}^{P+})((u, v_1)(u, v_2)) = r\max(F_{A_1}^{P+}(u), F_{B_2}^{P+}(v_1 v_2)),$$
$$(F_{\mu_1}^{P+} \circ F_{\mu_2}^{P+})((u, v_1)(u, v_2)) = \min(F_{\lambda_1}^{P+}(u), F_{\mu_2}^{P+}(v_1 v_2))$$

$$(F_{B_1}^{P-} \circ F_{B_2}^{P-})((u, v_1)(u, v_2)) = r\min(F_{A_1}^{P-}(u), F_{B_2}^{P-}(v_1 v_2)),$$
$$(F_{\mu_1}^{P-} \circ F_{\mu_2}^{P-})((u, v_1)(u, v_2)) = \max(F_{\lambda_1}^{P-}(u), F_{\mu_2}^{P-}(v_1 v_2))$$

3. $\forall v \in V_2$ and $u_1 u_2 \in E_1$

$$(T_{B_1}^{P+} \circ T_{B_2}^{P+})((u_1, v)(u_2, v)) = r\min(T_{B_1}^{P+}(u_1 u_2), T_{A_2}^{P+}(v)),$$
$$(T_{\mu_1}^{P+} \circ T_{\mu_2}^{P+})((u_1, v)(u_2, v)) = \max(T_{\mu_1}^{P+}(u_1 u_2), T_{\lambda_2}^{P+}(v))$$

$$(T_{B_1}^{P-} \circ T_{B_2}^{P-})((u_1, v)(u_2, v)) = r\max(T_{B_1}^{P-}(u_1 u_2), T_{A_2}^{P-}(v)),$$
$$(T_{\mu_1}^{P-} \circ T_{\mu_2}^{P-})((u_1, v)(u_2, v)) = \min(T_{\mu_1}^{P-}(u_1 u_2), T_{\lambda_2}^{P-}(v))$$

$$(I_{B_1}^{P+} \circ I_{B_2}^{P+})((u_1, v)(u_2, v)) = r\min(I_{B_1}^{P+}(u_1 u_2), I_{A_2}^{P+}(v)),$$
$$(I_{\mu_1}^{P+} \circ I_{\mu_2}^{P+})((u_1, v)(u_2, v)) = \max(I_{\mu_1}^{P+}(u_1 u_2), I_{\lambda_2}^{P+}(v))$$

$$(I_{B_1}^{P-} \circ I_{B_2}^{P-})((u_1, v)(u_2, v)) = r\max(I_{B_1}^{P-}(u_1 u_2), I_{A_2}^{P-}(v)),$$
$$(I_{\mu_1}^{P-} \circ I_{\mu_2}^{P-})((u_1, v)(u_2, v)) = \min(I_{\mu_1}^{P-}(u_1 u_2), I_{\lambda_2}^{P-}(v))$$

$$(F_{B_1}^{P+} \circ F_{B_2}^{P+})((u_1, v)(u_2, v)) = r\max(F_{B_1}^{P+}(u_1 u_2), F_{A_2}^{P+}(v)),$$
$$(F_{\mu_1}^{P+} \circ F_{\mu_2}^{P+})((u_1, v)(u_2, v)) = \min(F_{\mu_1}^{P+}(u_1 u_2), F_{\lambda_2}^{P+}(v))$$

$$(F_{B_1}^{P-} \circ F_{B_2}^{P-})((u_1, v)(u_2, v)) = r\min(F_{B_1}^{P-}(u_1 u_2), F_{A_2}^{P-}(v)),$$
$$(F_{\mu_1}^{P-} \circ F_{\mu_2}^{P-})((u_1, v)(u_2, v)) = \max(F_{\mu_1}^{P-}(u_1 u_2), F_{\lambda_2}^{P-}(v))$$

4. $\forall(u_1,v_1)(u_2,v_2)\in E^{\circ}-E$

$$(T^{P+}_{B_1}\circ T^{P+}_{B_2})((u_1,v_1)(u_2,v_2)) = r\min(T^{P+}_{A_2}(v_1), T^{P+}_{A_2}(v_2), T^{P+}_{B_1}(u_1,u_2)),$$
$$(T^{P+}_{\mu_1}\circ T^{P+}_{\mu_2})((u_1,v_1)(u_2,v_2)) = \max(T^{P+}_{\lambda_2}(v_1), T^{P+}_{\lambda_2}(v_2), T^{P+}_{\mu_1}(u_1,u_2))$$

$$(T^{P-}_{B_1}\circ T^{P-}_{B_2})((u_1,v_1)(u_2,v_2)) = r\max(T^{P-}_{A_2}(v_1), T^{P-}_{A_2}(v_2), T^{P-}_{B_1}(u_1,u_2)),$$
$$(T^{P-}_{\mu_1}\circ T^{P-}_{\mu_2})((u_1,v_1)(u_2,v_2)) = \min(T^{P-}_{\lambda_2}(v_1), T^{P-}_{\lambda_2}(v_2), T^{P-}_{\mu_1}(u_1,u_2))$$

$$(I^{P+}_{B_1}\circ I^{P+}_{B_2})((u_1,v_1)(u_2,v_2)) = r\min(I^{P+}_{A_2}(v_1), I^{P+}_{A_2}(v_2), I^{P+}_{B_1}(u_1,u_2)),$$
$$(I^{P+}_{\mu_1}\circ I^{P+}_{\mu_2})((u_1,v_1)(u_2,v_2)) = \max(I^{P+}_{\lambda_2}(v_1), I^{P+}_{\lambda_2}(v_2), I^{P+}_{\mu_1}(u_1,u_2))$$

$$(I^{P-}_{B_1}\circ I^{P-}_{B_2})((u_1,v_1)(u_2,v_2)) = r\max(I^{P-}_{A_2}(v_1), I^{P-}_{A_2}(v_2), I^{P-}_{B_1}(u_1,u_2)),$$
$$(I^{P-}_{\mu_1}\circ I^{P-}_{\mu_2})((u_1,v_1)(u_2,v_2)) = \min(I^{P-}_{\lambda_2}(v_1), I^{P-}_{\lambda_2}(v_2), I^{P-}_{\mu_1}(u_1,u_2))$$

$$(F^{P+}_{B_1}\circ F^{P+}_{B_2})((u_1,v_1)(u_2,v_2)) = r\max(F^{P+}_{A_2}(v_1), F^{P+}_{A_2}(v_2), F^{P+}_{B_1}(u_1,u_2)),$$
$$(F^{P+}_{\mu_1}\circ F^{P+}_{\mu_2})((u_1,v_1)(u_2,v_2)) = \min(F^{P+}_{\lambda_2}(v_1), F^{P+}_{\lambda_2}(v_2), F^{P+}_{\mu_1}(u_1,u_2))$$

$$(F^{P-}_{B_1}\circ F^{P-}_{B_2})((u_1,v_1)(u_2,v_2)) = r\min(F^{P-}_{A_2}(v_1), F^{P-}_{A_2}(v_2), F^{P-}_{B_1}(u_1,u_2)),$$
$$(F^{P-}_{\mu_1}\circ F^{P-}_{\mu_2})((u_1,v_1)(u_2,v_2)) = \max(F^{P-}_{\lambda_2}(v_1), F^{P-}_{\lambda_2}(v_2), F^{P-}_{\mu_1}(u_1,u_2))$$

Example 2.12

Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two bipolar spherical fuzzy neutrosophic cubic graphs, where $V_1 = (a,b)$ and $V_2 = (c,d)$. Suppose P_1 and P_2 be the bipolar spherical fuzzy neutrosophic cubic set representation of V_1 and V_2. Also Q_1 and Q_2 be the bipolar spherical fuzzy neutrosophic cubic set representation of E_1 and E_2 defined as follows:

$$P_1 = \left\langle \begin{Bmatrix} a, ([0.4,0.7],0.3), ([0.3,0.5],0.6), ([0.8,0.4],0.7), \\ ([-0.4,-0.3],-0.1), ([-0.6,-0.5],-0.2), ([-0.8,-0.6],-0.2) \end{Bmatrix} \begin{Bmatrix} b, ([0.8,0.5],0.1), ([0.9,0.4],0.4), ([0.6,0.8],0.9), \\ ([-0.8,-0.3],-0.2), ([-0.9,-0.5],-0.5), ([-0.4,-0.5],-0.2) \end{Bmatrix} \right\rangle$$

$$Q_1 = \left\langle \begin{Bmatrix} ab, ([0.4,0.5],0.3), ([0.3,0.4],0.6), ([0.8,0.8],0.7), \\ ([-0.4,-0.3],-0.2), ([-0.6,-0.5],-0.5), ([-0.8,-0.6],-0.2) \end{Bmatrix} \right\rangle$$

$$P_2 = \left\langle \begin{array}{l} \left\{ \begin{array}{l} c, ([0.3, 0.6], 0.9), ([0.4, 0.7], 0.5), ([1.0, 0.2], 0.3), \\ ([-0.5, -0.6], -0.3), ([-0.4, -0.5], -0.2), ([-0.8, -0.6], -0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} d, ([0.5, 0.1], 0.2), (0.8, 0.3], 0.4), ([0.9, 0.4], 0.1), \\ ([-0.8, -0.7], -0.3), ([-0.7, -0.4], -0.2), ([-0.6, -0.5], -0.4) \end{array} \right\} \end{array} \right\rangle$$

$$Q_2 = \left\langle \left\{ \begin{array}{l} cd, ([0.3, 0.1], 0.9), ([0.4, 0.3], 0.5), ([0.9, 0.4], 0.1), \\ ([-0.5, -0.6], -0.3), ([-0.4, -0.4], -0.2), ([-0.8, -0.6], -0.1) \end{array} \right\} \right\rangle$$

For the two bipolar spherical fuzzy neutrosophic cubic graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ the vertex sets $V_1 = (a, b)$ and $V_2 = (c, d)$ and the edge sets E_1 and E_2 represented.

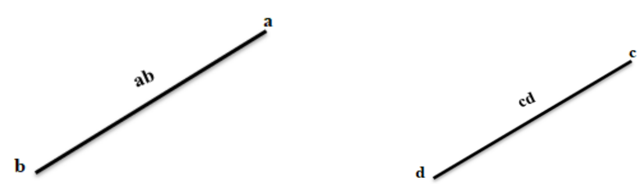

Figure 2.4

The composition of two bipolar spherical fuzzy neutrosophic cubic graphs G_1 and G_2 is again a bipolar spherical fuzzy neutrosophic cubic graphs, where

$$P_1[P_2] = \left\langle \begin{array}{l} \left\{ \begin{array}{l} (a,c),([0.3,0.6],0.9),([0.3,0.5],0.6),([1.0,0.4],0.3), \\ ([-0.4,-0.3],-0.3),([-0.4,-0.5],-0.2),([-0.8,-0.6],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} (a,d),([0.4,0.1],0.3),([0.3,0.3],0.6),([0.9,0.4],0.1), \\ ([-0.4,-0.3],-0.3),([-0.6,-0.4],-0.2),([-0.8,-0.6]-0.2) \end{array} \right\} \\ \left\{ \begin{array}{l} (b,c),([0.3,0.5],0.9),([0.4,0.4],0.5),([1.0,0.8],0.3), \\ ([-0.5,-0.3],-0.3),([-0.4,-0.5],-0.5),([-0.8,-0.6],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} (b,d)([0.5,0.1]0.2),([0.8,0.3],0.4),([0.9,0.8],0.1), \\ ([-0.8,-0.3],-0.3),([-0.7,-0.4],-0.5),([-0.6,-0.5],-0.2) \end{array} \right\} \end{array} \right\rangle$$

$$Q_1[Q_2] = \left\langle \begin{array}{l} \left\{ \begin{array}{l} ((a,c)(a,d)),([0.3,0.1],0.9),([0.3,0.3],0.6),([1.0,0.4],0.1), \\ ([-0.4,-0.3],-0.3),([-0.4,-0.4],-0.2),([-0.8,-0.6],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} ((a,d)(b,d)),([0.4,0.1],0.3),([0.3,0.3],0.6),([0.9,0.8],0.1), \\ ([-0.4,-0.3],-0.3),([-0.6,-0.4],-0.5),([-0.8,-0.6],-0.2) \end{array} \right\} \\ \left\{ \begin{array}{l} ((b,d)(b,c)),([0.3,0.1],0.9),([0.4,0.3],0.5),([1.0,0.8],0.1), \\ ([-0.5,-0.3],-0.3),([-0.4,-0.4],-0.5),([-0.8,-0.6],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} ((b,c)(a,c)),([0.3,0.5],0.9),([0.3,0.4],0.6),([1.0,0.8],0.3), \\ ([-0.4,-0.3],-0.3),([-0.4,-0.5],-0.5),([-0.8,-0.6],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} ((a,c)(b,d)),([0.3,0.1],0.9),([0.3,0.3],0.6),([1.0,0.8],0.1), \\ ([-0.4,-0.3],-0.3),([-0.4,-0.4],-0.5),([-0.8,-0.6],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} ((a,d)(b,c)),([0.3,0.1],0.9),([0.3,0.3],0.6),([1.0,0.8],0.1), \\ ([-0.4,-0.3],-0.3),([-0.4,-0.4],-0.5),([-0.8,-0.6],-0.1) \end{array} \right\} \end{array} \right\rangle$$

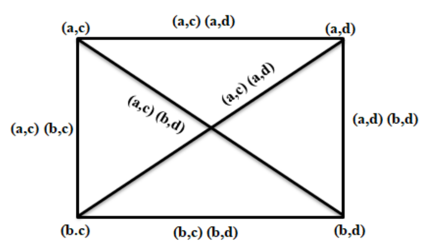

Figure 2.5

The composition of two bipolar spherical fuzzy neutrosophic cubic graphs G_1 and G_2

Definition 2.13

Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two bipolar spherical fuzzy neutrosophic cubic graphs of the graph G_1^* and G_2^* respectively. Then M- union is denoted by $G_1 \cup_M G_2$ and is defined by as

$$G_1 \cup_M G_2 = \{(P_1, Q_1) \cup_M (P_2, Q_2)\} = \{P_1 \cup_M P_2, Q_1 \cup_M Q_2\}$$

$$= \left\{ \begin{array}{l} \left\langle \begin{array}{l} ((T_{A_1}^{P+} \cup_M T_{A_2}^{P+}), (T_{\lambda_1}^{P+} \cup_M T_{\lambda_1}^{P+})), ((I_{A_1}^{P+} \cup_M I_{A_2}^{P+}), (I_{\lambda_1}^{P+} \cup_M I_{\lambda_2}^{P+})), ((F_{A_1}^{P+} \cup_M F_{A_2}^{P+}), (F_{\lambda_1}^{P+} \cup_M F_{\lambda_2}^{P+})) \\ ((T_{A_1}^{P-} \cup_M T_{A_2}^{P-}), (T_{\lambda_1}^{P-} \cup_M T_{\lambda_1}^{P-})), ((I_{A_1}^{P-} \cup_M I_{A_2}^{P-}), (I_{\lambda_1}^{P-} \cup_M I_{\lambda_2}^{P-})), ((F_{A_1}^{P-} \cup_M F_{A_2}^{P-}), (F_{\lambda_1}^{P-} \cup_M F_{\lambda_2}^{P-})) \end{array} \right\rangle \\ \left\langle \begin{array}{l} ((T_{B_1}^{P+} \cup_M T_{B_2}^{P+}), (T_{\mu_1}^{P+} \cup_M T_{\mu_2}^{P+})), ((I_{B_1}^{P+} \cup_M I_{B_2}^{P+}), (I_{\mu_1}^{P+} \cup_M I_{\mu_2}^{P+})), ((F_{B_1}^{P+} \cup_M F_{B_2}^{P+}), (F_{\mu_1}^{P+} \cup_M F_{\mu_2}^{P+})) \\ ((T_{B_1}^{P-} \cup_M T_{B_2}^{P-}), (T_{\mu_1}^{P-} \cup_M T_{\mu_2}^{P-})), ((I_{B_1}^{P-} \cup_M I_{B_2}^{P-}), (I_{\mu_1}^{P-} \cup_M I_{\mu_2}^{P-})), ((F_{B_1}^{P-} \cup_M F_{B_2}^{P-}), (F_{\mu_1}^{P-} \cup_M F_{\mu_2}^{P-})) \end{array} \right\rangle \end{array} \right\}$$

where

$$\left(T_{A_1}^{P+} \cup_M T_{A_2}^{P+}\right)(u) = \begin{cases} T_{A_1}^{P+}(u), & \text{, if } u \in v_1 - v_2 \\ T_{A_2}^{P+}(u), & \text{, if } u \in v_2 - v_1 \\ r\max\{T_{A_1}^{P+}(u), T_{A_2}^{P+}(u)\} & \text{, if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(T_{A_1}^{P-} \cup_M T_{A_2}^{P-}\right)(u) = \begin{cases} T_{A_1}^{P-}(u), & \text{, if } u \in v_1 - v_2 \\ T_{A_2}^{P-}(u), & \text{, if } u \in v_2 - v_1 \\ r\min\{T_{A_1}^{P-}(u), T_{A_2}^{P-}(u)\} & \text{, if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(T_{\lambda_1}^{P+} \cup_M T_{\lambda_2}^{P+}\right)(u) = \begin{cases} T_{\lambda_1}^{P+}(u), & \text{, if } u \in v_1 - v_2 \\ T_{\lambda_2}^{P+}(u), & \text{, if } u \in v_2 - v_1 \\ r\max\{T_{\lambda_1}^{P+}(u), T_{\lambda_2}^{P+}(u)\} & \text{, if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(T_{\lambda_1}^{P-} \cup_M T_{\lambda_2}^{P-}\right)(u) = \begin{cases} T_{\lambda_1}^{P-}(u), & \text{, if } u \in v_1 - v_2 \\ T_{\lambda_2}^{P-}(u), & \text{, if } u \in v_2 - v_1 \\ r\min\{T_{\lambda_1}^{P-}(u), T_{\lambda_2}^{P-}(u)\} & \text{, if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(I_{A_1}^{P+} \cup_M I_{A_2}^{P+}\right)(u) = \begin{cases} I_{A_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ I_{A_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ r\max\left\{I_{A_1}^{P+}(u), I_{A_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(I_{A_1}^{P-} \cup_M I_{A_2}^{P-}\right)(u) = \begin{cases} I_{A_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ I_{A_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ r\min\left\{I_{A_1}^{P-}(u), I_{A_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(I_{\lambda_1}^{P+} \cup_M I_{\lambda_2}^{P+}\right)(u) = \begin{cases} I_{\lambda_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ I_{\lambda_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ r\max\left\{I_{\lambda_1}^{P+}(u), I_{\lambda_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(I_{\lambda_1}^{P-} \cup_M I_{\lambda_2}^{P-}\right)(u) = \begin{cases} I_{\lambda_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ I_{\lambda_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ r\min\left\{I_{\lambda_1}^{P-}(u), I_{\lambda_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(F_{A_1}^{P+} \cup_M F_{A_2}^{P+}\right)(u) = \begin{cases} F_{A_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ F_{A_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ r\max\left\{F_{A_1}^{P+}(u), F_{A_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(F_{A_1}^{P-} \cup_M F_{A_2}^{P-}\right)(u) = \begin{cases} F_{A_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ F_{A_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ r\max\left\{F_{A_1}^{P-}(u), F_{A_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(F_{\lambda_1}^{P+} \cup_M F_{\lambda_2}^{P+}\right)(u) = \begin{cases} F_{\lambda_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ F_{\lambda_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ r\min\left\{F_{\lambda_1}^{P+}(u), F_{\lambda_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$(T_{B_1}^{P+} \cup_M T_{B_2}^{P+})(u_2v_2) = \begin{cases} T_{B_1}^{P+}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ T_{B_2}^{P+}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ r\max\left\{T_{B_1}^{P+}(u_2v_2)\, T_{B_2}^{P+}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$(T_{B_1}^{P-} \cup_M T_{B_2}^{P-})(u_2v_2) = \begin{cases} T_{B_1}^{P-}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ T_{B_2}^{P-}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ r\min\{T_{B_1}^{P+}(u_2v_2)\, T_{B_2}^{P+}(u_2v_2)\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$(T_{\mu_1}^{P+} \cup_M T_{\mu_2}^{P+})(u_2v_2) = \begin{cases} T_{\mu_1}^{P+}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ T_{\mu_2}^{P+}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ \max\{T_{\mu_1}^{P+}(u_2v_2)\, T_{\mu_2}^{P+}(u_2v_2)\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$(T_{\mu_1}^{P-} \cup_M T_{\mu_2}^{P-})(u_2v_2) = \begin{cases} T_{\mu_1}^{P-}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ T_{\mu_2}^{P-}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ \min\{T_{\mu_1}^{P-}(u_2v_2)\, T_{\mu_2}^{P-}(u_2v_2)\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$(I_{B_1}^{P+} \cup_M I_{B_2}^{P+})(u_2v_2) = \begin{cases} I_{B_1}^{P+}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ I_{B_2}^{P+}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ r\max\{I_{B_1}^{P+}(u_2v_2)\, I_{B_2}^{P+}(u_2v_2)\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$(I_{B_1}^{P-} \cup_M I_{B_2}^{P-})(u_2v_2) = \begin{cases} I_{B_1}^{P-}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ I_{B_2}^{P-}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ r\min\{I_{B_1}^{P+}(u_2v_2)\, I_{B_2}^{P+}(u_2v_2)\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$(I_{\mu_1}^{P+} \cup_M I_{\mu_2}^{P+})(u_2v_2) = \begin{cases} I_{\mu_1}^{P+}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ I_{\mu_2}^{P+}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ \max\{I_{\mu_1}^{P+}(u_2v_2)\, I_{\mu_2}^{P+}(u_2v_2)\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$(I_{\mu_1}^{P-} \cup_M I_{\mu_2}^{P-})(u_2v_2) = \begin{cases} I_{\mu_1}^{P-}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ I_{\mu_2}^{P-}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ \min\{I_{\mu_1}^{P-}(u_2v_2)\, I_{\mu_2}^{P-}(u_2v_2)\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$(F^{P+}_{B_1} \cup_M F^{P+}_{B_2})(u_2v_2) = \left\{ \begin{array}{ll} F^{P+}_{B_1}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ F^{P+}_{B_2}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ r\min\left\{F^{P+}_{B_1}(u_2v_2)\, F^{P+}_{B_2}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{array} \right\}$$

$$(F^{P-}_{B_1} \cup_M F^{P-}_{B_2})(u_2v_2) = \left\{ \begin{array}{ll} F^{P-}_{B_1}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ F^{P-}_{B_2}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ r\max\left\{F^{P+}_{B_1}(u_2v_2)\, F^{P+}_{B_2}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{array} \right\}$$

$$(F^{P+}_{\mu_1} \cup_M F^{P+}_{\mu_2})(u_2v_2) = \left\{ \begin{array}{ll} F^{P+}_{\mu_1}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ F^{P+}_{\mu_2}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ \min\left\{F^{P+}_{\mu_1}(u_2v_2)\, F^{P+}_{\mu_2}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{array} \right\}$$

$$(F^{P-}_{\mu_1} \cup_M F^{P-}_{\mu_2})(u_2v_2) = \left\{ \begin{array}{ll} F^{P-}_{\mu_1}(u_2v_2) & ,\text{if } u_2v_2 \in v_1 - v_2 \\ F^{P-}_{\mu_2}(u_2v_2) & ,\text{if } u_2v_2 \in v_2 - v_1 \\ \max\left\{F^{P-}_{\mu_1}(u_2v_2)\, F^{P-}_{\mu_2}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{array} \right\}$$

And the N-union is denoted by $G_1 \cup_N G_2$ and is defined as follows

$$G_1 \cup_N G_2 = \{(P_1, Q_1) \cup_N (P_2, Q_2)\} = \{P_1 \cup_N P_2, Q_1 \cup_N Q_2\}$$

$$= \left\{ \begin{array}{c} \left\langle \begin{array}{c} ((T^{P+}_{A_1} \cup_N T^{P+}_{A_2}), (T^{P+}_{\lambda_1} \cup_N T^{P+}_{\lambda_1})), ((I^{P+}_{A_1} \cup_N I^{P+}_{A_2}), (I^{P+}_{\lambda_1} \cup_N I^{P+}_{\lambda_2})), ((F^{P+}_{A_1} \cup_N F^{P+}_{A_2}), (F^{P+}_{\lambda_1} \cup_N F^{P+}_{\lambda_2})) \\ ((T^{P-}_{A_1} \cup_N T^{P-}_{A_2}), (T^{P-}_{\lambda_1} \cup_N T^{P-}_{\lambda_1})), ((I^{P-}_{A_1} \cup_N I^{P-}_{A_2}), (I^{P-}_{\lambda_1} \cup_N I^{P-}_{\lambda_2})), ((F^{P-}_{A_1} \cup_N F^{P-}_{A_2}), (F^{P-}_{\lambda_1} \cup_N F^{P-}_{\lambda_2})) \end{array} \right\rangle \\ \left\langle \begin{array}{c} ((T^{P+}_{B_1} \cup_N T^{P+}_{B_2}), (T^{P+}_{\mu_1} \cup_N T^{P+}_{\mu_2})), ((I^{P+}_{B_1} \cup_N I^{P+}_{B_2}), (I^{P+}_{\mu_1} \cup_N I^{P+}_{\mu_2})), ((F^{P+}_{B_1} \cup_N F^{P+}_{B_2}), (F^{P+}_{\mu_1} \cup_N F^{P+}_{\mu_2})) \\ ((T^{P-}_{B_1} \cup_N T^{P-}_{B_2}), (T^{P-}_{\mu_1} \cup_N T^{P-}_{\mu_2})), ((I^{P-}_{B_1} \cup_N I^{P-}_{B_2}), (I^{P-}_{\mu_1} \cup_N I^{P-}_{\mu_2})), ((F^{P-}_{B_1} \cup_N F^{P-}_{B_2}), (F^{P-}_{\mu_1} \cup_N F^{P-}_{\mu_2})) \end{array} \right\rangle \end{array} \right\}$$

Where

$$\left(T^{P+}_{A_1} \cup_N T^{P+}_{A_2}\right)(u) = \left\{ \begin{array}{ll} T^{P+}_{A_1}(u), & ,\text{if } u \in v_1 - v_2 \\ T^{P+}_{A_2}(u), & ,\text{if } u \in v_2 - v_1 \\ r\max\left\{T^{P+}_{A_1}(u), T^{P+}_{A_2}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{array} \right\}$$

$$\left(T_{A_1}^{P-} \cup_N T_{A_2}^{P-}\right)(u) = \begin{cases} T_{A_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ T_{A_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ r\min\left\{T_{A_1}^{P-}(u), T_{A_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(T_{\lambda_1}^{P+} \cup_N T_{\lambda_2}^{P+}\right)(u) = \begin{cases} T_{\lambda_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ T_{\lambda_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ \max\left\{T_{\lambda_1}^{P+}(u), T_{\lambda_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(T_{\lambda_1}^{P-} \cup_N T_{\lambda_2}^{P-}\right)(u) = \begin{cases} T_{\lambda_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ T_{\lambda_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ \min\left\{T_{\lambda_1}^{P-}(u), T_{\lambda_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(I_{A_1}^{P+} \cup_N I_{A_2}^{P+}\right)(u) = \begin{cases} I_{A_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ I_{A_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ r\max\left\{I_{A_1}^{P+}(u), I_{A_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(I_{A_1}^{P-} \cup_N I_{A_2}^{P-}\right)(u) = \begin{cases} I_{A_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ I_{A_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ r\min\left\{I_{A_1}^{P-}(u), I_{A_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(I_{\lambda_1}^{P+} \cup_N I_{\lambda_2}^{P+}\right)(u) = \begin{cases} I_{\lambda_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ I_{\lambda_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ \max\left\{I_{\lambda_1}^{P+}(u), I_{\lambda_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(I_{\lambda_1}^{P-} \cup_N I_{\lambda_2}^{P-}\right)(u) = \begin{cases} I_{\lambda_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ I_{\lambda_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ \min\left\{I_{\lambda_1}^{P-}(u), I_{\lambda_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(F_{A_1}^{P+} \cup_N F_{A_2}^{P+}\right)(u) = \begin{cases} F_{A_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ F_{A_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ r\min\left\{F_{A_1}^{P+}(u), F_{A_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(F_{A_1}^{P-} \cup_N F_{A_2}^{P-}\right)(u) = \begin{cases} F_{A_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ F_{A_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ r\max\left\{F_{A_1}^{P-}(u), F_{A_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(F_{\lambda_1}^{P+} \cup_N F_{\lambda_2}^{P+}\right)(u) = \begin{cases} F_{\lambda_1}^{P+}(u), & ,\text{if } u \in v_1 - v_2 \\ F_{\lambda_2}^{P+}(u), & ,\text{if } u \in v_2 - v_1 \\ \min\left\{F_{\lambda_1}^{P+}(u), F_{\lambda_2}^{P+}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(F_{\lambda_1}^{P-} \cup_N F_{\lambda_2}^{P-}\right)(u) = \begin{cases} F_{\lambda_1}^{P-}(u), & ,\text{if } u \in v_1 - v_2 \\ F_{\lambda_2}^{P-}(u), & ,\text{if } u \in v_2 - v_1 \\ \max\left\{F_{\lambda_1}^{P-}(u), F_{\lambda_2}^{P-}(u)\right\} & ,\text{if } u \in v_1 \cap v_2 \end{cases}$$

$$\left(T_{B_1}^{P+} \cup_N T_{B_2}^{P+}\right)(u_2v_2) = \begin{cases} T_{B_1}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ T_{B_2}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ r\max\left\{T_{B_1}^{P+}(u_2v_2), T_{B_2}^{P+}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(T_{B_1}^{P-} \cup_N T_{B_2}^{P-}\right)(u_2v_2) = \begin{cases} T_{B_1}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ T_{B_2}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ r\min\left\{T_{B_1}^{P-}(u_2v_2), T_{B_2}^{P-}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(T_{\mu_1}^{P+} \cup_N T_{\mu_2}^{P+}\right)(u_2v_2) = \begin{cases} T_{\mu_1}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ T_{\mu_2}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ \max\left\{T_{\mu_1}^{P+}(u_2v_2), T_{\mu_2}^{P+}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(T_{\mu_1}^{P-} \cup_N T_{\mu_2}^{P-}\right)(u_2v_2) = \begin{cases} T_{\mu_1}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ T_{\mu_2}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ \max\left\{T_{\mu_1}^{P-}(u_2v_2), T_{\mu_2}^{P-}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(I_{B_1}^{P+} \cup_N I_{B_2}^{P+}\right)(u_2v_2) = \begin{cases} I_{B_1}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ I_{B_2}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ r\max\left\{I_{B_1}^{P+}(u_2v_2), I_{B_2}^{P+}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(I_{B_1}^{P-} \cup_N I_{B_2}^{P-}\right)(u_2v_2) = \begin{cases} I_{B_1}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ I_{B_2}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ r\min\left\{I_{B_1}^{P-}(u_2v_2), I_{B_2}^{P-}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(I_{\mu_1}^{P+} \cup_N I_{\mu_2}^{P+}\right)(u_2v_2) = \begin{cases} I_{\mu_1}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ I_{\mu_2}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ \max\left\{I_{\mu_1}^{P+}(u_2v_2), I_{\mu_2}^{P+}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(I_{\mu_1}^{P-} \cup_N I_{\mu_2}^{P-}\right)(u_2v_2) = \begin{cases} I_{\mu_1}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ I_{\mu_2}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ \min\left\{I_{\mu_1}^{P-}(u_2v_2), I_{\mu_2}^{P-}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(F_{B_1}^{P+} \cup_N F_{B_2}^{P+}\right)(u_2v_2) = \begin{cases} F_{B_1}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ F_{B_2}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ r\min\left\{F_{B_1}^{P+}(u_2v_2), F_{B_2}^{P+}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(F_{B_1}^{P-} \cup_N F_{B_2}^{P-}\right)(u_2v_2) = \begin{cases} F_{B_1}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ F_{B_2}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ r\max\left\{F_{B_1}^{P-}(u_2v_2), F_{B_2}^{P-}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(F_{\mu_1}^{P+} \cup_N F_{\mu_2}^{P+}\right)(u_2v_2) = \begin{cases} F_{\mu_1}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ F_{\mu_2}^{P+}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ \min\left\{F_{\mu_1}^{P+}(u_2v_2), F_{\mu_2}^{P+}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

$$\left(F_{\mu_1}^{P-} \cup_N F_{\mu_2}^{P-}\right)(u_2v_2) = \begin{cases} F_{\mu_1}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ F_{\mu_2}^{P-}(u_2v_2), & ,\text{if } u_2v_2 \in v_1 - v_2 \\ \max\left\{F_{\mu_1}^{P-}(u_2v_2), F_{\mu_2}^{P-}(u_2v_2)\right\} & ,\text{if } u_2v_2 \in E_1 \cap E_2 \end{cases}$$

Example 2.14

Let us consider the two bipolar spherical fuzzy neutrosophic cubic graphs as $G_1 = (P_1, Q_1)$ and $G_1 = (P_2, Q_2)$

$$P_1 = \left\langle \begin{matrix} \begin{Bmatrix} a, ([0.3,0.4],0.7), ([0.7,0.5],0.1), ([0.9,0.1],0.3), \\ ([-0.6,-0.2],-0.3), ([-0.7,-0.8],-0.1), ([-0.4,-0.7],-0.5) \end{Bmatrix} \\ \begin{Bmatrix} b, ([0.2,0.6],0.8), ([0.4,0.6],0.3), ([0.7,0.8],0.4), \\ ([-0.5,-0.8],-0.6), ([-0.3,-0.5],-0.2), ([-0.4,-0.9],-0.3) \end{Bmatrix} \\ \begin{Bmatrix} c, ([0.7,0.5],0.1), ([0.8,0.3],0.7), ([0.6,0.7],0.4), \\ ([-0.4,-0.2],-0.9), ([-0.1,-0.2],-0.9), ([-0.8,-0.5],-0.1) \end{Bmatrix} \end{matrix} \right\rangle$$

$$Q_1 = \left\langle \begin{matrix} \begin{Bmatrix} ab, ([0.2,0.4],0.8), ([0.4,0.5],0.3), ([0.9,0.8],0.3), \\ ([-0.5,-0.2],-0.6), ([-0.3,-0.5],-0.2), ([-0.4,-0.9],-0.3) \end{Bmatrix} \\ \begin{Bmatrix} ac, ([0.3,0.4],0.7), ([0.7,0.3],0.7), ([0.9,0.7],0.3), \\ ([-0.4,-0.2],-0.9), ([-0.1,-0.2],-0.9), ([-0.8,-0.7],-0.1) \end{Bmatrix} \\ \begin{Bmatrix} bc, ([0.2,0.5],0.8), ([0.4,0.3],0.7), ([0.7,0.8],0.4), \\ ([-0.4,-0.2],-0.9), ([-0.1,-0.2],-0.9), ([-0.8,-0.9],-0.1) \end{Bmatrix} \end{matrix} \right\rangle$$

$$P_2 = \left\langle \begin{matrix} \begin{Bmatrix} a, ([0.5,0.7],0.3), ([0.2,0.7],0.8), ([0.2,0.4],0.3), \\ ([-0.9,-0.8],-0.1), ([-0.6,-0.3],-0.2), ([-0.3,-0.7],-0.8) \end{Bmatrix} \\ \begin{Bmatrix} b, ([0.5,0.4],0.3), ([0.3,0.5],0.6), ([0.7,0.5],0.3), \\ ([-0.9,-0.4],-0.2), ([-0.7,-0.4],-0.3), ([-0.1,-0.4],-0.6) \end{Bmatrix} \\ \begin{Bmatrix} c, ([0.2,0.4],0.8), ([0.2,0.4],0.3), ([0.3,0.5],0.3), \\ ([-0.2,-0.5],-0.4), ([-0.7,-0.4],-0.5), ([-0.5,-0.4],-0.3) \end{Bmatrix} \end{matrix} \right\rangle$$

$$Q_2 = \left\langle \begin{matrix} \begin{Bmatrix} ab, ([0.5,0.4],0.3), ([0.2,0.5],0.8), ([0.7,0.5],0.3), \\ ([-0.9,-0.4],-0.2), ([-0.6,-0.3],-0.3), ([-0.3,-0.7],-0.6) \end{Bmatrix} \\ \begin{Bmatrix} ac, ([0.2,0.4],0.8), ([0.2,0.4],0.8), ([0.3,0.5],0.3), \\ ([-0.2,-0.5],-0.4), ([-0.6,-0.3],-0.5), ([-0.5,-0.7],-0.3) \end{Bmatrix} \\ \begin{Bmatrix} bc, ([0.2,0.4],0.8), ([0.2,0.4],0.6), ([0.7,0.5],0.3), \\ ([-0.2,-0.4],-0.4), ([-0.7,-0.4],-0.5), ([-0.5,-0.4],-0.3) \end{Bmatrix} \end{matrix} \right\rangle$$

Here M-union of the bipolar spherical neutrosophic cubic graph $G_1 \cup_M G_2$ as follows:

$$P_1 \cup_M P_2 = \left\langle \begin{array}{l} \left\{ \begin{array}{l} a, ([0.5,0.7],0.7), ([0.7,0.7],0.8), ([0.9,0.4],0.3), \\ ([-0.9,-0.8],-0.3), ([-0.7,-0.8],-0.2), ([-0.4,-0.7],-0.8) \end{array} \right\} \\ \left\{ \begin{array}{l} b, ([0.5,0.6],0.8), ([0.4,0.6],0.6), ([0.7,0.8],0.4), \\ ([-0.9,-0.8],-0.6), ([-0.7,-0.5],-0.3), ([-0.4,-0.9],-0.6) \end{array} \right\} \\ \left\{ \begin{array}{l} c, ([0.7,0.5],0.8), ([0.8,0.4],0.7), ([0.6,0.7],0.4), \\ ([-0.4,-0.5],-0.9), ([-0.7,-0.4],-0.9), ([-0.8,-0.5],-0.3) \end{array} \right\} \end{array} \right\rangle$$

$$Q_1 \cup_M Q_2 = \left\langle \begin{array}{l} \left\{ \begin{array}{l} ab, ([0.5,0.4],0.8), ([0.4,0.5],0.8), ([0.9,0.8],0.3), \\ ([-0.9,-0.4],-0.6), ([-0.6,-0.5],-0.3), ([-0.4,-0.9],-0.6) \end{array} \right\} \\ \left\{ \begin{array}{l} ac, ([0.3,0.4],0.8), ([0.7,0.4],0.8), ([0.9,0.7],0.3), \\ ([-0.4,-0.5],-0.9), ([-0.6,-0.3],-0.9), ([-0.8,-0.7],-0.3) \end{array} \right\} \\ \left\{ \begin{array}{l} bc, ([0.2,0.5],0.8), ([0.4,0.4],0.7), ([0.7,0.8],0.4), \\ ([-0.4,-0.4],-0.9), ([-0.7,-0.4],-0.9), ([-0.8,-0.9],-0.3) \end{array} \right\} \end{array} \right\rangle$$

Here N-union of the bipolar spherical neutrosophic cubic graph $G_1 \cup_N G_2$ as follows:

$$P_1 \cup_N P_2 = \left\langle \begin{array}{l} \left\{ \begin{array}{l} a, ([0.5,0.7],0.3), ([0.7,0.7],0.1), ([0.9,0.4],0.3), \\ ([-0.9,-0.1],-0.3), ([-0.7,-0.8],-0.1), ([-0.4,-0.7],-0.5) \end{array} \right\} \\ \left\{ \begin{array}{l} b, ([0.5,0.6],0.3), ([0.4,0.6],0.3), ([0.7,0.8],0.3), \\ ([-0.9,-0.8],-0.2), ([-0.7,-0.5],-0.2), ([-0.4,-0.9],-0.3) \end{array} \right\} \\ \left\{ \begin{array}{l} c, ([0.7,0.5],0.1), ([0.8,0.4],0.3), ([0.6,0.7],0.3), \\ ([-0.4,-0.5],-0.4), ([-0.7,-0.4],-0.5), ([-0.8,-0.5],-0.1) \end{array} \right\} \end{array} \right\rangle$$

$$Q_1 \cup_N Q_2 = \left\langle \begin{array}{l} \left\{ \begin{array}{l} ab, ([0.5,0.4],0.3), ([0.4,0.5],0.3), ([0.9,0.8],0.3), \\ ([-0.9,-0.4],-0.2), ([-0.6,-0.5],-0.2), ([-0.4,-0.9],-0.3) \end{array} \right\} \\ \left\{ \begin{array}{l} ac, ([0.3,0.4],0.7), ([0.7,0.4],0.7), ([0.9,0.7],0.3), \\ ([-0.4,-0.5],-0.4), ([-0.6,-0.3],-0.5), ([-0.8,-0.7],-0.1) \end{array} \right\} \\ \left\{ \begin{array}{l} bc, ([0.2,0.5],0.8), ([0.4,0.4],0.6), ([0.7,0.8],0.3), \\ ([-0.4,-0.4],-0.4), ([-0.7,-0.4],-0.5), ([-0.8,-0.9],-0.1) \end{array} \right\} \end{array} \right\rangle$$

Proposition 2.15

The M-union of the two bipolar spherical fuzzy neutrosophic cubic graphs is again a bipolar spherical fuzzy neutrosophic cubic graph.

Definition 2.16

Let $G_1=(P_1,Q_1)$ and $G_2=(P_2,Q_2)$ be two bipolar spherical fuzzy neutrosophic cubic graphs of the graphs G_1^* and G_2^* respectively, the M-join is denoted by $G_1+_M G_2$ and is defined as follows:

$$G_1+_M G_2=(P_1,Q_1)+_M(P_2Q_2)=(P_1+_M P_2,Q_1+_M Q_2)$$

$$=\left\{\begin{array}{l}\left\langle\begin{array}{l}((T_{A_1}^{P+}+_M T_{A_2}^{P+}),(T_{\lambda_1}^{P+}+_M T_{\lambda_2}^{P+})),((I_{A_1}^{P+}+_M I_{A_2}^{P+}),(I_{\lambda_1}^{P+}+_M I_{\lambda_2}^{P+})),((F_{A_1}^{P+}+_M F_{A_2}^{P+}),(F_{\lambda_1}^{P+}+_M F_{\lambda_2}^{P+}))\\((T_{A_1}^{P-}+_M T_{A_2}^{P-}),(T_{\lambda_1}^{P-}+_M T_{\lambda_2}^{P-})),((I_{A_1}^{P-}+_M I_{A_2}^{P-}),(I_{\lambda_1}^{P-}+_M I_{\lambda_2}^{P-})),((F_{A_1}^{P-}+_M F_{A_2}^{P-}),(F_{\lambda_1}^{P-}+_M F_{\lambda_2}^{P-}))\end{array}\right\rangle\\\left\langle\begin{array}{l}((T_{B_1}^{P+}+_M T_{B_2}^{P+}),(T_{\mu_1}^{P+}+_M T_{\mu_2}^{P+})),((I_{B_1}^{P+}+_M I_{B_2}^{P+}),(I_{\mu_1}^{P+}+_M I_{\mu_2}^{P+})),((F_{B_1}^{P+}+_M F_{B_2}^{P+}),(F_{\mu_1}^{P+}+_M F_{\mu_2}^{P+}))\\((T_{B_1}^{P-}+_M T_{B_2}^{P-}),(T_{\mu_1}^{P-}+_M T_{\mu_2}^{P-})),((I_{B_1}^{P-}+_M I_{B_2}^{P-}),(I_{\mu_1}^{P-}+_M I_{\mu_2}^{P-})),((F_{B_1}^{P-}+_M F_{B_2}^{P-}),(F_{\mu_1}^{P-}+_M F_{\mu_2}^{P-}))\end{array}\right\rangle\end{array}\right\}$$

Where

(i)if $u\in v_1\cup v_2$

$$(T_{A_1}^{P+}+_M T_{A_2}^{P+})(u)=(T_{A_1}^{P+}\cup_M T_{A_2}^{P+})(u),(T_{\lambda_1}^{P+}+_M T_{\lambda_2}^{P+})(u)=(T_{\lambda_1}^{P+}\cup_M T_{\lambda_2}^{P+})(u)$$

$$(T_{A_1}^{P-}+_M T_{A_2}^{P-})(u)=(T_{A_1}^{P-}\cup_M T_{A_2}^{P-})(u),(T_{\lambda_1}^{P-}+_M T_{\lambda_2}^{P-})(u)=(T_{\lambda_1}^{P-}\cup_M T_{\lambda_2}^{P-})(u)$$

$$(I_{A_1}^{P+}+_M I_{A_2}^{P+})(u)=(I_{A_1}^{P+}\cup_M I_{A_2}^{P+})(u),(I_{\lambda_1}^{P+}+_M I_{\lambda_2}^{P+})(u)=(I_{\lambda_1}^{P+}\cup_M I_{\lambda_2}^{P+})(u)$$

$$(I_{A_1}^{P-}+_M I_{A_2}^{P-})(u)=(I_{A_1}^{P-}\cup_M I_{A_2}^{P-})(u),(I_{\lambda_1}^{P-}+_M I_{\lambda_2}^{P-})(u)=(I_{\lambda_1}^{P-}\cup_M I_{\lambda_2}^{P-})(u)$$

$$(F_{A_1}^{P+}+_M F_{A_2}^{P+})(u)=(F_{A_1}^{P+}\cup_M F_{A_2}^{P+})(u),(F_{\lambda_1}^{P+}+_M F_{\lambda_2}^{P+})(u)=(F_{\lambda_1}^{P+}\cup_M F_{\lambda_2}^{P+})(u)$$

$$(F_{A_1}^{P-}+_M F_{A_2}^{P-})(u)=(F_{A_1}^{P-}\cup_M F_{A_2}^{P-})(u),(F_{\lambda_1}^{P-}+_M F_{\lambda_2}^{P-})(u)=(F_{\lambda_1}^{P-}\cup_M F_{\lambda_2}^{P-})(u)$$

(ii) if $uv\in E_1\cup E_2$

$$(T_{B_1}^{P+}+_M T_{B_2}^{P+})(uv)=(T_{B_1}^{P+}\cup_M T_{B_2}^{P+})(uv),(T_{\mu_1}^{P+}+_M T_{\mu_2}^{P+})(uv)=(T_{\mu_1}^{P+}\cup_M T_{\mu_2}^{P+})(uv)$$

$$(T_{B_1}^{P-} +_M T_{B_2}^{P-})(uv) = \left(T_{B_1}^{P-} \cup_M T_{B_2}^{P-}\right)(uv), (T_{\mu_1}^{P-} +_M T_{\mu_2}^{P-})(uv) = \left(T_{\mu_1}^{P-} \cup_M T_{\mu_2}^{P-}\right)(uv)$$

$$(I_{B_1}^{P+} +_M I_{B_2}^{P+})(uv) = \left(I_{B_1}^{P+} \cup_M I_{B_2}^{P+}\right)(uv), (I_{\mu_1}^{P+} +_M I_{\mu_2}^{P+})(uv) = \left(I_{\mu_1}^{P+} \cup_M I_{\mu_2}^{P+}\right)(uv)$$

$$(I_{B_1}^{P-} +_M I_{B_2}^{P-})(uv) = \left(I_{B_1}^{P-} \cup_M I_{B_2}^{P-}\right)(uv), (I_{\mu_1}^{P-} +_M I_{\mu_2}^{P-})(uv) = \left(I_{\mu_1}^{P-} \cup_M I_{\mu_2}^{P-}\right)(uv)$$

$$(F_{B_1}^{P+} +_M F_{B_2}^{P+})(uv) = \left(F_{B_1}^{P+} \cup_M F_{B_2}^{P+}\right)(uv), (F_{\mu_1}^{P+} +_M F_{\mu_2}^{P+})(uv) = \left(F_{\mu_1}^{P+} \cup_M F_{\mu_2}^{P+}\right)(uv)$$

$$(F_{B_1}^{P-} +_M F_{B_2}^{P-})(uv) = \left(F_{B_1}^{P-} \cup_M F_{B_2}^{P-}\right)(uv), (F_{\mu_1}^{P-} +_M F_{\mu_2}^{P-})(uv) = \left(F_{\mu_1}^{P-} \cup_M F_{\mu_2}^{P-}\right)(uv)$$

(iii) if $uv \in E^*$, where E^* is the set of all edges joining the vertices of $v_1 \,\&\, v_2$.

$$(T_{B_1}^{P+} +_M T_{B_2}^{P+})(uv) = r\min\left\{T_{A_1}^{P+}(u), T_{A_2}^{P+}(v)\right\}, \left(T_{\mu_1}^{P+} +_M T_{\mu_2}^{P+}\right)(uv) = \min\left\{T_{\lambda_1}^{P+}(u), T_{\lambda_2}^{P+}(v)\right\}$$

$$(T_{B_1}^{P-} +_M T_{B_2}^{P-})(uv) = r\max\left\{T_{A_1}^{P-}(u), T_{A_2}^{P-}(v)\right\}, \left(T_{\mu_1}^{P-} +_M T_{\mu_2}^{P-}\right)(uv) = \max\left\{T_{\lambda_1}^{P-}(u), T_{\lambda_2}^{P-}(v)\right\}$$

$$(I_{B_1}^{P+} +_M I_{B_2}^{P+})(uv) = r\min\left\{I_{A_1}^{P+}(u), I_{A_2}^{P+}(v)\right\}, \left(I_{\mu_1}^{P+} +_M I_{\mu_2}^{P+}\right)(uv) = \min\left\{I_{\lambda_1}^{P+}(u), I_{\lambda_2}^{P+}(v)\right\}$$

$$(I_{B_1}^{P-} +_M I_{B_2}^{P-})(uv) = r\max\left\{I_{A_1}^{P-}(u), I_{A_2}^{P-}(v)\right\}, \left(I_{\mu_1}^{P-} +_M I_{\mu_2}^{P-}\right)(uv) = \max\left\{I_{\lambda_1}^{P-}(u), I_{\lambda_2}^{P-}(v)\right\}$$

$$(F_{B_1}^{P+} +_M F_{B_2}^{P+})(uv) = r\min\left\{F_{A_1}^{P+}(u), F_{A_2}^{P+}(v)\right\}, \left(F_{\mu_1}^{P+} +_M F_{\mu_2}^{P+}\right)(uv) = \min\left\{F_{\lambda_1}^{P+}(u), F_{\lambda_2}^{P+}(v)\right\}$$

$$(F_{B_1}^{P-} +_M F_{B_2}^{P-})(uv) = r\max\left\{F_{A_1}^{P-}(u), F_{A_2}^{P-}(v)\right\}, \left(F_{\mu_1}^{P-} +_M F_{\mu_2}^{P-}\right)(uv) = \max\left\{F_{\lambda_1}^{P-}(u), F_{\lambda_2}^{P-}(v)\right\}$$

Definition 2.17

Let $G_1 = (P_1, Q_1)$ and $G_2 = (P_2, Q_2)$ be two bipolar spherical fuzzy neutrosophic cubic graphs of the graphs G_1^* and G_2^* respectively, the N-join is denoted by $G_1 +_N G_2$ and is defined as follows:

$$G_1 +_N G_2 = (P_1, Q_1) +_N (P_2 Q_2) = (P_1 +_N P_2, Q_1 +_N Q_2)$$

$$=\left\{\begin{array}{l}\left\langle\begin{array}{l}((T^{P+}_{A_1}+_N T^{P+}_{A_2}),(T^{P+}_{\lambda_1}+_N T^{P+}_{\lambda_2})),((I^{P+}_{A_1}+_N I^{P+}_{A_2}),(I^{P+}_{\lambda_1}+_N I^{P+}_{\lambda_2})),((F^{P+}_{A_1}+_N F^{P+}_{A_2}),(F^{P+}_{\lambda_1}+_N F^{P+}_{\lambda_2}))\\((T^{P-}_{A_1}+_N T^{P-}_{A_2}),(T^{P-}_{\lambda_1}+_N T^{P-}_{\lambda_2})),((I^{P-}_{A_1}+_N I^{P-}_{A_2}),(I^{P-}_{\lambda_1}+_N I^{P-}_{\lambda_2})),((F^{P-}_{A_1}+_N F^{P-}_{A_2}),(F^{P-}_{\lambda_1}+_N F^{P-}_{\lambda_2}))\end{array}\right\rangle\\\left\langle\begin{array}{l}((T^{P+}_{B_1}+_N T^{P+}_{B_2}),(T^{P+}_{\mu_1}+_N T^{P+}_{\mu_2})),((I^{P+}_{B_1}+_N I^{P+}_{B_2}),(I^{P+}_{\mu_1}+_N I^{P+}_{\mu_2})),((F^{P+}_{B_1}+_N F^{P+}_{B_2}),(F^{P+}_{\mu_1}+_N F^{P+}_{\mu_2}))\\((T^{P-}_{B_1}+_N T^{P-}_{B_2}),(T^{P-}_{\mu_1}+_N T^{P-}_{\mu_2})),((I^{P-}_{B_1}+_N I^{P-}_{B_2}),(I^{P-}_{\mu_1}+_N I^{P-}_{\mu_2})),((F^{P-}_{B_1}+_N F^{P-}_{B_2}),(F^{P-}_{\mu_1}+_N F^{P-}_{\mu_2}))\end{array}\right\rangle\end{array}\right\}$$

where

(i) if $u\in v_1\bigcup v_2$

$$(T^{P+}_{A_1}+_N T^{P+}_{A_2})(u)=(T^{P+}_{A_1}\bigcup_N T^{P+}_{A_2})(u),(T^{P+}_{\lambda_1}+_N T^{P+}_{\lambda_2})(u)=(T^{P+}_{\lambda_1}\bigcup_N T^{P+}_{\lambda_2})(u)$$

$$(T^{P-}_{A_1}+_N T^{P-}_{A_2})(u)=(T^{P-}_{A_1}\bigcup_N T^{P-}_{A_2})(u),(T^{P-}_{\lambda_1}+_N T^{P-}_{\lambda_2})(u)=(T^{P-}_{\lambda_1}\bigcup_N T^{P-}_{\lambda_2})(u)$$

$$(I^{P+}_{A_1}+_N I^{P+}_{A_2})(u)=(I^{P+}_{A_1}\bigcup_N I^{P+}_{A_2})(u),(I^{P+}_{\lambda_1}+_N I^{P+}_{\lambda_2})(u)=(I^{P+}_{\lambda_1}\bigcup_N I^{P+}_{\lambda_2})(u)$$

$$(I^{P-}_{A_1}+_N I^{P-}_{A_2})(u)=(I^{P-}_{A_1}\bigcup_N I^{P-}_{A_2})(u),(I^{P-}_{\lambda_1}+_N I^{P-}_{\lambda_2})(u)=(I^{P-}_{\lambda_1}\bigcup_N I^{P-}_{\lambda_2})(u)$$

$$(F^{P+}_{A_1}+_N F^{P+}_{A_2})(u)=(F^{P+}_{A_1}\bigcup_N F^{P+}_{A_2})(u),(F^{P+}_{\lambda_1}+_N F^{P+}_{\lambda_2})(u)=(F^{P+}_{\lambda_1}\bigcup_N F^{P+}_{\lambda_2})(u)$$

$$(F^{P-}_{A_1}+_N F^{P-}_{A_2})(u)=(F^{P-}_{A_1}\bigcup_N F^{P-}_{A_2})(u),(F^{P-}_{\lambda_1}+_N F^{P-}_{\lambda_2})(u)=(F^{P-}_{\lambda_1}\bigcup_N F^{P-}_{\lambda_2})(u)$$

(ii) if $uv\in E_1\bigcup E_2$

$$(T^{P+}_{B_1}+_N T^{P+}_{B_2})(uv)=\left(T^{P+}_{B_1}\bigcup_N T^{P+}_{B_2}\right)(uv),(T^{P+}_{\mu_1}+_N T^{P+}_{\mu_2})(uv)=\left(T^{P+}_{\mu_1}\bigcup_N T^{P+}_{\mu_2}\right)(uv)$$

$$(T^{P-}_{B_1}+_N T^{P-}_{B_2})(uv)=\left(T^{P-}_{B_1}\bigcup_N T^{P-}_{B_2}\right)(uv),(T^{P-}_{\mu_1}+_N T^{P-}_{\mu_2})(uv)=\left(T^{P-}_{\mu_1}\bigcup_N T^{P-}_{\mu_2}\right)(uv)$$

$$(I^{P+}_{B_1}+_N I^{P+}_{B_2})(uv)=\left(I^{P+}_{B_1}\bigcup_N I^{P+}_{B_2}\right)(uv),(I^{P+}_{\mu_1}+_N I^{P+}_{\mu_2})(uv)=\left(I^{P+}_{\mu_1}\bigcup_N I^{P+}_{\mu_2}\right)(uv)$$

$$(I^{P-}_{B_1}+_N I^{P-}_{B_2})(uv)=\left(I^{P-}_{B_1}\bigcup_N I^{P-}_{B_2}\right)(uv),(I^{P-}_{\mu_1}+_N I^{P-}_{\mu_2})(uv)=\left(I^{P-}_{\mu_1}\bigcup_N I^{P-}_{\mu_2}\right)(uv)$$

$$(F^{P+}_{B_1}+_N F^{P+}_{B_2})(uv)=\left(F^{P+}_{B_1}\bigcup_N F^{P+}_{B_2}\right)(uv),(F^{P+}_{\mu_1}+_N F^{P+}_{\mu_2})(uv)=\left(F^{P+}_{\mu_1}\bigcup_N F^{P+}_{\mu_2}\right)(uv)$$

$$(F^{P-}_{B_1}+_N F^{P-}_{B_2})(uv)=\left(F^{P-}_{B_1}\bigcup_N F^{P-}_{B_2}\right)(uv),(F^{P-}_{\mu_1}+_N F^{P-}_{\mu_2})(uv)=\left(F^{P-}_{\mu_1}\bigcup_N F^{P-}_{\mu_2}\right)(uv)$$

(iii) if $uv \in E^*$, where E^* is the set of all edges joining the vertices of v_1 & v_2.

$$(T_{B_1}^{P+} +_N T_{B_2}^{P+})(uv) = r\min\left\{T_{A_1}^{P+}(u), T_{A_2}^{P+}(v)\right\}, \left(T_{\mu_1}^{P+} +_N T_{\mu_2}^{P+}\right)(uv) = \min\left\{T_{\lambda_1}^{P+}(u), T_{\lambda_2}^{P+}(v)\right\}$$

$$(T_{B_1}^{P-} +_N T_{B_2}^{P-})(uv) = r\max\left\{T_{A_1}^{P-}(u), T_{A_2}^{P-}(v)\right\}, \left(T_{\mu_1}^{P-} +_N T_{\mu_2}^{P-}\right)(uv) = \max\left\{T_{\lambda_1}^{P-}(u), T_{\lambda_2}^{P-}(v)\right\}$$

$$(I_{B_1}^{P+} +_N I_{B_2}^{P+})(uv) = r\min\left\{I_{A_1}^{P+}(u), I_{A_2}^{P+}(v)\right\}, \left(I_{\mu_1}^{P+} +_N I_{\mu_2}^{P+}\right)(uv) = \min\left\{I_{\lambda_1}^{P+}(u), I_{\lambda_2}^{P+}(v)\right\}$$

$$(I_{B_1}^{P-} +_N I_{B_2}^{P-})(uv) = r\max\left\{I_{A_1}^{P-}(u), I_{A_2}^{P-}(v)\right\}, \left(I_{\mu_1}^{P-} +_N I_{\mu_2}^{P-}\right)(uv) = \max\left\{I_{\lambda_1}^{P-}(u), I_{\lambda_2}^{P-}(v)\right\}$$

$$(F_{B_1}^{P+} +_N F_{B_2}^{P+})(uv) = r\min\left\{F_{A_1}^{P+}(u), F_{A_2}^{P+}(v)\right\}, \left(F_{\mu_1}^{P+} +_N F_{\mu_2}^{P+}\right)(uv) = \min\left\{F_{\lambda_1}^{P+}(u), F_{\lambda_2}^{P+}(v)\right\}$$

$$(F_{B_1}^{P-} +_N F_{B_2}^{P-})(uv) = r\max\left\{F_{A_1}^{P-}(u), F_{A_2}^{P-}(v)\right\}, \left(F_{\mu_1}^{P-} +_N F_{\mu_2}^{P-}\right)(uv) = \max\left\{F_{\lambda_1}^{P-}(u), F_{\lambda_2}^{P-}(v)\right\}$$

Proposition 2.18

The M-join and N-join of two bipolar spherical fuzzy neutrosophic cubic graphs is again a bipolar spherical fuzzy neutrosophic cubic graph.

CHAPTER 3

CHAPTER-3

Spherical Fuzzy Neutrosophic Cubic Graph

Definition 3.1

Let $G^* = (V, E)$ be a graph and $G(K, L)$ is a spherical fuzzy neutrosophic cubic graph of G^*, if

$S = (M, \varphi) = \{V, (T_M^P, I_M^P, F_M^P), \varphi_M\}$ is the SFNCS representation of vertex set V and

$R = (N, \delta) = \{E, (T_N^P, I_N^P, F_N^P), \delta_N\}$ is the SFNCS representation of edge set E such that

$$T_N^P(\mu_i\, \nu_i) \leq \min\{\, T_M^P(\mu_i), T_M^P(\nu_i)\},\ T_\delta^P(\mu_i\, \nu_i) \geq \max\{\, T_\delta^P(\mu_i), T_\delta^P(\nu_i)\}$$

$$I_N^P(\mu_i\, \nu_i) \leq \min\{\, I_M^P(\mu_i), I_M^P(\nu_i)\},\ I_\delta^P(\mu_i\, \nu_i) \geq \max\{\, I_\delta^P(\mu_i), I_\delta^P(\nu_i)\}$$

$$F_N^P(\mu_i\, \nu_i) \leq \max\{\, F_M^P(\mu_i), F_M^P(\nu_i)\},\ F_\delta^P(\mu_i\, \nu_i) \geq \min\{\, F_\delta^P(\mu_i), F_\delta^P(\nu_i)\}$$

Let $G^* = (V, E)$ be a graph and $G(K, L)$ is a spherical fuzzy neutrosophic cubic graph (SFNCG) of G^*, if

$S = (M, \varphi) = \{V, (T_M^P, I_M^P, F_M^P), \varphi_M\}$ is the SFNCS representation of vertex set V and

$R = (N, \delta) = \{E, (T_N^P, I_N^P, F_N^P), \delta_N\}$ is the SFNCS representation of edge set E and φ and δ are spherical fuzzy neotrosophic cubic sets.

Example 3.2

Let $G^* = (V, E)$ be a graph where $V = \{a,b,c,d\}$ and $E = \{ab,ac,ad,bc,bd,cd\}$ where K and L are as follows

$$K = \left\langle \begin{array}{l} \{a, ([0.2,0.4],0.3), ([0.1,0.3],0.5), ([0.5,0.4],0.8)\} \\ \{b, ([0.5,0.4],0.7), ([0.1,0.5],0.6), ([0.3,0.6],0.9)\} \\ \{c, ([0.4,0.6],0.8), ([0.2,0.4],0.7), ([0.3,0.5],0.9)\} \\ \{d, ([0.1,0.2],0.9), ([0.3,0.6],0.8), ([0.4,0.6],0.7)\} \end{array} \right\rangle$$

$$L = \left\langle \begin{array}{l} \{ab, ([0.2,0.4],0.7), ([0.1,0.3],0.6), ([0.3,0.4],0.9)\} \\ \{ac, ([0.2,0.4],0.8), ([0.1,0.3],0.7), ([0.30.4],0.9)\} \\ \{ad, ([0.1,0.2],0.9), ([0.1,0.3],0.8), ([0.4,0.4],0.8)\} \\ \{bc, (0.4,0.4],0.8), ([0.1,0.4),0.7), ([0.3,0.5],0.9)\} \\ \{bd, ([0.1,0.2],0.9), ([0.1,0.5],0.8), ([0.3,0.6],0.9)\} \\ \{cd, ([0.1,0.2],0.9), ([0.2,0.4],0.8), ([0.3,0.5],0.9)\} \end{array} \right\rangle$$

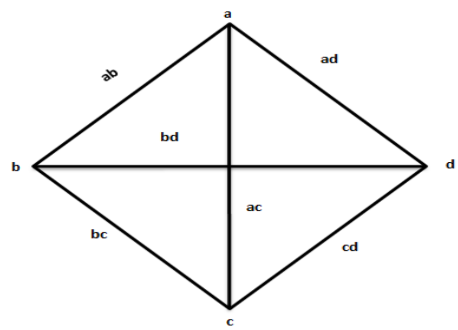

Figure 3.1

Definition 3.3

Let $G = (K, L)$ be a spherical fuzzy neutrosophic cubic graph. The order of spherical fuzzy neutrosophic cubic graph is defined by

$O(G) = \sum_{\mu \in V} \{(T^P_M, T^P_\varphi)(\mu), (I^P_M, I^P_\varphi)(\mu), (F^P_M, F^P_\varphi)(\mu)\}$ and the degree of a vertex μ and G is defined by

$$\deg(\mu) = \sum_{\mu\nu \in E} \{(T^P_N, T^P_\delta)(\mu\nu), (I^P_N, I^P_\delta)(\mu\nu), (F^P_N, F^P_\delta)(\mu\nu)\}$$

Example 3.4

In the above example, the order of a spherical fuzzy neutrosophic cubic graph is

$$\deg(a) = \{([0.5,0.7],2), ([1.3,0.9],2.1), ([0.8,0.9],0.9)\}$$

$$\deg(b) = \{([1.2,0.9],2), ([0.9,1.5],2), ([1.8,2],0.9)\}$$

$$\deg(c) = \{([0.9,1.5],2.2), ([0.5,0.9],1.5), ([0.5,1],1.5)\}$$

$$\deg(d) = \{([1,0.8],2.5), ([0.6,0.9],1.8), ([0.9,1.5],2.5)\}$$

Definition 3.5

Let $G_1 = (K_1, L_1)$ be a spherical fuzzy neutrosophic cubic graph of $G_1^* = (V_1, E_1)$ and $G_2 = (K_2, L_2)$ be a spherical fuzzy neutrosophic cubic graph of $G_2^* = (V_2, E_2)$. Then Cartesian product of G_1 and G_2 is denoted by

$$(G_1 \times G_2) = (K_1 \times K_2, L_1 \times L_2)$$

$$= \begin{pmatrix} (M_1^P, \varphi_1^P) \times (M_2^P, \varphi_2^P) \\ (N_1^P, \delta_1^P) \times (N_2^P, \delta_2^P) \end{pmatrix} \quad = \begin{pmatrix} (M_1^P \times M_2^P, \varphi_1^P \times \varphi_2^P) \\ (N_1^{P^+} \times N_2^{P^+}, \delta_1^{P^+} \times \delta_2^{P^+}) \end{pmatrix}$$

$$G_1 \times G_2 = \left\langle \begin{matrix} (T^P_{M_1\times M_2}, T^P_{\varphi_1\times\varphi_2}), (I^P_{M_1\times M_2}, I^P_{\varphi_1\times\varphi_2}), (F^P_{M_1\times M_2}, F^P_{\varphi_1\times\varphi_2}) \\ (T^P_{N_1\times N_2}, T^P_{\delta_1\times\delta_2}), (I^P_{N_1\times N_2}, I^P_{\delta_1\times\delta_2}), (F^P_{N_1\times N_2}, F^P_{\delta_1\times\delta_2}) \end{matrix} \right\rangle$$

and is defined as follows

1. $\left(T^P_{M_1\times M_2}(\mu\nu) = \min\{ T^P_{M_1}(\mu), T^P_{M_2}(\nu)\}, T^P_{\varphi_1\times\varphi_2}(\mu\nu) = \max\{ T^P_{\varphi_1}(\mu), T^P_{\varphi_2}(\nu)\}\right)$

2. $\left(I^P_{M_1\times M_2}(\mu\nu) = \min\{ I^P_{M_1}(\mu), I^P_{M_2}(\nu)\}, I^P_{\varphi_1\times\varphi_2}(\mu\nu) = \max\{ I^P_{\varphi_1}(\mu), I^P_{\varphi_2}(\nu)\}\right)$

3. $\left(F^P_{M_1\times M_2}(\mu\nu) = \max\{ F^P_{M_1}(\mu), F^P_{M_2}(\nu)\}, F^P_{\varphi_1\times\varphi_2}(\mu\nu) = \min\{ F^P_{\varphi_1}(\mu), F^P_{\varphi_2}(\nu)\}\right)$

4. $\begin{pmatrix} T^P_{N_1\times N_2}((\mu,\nu_1)(\mu,\nu_2)) = \min\{ T^P_{M_1}(\mu), T^P_{N_2}(\nu_1,\nu_2)\}, \\ T^P_{\varphi_1\times\varphi_2}((\mu,\nu_1)(\mu,\nu_2)) = \max\{ T^P_{\varphi_1}(\mu), T^P_{\delta_2}(\nu_1,\nu_2)\} \end{pmatrix}$

5. $\begin{pmatrix} I^P_{N_1\times N_2}((\mu,\nu_1)(\mu,\nu_2)) = \min\{ I^P_{M_1}(\mu), I^P_{N_2}(\nu_1,\nu_2)\}, \\ I^P_{\varphi_1\times\varphi_2}((\mu,\nu_1)(\mu,\nu_2)) = \max\{ I^P_{\varphi_1}(\mu), I^P_{\delta_2}(\nu_1,\nu_2)\} \end{pmatrix}$

6. $\begin{pmatrix} F^P_{N_1\times N_2}((\mu,\nu_1)(\mu,\nu_2)) = \max\{ F^P_{M_1}(\mu), F^P_{N_2}(\nu_1,\nu_2)\}, \\ F^P_{\varphi_1\times\varphi_2}((\mu,\nu_1)(\mu,\nu_2)) = \min\{ F^P_{\varphi_1}(\mu), F^P_{\delta_2}(\nu_1,\nu_2)\} \end{pmatrix}$

7. $\begin{pmatrix} T^P_{N_1\times N_2}((\mu_1,\nu)(\mu_2,\nu)) = \min\{ T^P_{M_1}(\mu_1,\mu_2), T^P_{N_2}(\nu)\}, \\ T^P_{\varphi_1\times\varphi_2}((\mu,\nu_1)(\mu,\nu_2)) = \max\{ T^P_{\varphi_1}(\mu_1,\mu_2), T^P_{\delta_2}(\nu)\} \end{pmatrix}$

8. $\begin{pmatrix} I^P_{N_1\times N_2}((\mu_1,\nu)(\mu_2,\nu)) = \min\{ I^P_{M_1}(\mu_1,\mu_2), I^P_{N_2}(\nu)\}, \\ I^P_{\varphi_1\times\varphi_2}((\mu,\nu_1)(\mu,\nu_2)) = \max\{ I^P_{\varphi_1}(\mu_1,\mu_2), I^P_{\delta_2}(\nu)\} \end{pmatrix}$

9. $\begin{pmatrix} F^{P}_{N_1\times N_2}((\mu_1,\nu)(\mu_2,\nu)) = \max\{ F^{P}_{M_1}(\mu_1,\mu_2), F^{P}_{N_2}(\nu)\}, \\ F^{P}_{\varphi_1\times\varphi_2}((\mu,\nu_1)(\mu,\nu_2)) = \min\{ F^{P}_{\varphi_1}(\mu_1,\mu_2), F^{P}_{\delta_2}(\nu)\} \end{pmatrix}$

Example 3.6

Let $G_1 = (K_1, L_1)$ be a spherical fuzzy neutrosophic cubic graph of $G_1^* = (V_1, E_1)$,where

V_1 = {a, b, c} and E_1 = {ab, bc, ac}.

$$K_1 = \left\langle \begin{matrix} \{a, ([0.2,0.5],0.7), ([0.1,0.4],0,7), ([0.3,0.6],0.8)\} \\ \{b, ([0.1,0,3],0.9), ([0.2,0.5],0.6), ([0.4,0.7],0.9)\} \\ \{c, ([0.2,0.3],0.8), ([0.3,0.5],0.7), ([0.1,0.4\},0.8)\} \end{matrix} \right\rangle$$

$$L_1 = \left\langle \begin{matrix} \{ab, ([0.1,0.3],0.9), ([0.1,0.4],0.7), ([0.3,0.6],0.9)\} \\ \{bc, ([0.1,0.3],0.9), ([0.2,0.5],0.7), ([0.1,0.4],0.9)\} \\ \{ac, ([0.2,0.3],0.8), ([0.1,0.4],0.7), ([0.1,0.4],0.8)\} \end{matrix} \right\rangle$$

and $G_2 = (K_2, L_2)$ be a spherical fuzzy neutrosophic cubic graph if $G_2^* = (V_2, E_2)$, where

V_2 ={x, y, z} and E_2 = { xy, yz, zx }

$$K_2 = \left\langle \begin{matrix} \{x, ([0.1,0.2],0.7), ([0.2,0.4],0.8), ([0.3,0.5],0.8)\} \\ \{y, ([0.3,0.5],0.8), ([0.4,0.6],0.9), ([0.2,0.3],0.8)\} \\ \{z, ([0.2,0.6],0.9), ([0.3,0.5],0.7), ([0.1,0.3],0.7)\} \end{matrix} \right\rangle$$

$$L_2 = \left\langle \begin{matrix} \{xy, ([0.1,0.2],0.8), ([0.2,0.4],0.9), ([0.2,0.3],0.8)\} \\ \{yz, ([0.2,0.5],0.9), ([0.3,0.5],0.9), ([0.1,0.3],0.8)\} \\ \{zx, ([0.1,0.2],0.9), ([0.2,0.4],0.8), ([0.1,0.3],0.8)\} \end{matrix} \right\rangle$$

Then $G_1 \times G_2$ is a spherical fuzzy neutrosophic cubic graph of $G_1^* \times G_2^*$, where

$V_1 \times V_2$ ={ (a,x), (a,y), (a,z), (b,x), (b,y), (b,z), (c,x), (c,y), (c,z) } and

$$K_1 \times K_2 = \left\langle \begin{array}{l} \{(a,x),([0.1,0.2],0.7),([0.1,0.4],0.8),([0.3,0.5],0.8)\} \\ \{(a,y),([0.2,0.5],0.8),([0.1,0.4],0.9),([0.2,0.3],0.8)\} \\ \{(a,z),([0.2,0.5],0.9),([0.1,0.4],0.7),([0.1,0.3],0.8)\} \\ \{(b,x),([0.1,0.2],0.9),([0.2,0.4],0.8),([0.2,0.3],0.9)\} \\ \{(b,y),([0.1,0.3],0.9),([0.2,0.5],0.9),([0.2,0.3],0.9)\} \\ \{(b,z),([0.1,0.3],0.9),([0.2,0.5],0.9),([0.1,0.3],0.9)\} \\ \{(c,x),([0.1,0.2],0.8),([0.2,0.4],0.8),([0.1,0.4],0.8)\} \\ \{(c,y),([0.2,0.3],0.8),([0.3,0.5],0.9),([0.1,0.3],0.8)\} \\ \{(c,z),([0.2,0.3],0.9),([0.3,0.5],0.7),([0.1,0.3],0.8)\} \end{array} \right\rangle$$

$$L_1 \times L_2 = \left\langle \begin{array}{l} \{(a,x)\,(a,y),([0.1,0.2],0.8),([0.1,0.4],0.9),([0.2,0.3],0.8)\} \\ \{(a,y)(a,z),([0.2,0.5],0.9),([0.1,0.4],0.9),([0.1,0.3],0.8)\} \\ \{(b,x)(b,y),([0.1,0.2],0.9),([0.2,0.4],0.9),([0.2,0.3],0.9)\} \\ \{(b,x)(b,z),([0.1,0.2],0.9),([0.2,0.4],0.9),([0.1,0.3],0.9)\} \\ \{(a,z)(b,z),([0.1,0.3],0.9),([0.1,0.4],0.9),([0.1,0.3],0.9)\} \\ \{(c,x)(c,z),([0.1,0.2],0.8),([0.2,0.4],0.8),([0.1,0.3],0.8)\} \\ \{(c,y)(c,z),([0.2,0.3],0.9),([0.3,0.5],0.9),([0.1,0.3],0.8)\} \\ \{(a,x)(c,x),([0.1,0.2],0.8),([0.1,0.4],0.8),([0.1,0.4],0.8)\} \end{array} \right\rangle$$

The vertex set in K_1 and the edge set in L_1 are represented for the graph $G_1 = (K_1, L_1)$

and

The vertex set in K_2 and the edge set in L_2 are represented for the graph $G_2 = (K_2, L_2)$

Figure 3.2

Proposition 3.7

The Cartesian product of two spherical fuzzy neutrosophic cubic graph is again a spherical fuzzy neutrosophic cubic graph.

Proof

For $K_1 \times K_2$ the condition is obvious. Now we verify the condition for only $L_1 \times L_2$,

Where, $L_1 \times L_2 = \{(T^P_{N_1 \times N_2}, T^P_{\delta_1 \times \delta_2}), (I^P_{N_1 \times N_2}, I^P_{\delta_1 \times \delta_2}), (F^P_{N_1 \times N_2}, F^P_{\delta_1 \times \delta_2})\}$

then

$$\begin{pmatrix} T^P_{N_1 \times N_2}((\mu,\mu_2)(\mu,\nu_2)) = \min\{ T^P_{M_1}(\mu), T^P_{N_2}(\mu_2\,\nu_2)\}, \\ \leq \min\{(T^P_{M_1}(\mu), (\min(T^P_{M_2}(\mu_2), T^P_{M_2}(\nu_2)))\} \\ = \min\{ \min(T^P_{M_1}(\mu), T^P_{M_2}(\mu_2)), \min(T^P_{M_1}(\mu), T^P_{M_2}(\nu_2))\} \\ = \min\{((T^P_{M_1} \times T^P_{M_2})(\mu\mu_2), ((T^P_{M_1} \times T^P_{M_2})(\mu\nu_2))\} \end{pmatrix}$$

$$\begin{pmatrix} T^P_{\delta_1 \times \delta_2}((\mu,\mu_2)(\mu,\nu_2)) = \max\{ T^P_{\varphi_1}(\mu), T^P_{\delta_2}(\mu_2\,\nu_2)\}, \\ \leq \max\{(T^P_{\varphi_1}(\mu), (\max(T^P_{\varphi_2}(\mu_2), T^P_{\varphi_2}(\nu_2)))\} \\ = \max\{ \max(T^P_{\delta_1}(\mu), T^P_{\delta_2}(\mu_2)), \max(T^P_{\delta_1}(\mu), T^P_{\delta_2}(\nu_2))\} \\ = \max\{((T^P_{\delta_1} \times T^P_{\delta_2})(\mu\mu_2), ((T^P_{\delta_1} \times T^P_{\delta_2})(\mu\nu_2))\} \end{pmatrix}$$

$$\begin{pmatrix} I^P_{N_1 \times N_2}((\mu,\mu_2)(\mu,\nu_2)) = \min\{ I^P_{M_1}(\mu), I^P_{N_2}(\mu_2\,\nu_2)\}, \\ \leq \min\{(I^P_{M_1}(\mu), (\min(I^P_{M_2}(\mu_2), I^P_{M_2}(\nu_2)))\} \\ = \min\{ \min(I^P_{M_1}(\mu), I^P_{M_2}(\mu_2)), \min(I^P_{M_1}(\mu), I^P_{M_2}(\nu_2))\} \\ = \min\{((I^P_{M_1} \times I^P_{M_2})(\mu\mu_2), ((I^P_{M_1} \times I^P_{M_2})(\mu\nu_2))\} \end{pmatrix}$$

$$\begin{pmatrix} I^P_{\delta_1 \times \delta_2}((\mu,\mu_2)(\mu,\nu_2)) = \max\{ I^P_{\varphi_1}(\mu), I^P_{\delta_2}(\mu_2\,\nu_2)\}, \\ \leq \max\{(I^P_{\varphi_1}(\mu), (\max(I^P_{\varphi_2}(\mu_2), I^P_{\varphi_2}(\nu_2)))\} \\ = \max\{ \max(I^P_{\delta_1}(\mu), I^P_{\delta_2}(\mu_2)), \max(I^P_{\delta_1}(\mu), I^P_{\delta_2}(\nu_2))\} \\ = \max\{((I^P_{\delta_1} \times I^P_{\delta_2})(\mu\mu_2), ((I^P_{\delta_1} \times I^P_{\delta_2})(\mu\nu_2))\} \end{pmatrix}$$

$$\begin{pmatrix} F^P_{N_1\times N_2}((\mu,\mu_2)(\mu,\nu_2)) = \max\{F^P_{M_1}(\mu), F^P_{N_2}(\mu_2\,\nu_2)\}, \\ \leq \max\{(F^P_{M_1}(\mu), (\max(F^P_{M_2}(\mu_2), F^P_{M_2}(\nu_2)))\} \\ = \max\{\max(F^P_{M_1}(\mu), F^P_{M_2}(\mu_2)), \max(F^P_{M_1}(\mu), F^P_{M_2}(\nu_2))\} \\ = \max\{((F^P_{M_1}\times F^P_{M_2})(\mu\mu_2), ((F^P_{M_1}\times F^P_{M_2})(\mu\nu_2))\} \end{pmatrix}$$

$$\begin{pmatrix} F^P_{\delta_1\times\delta_2}((\mu,\mu_2)(\mu,\nu_2)) = \min\{F^P_{\varphi_1}(\mu), F^P_{\delta_2}(\mu_2\,\nu_2)\}, \\ \leq \min\{(F^P_{\varphi_1}(\mu), (\min(F^P_{\varphi_2}(\mu_2), F^P_{\varphi_2}(\nu_2)))\} \\ = \min\{\min(F^P_{\delta_1}(\mu), F^P_{\delta_2}(\mu_2)), \min(F^P_{\delta_1}(\mu), F^P_{\delta_2}(\nu_2))\} \\ = \min\{((F^P_{\delta_1}\times F^P_{\delta_2})(\mu\mu_2), ((F^P_{\delta_1}\times F^P_{\delta_2})(\mu\nu_2))\} \end{pmatrix}$$

Similarly, we can prove it for $w \in V_2$ and $\mu_1, \mu_2 \in E_2$.

Definition 3.8

Let $G_1 = (K_1, L_1)$ and $G_2 = (K_2, L_2)$ be two spherical fuzzy neutrosophic cubic graphs. The degree of a vertex in $G_1 \times G_2$ can be defined as follows for any $(\mu_1 \times \mu_2) \in \nu_1 \times \nu_2$

$$\deg(T^P_{M_1}\times T^P_{M_2})(\mu_1,\mu_2)$$

$$= \sum_{(\mu_1\mu_2)(\nu_1\nu_2)\in E_2} \max(T^P_{N_1}\times T^P_{N_2})((\mu_1,\mu_2)(\nu_1,\nu_2)) = \sum_{\mu_1=\nu_1=\mu,\mu_2\nu_2\in E_2} \max(T^P_{M_1}(\mu), T^P_{M_2}(\mu_2,\nu_2))$$
$$+ \sum_{\mu_2=\nu_2=w,\mu L\nu_1\in E1} \max(T^P_{M_2}(w), T^P_{N_1}(\mu_1,\nu_1) + \sum_{\mu_1=\nu_1\in E,\mu_2\nu_2\in E_2} \max(T^P_{N_1}(\mu_1,\nu_1), T^P_{N_2}(\mu_2,\nu_2))$$

$$\deg(T^P_{\varphi_1}\times T^P_{\varphi_2})(\mu_1,\mu_2)$$

$$= \sum_{(\mu_1\mu_2)(\nu_1\nu_2)\in E_2} \min(T^P_{\delta_1}\times T^P_{\delta_2})((\mu_1,\mu_2)(\nu_1,\nu_2)) = \sum_{\mu_1=\nu_1=\mu,\mu_2\nu_2\in E_2} \min(T^P_{\varphi_1}(\mu), T^P_{\delta_2}(\mu_2,\nu_2))$$
$$+ \sum_{\mu_2=\nu_2=w,\mu L\nu_1\in E1} \min(T^P_{\varphi_2}(w), T^P_{\delta_1}(\mu_1,\nu_1) + \sum_{\mu_1=\nu_1\in E,\mu_2\nu_2\in E_2} \min(T^P_{\delta_1}(\mu_1,\nu_1), T^P_{\delta_2}(\mu_2,\nu_2))$$

$$\deg(I^P_{M_1}\times I^P_{M_2})(\mu_1,\mu_2)$$

$$= \sum_{(\mu_1\mu_2)(\nu_1\nu_2)\in E_2} \max(I^P_{N_1}\times I^P_{N_2})((\mu_1,\mu_2)(\nu_1,\nu_2)) = \sum_{\mu_1=\nu_1=\mu,\mu_2\nu_2\in E_2} \max(I^P_{M_1}(\mu), I^P_{M_2}(\mu_2,\nu_2))$$
$$+ \sum_{\mu_2=\nu_2=w,\mu L\nu_1\in E1} \max(I^P_{M_2}(w), I^P_{N_1}(\mu_1,\nu_1) + \sum_{\mu_1=\nu_1\in E,\mu_2\nu_2\in E_2} \max(I^P_{N_1}(\mu_1,\nu_1), I^P_{N_2}(\mu_2,\nu_2))$$

$$\deg(I^P_{\varphi_1}\times I^P_{\varphi_2})(\mu_1,\mu_2)$$

$$=\sum_{(\mu_1\mu_2)(\nu_1\nu_2)\in E_2}\min(I^P_{\delta_1}\times I^P_{\delta_2})((\mu_1,\mu_2)(\nu_1,\nu_2))=\sum_{\mu_1=\nu_1=\mu,\mu_2\nu_2\in E_2}\min(I^P_{\varphi_1}(\mu),I^P_{\delta_2}(\mu_2,\nu_2))$$
$$+\sum_{\mu_2=\nu_2=w,\mu L\nu_1\in E1}\min(I^P_{\varphi_2}(w),I^P_{\delta_1}(\mu_1,\nu_1)+\sum_{\mu_1=\nu_1\in E,\mu_2\nu_2\in E_2}\min(I^P_{\delta_1}(\mu_1,\nu_1),I^P_{\delta_2}(\mu_2,\nu_2))$$

$$\deg(F^P_{M_1}\times F^P_{M_2})(\mu_1,\mu_2)$$

$$=\sum_{(\mu_1\mu_2)(\nu_1\nu_2)\in E_2}\min(F^P_{N_1}\times F^P_{N_2})((\mu_1,\mu_2)(\nu_1,\nu_2))=\sum_{\mu_1=\nu_1=\mu,\mu_2\nu_2\in E_2}\min(F^P_{M_1}(\mu),F^P_{M_2}(\mu_2,\nu_2))$$
$$+\sum_{\mu_2=\nu_2=w,\mu L\nu_1\in E1}\min(F^P_{M_2}(w),F^P_{N_1}(\mu_1,\nu_1)+\sum_{\mu_1=\nu_1\in E,\mu_2\nu_2\in E_2}\min(F^P_{N_1}(\mu_1,\nu_1),F^P_{N_2}(\mu_2,\nu_2))$$

$$\deg(F^P_{\varphi_1}\times F^P_{\varphi_2})(\mu_1,\mu_2)$$

$$=\sum_{(\mu_1\mu_2)(\nu_1\nu_2)\in E_2}\max(F^P_{\delta_1}\times F^P_{\delta_2})((\mu_1,\mu_2)(\nu_1,\nu_2))=\sum_{\mu_1=\nu_1=\mu,\mu_2\nu_2\in E_2}\max(F^P_{\varphi_1}(\mu),F^P_{\delta_2}(\mu_2,\nu_2))$$
$$+\sum_{\mu_2=\nu_2=w,\mu L\nu_1\in E1}\max(F^P_{\varphi_2}(w),F^P_{\delta_1}(\mu_1,\nu_1)+\sum_{\mu_1=\nu_1\in E,\mu_2\nu_2\in E_2}\max(F^P_{\delta_1}(\mu_1,\nu_1),F^P_{\delta_2}(\mu_2,\nu_2))$$

Definition 3.9

Let $G_1=(K_1,L_1)$ be a spherical fuzzy neutrosophic cubic graph of $G^*_1=(V^*_1,E^*_2)$ and $G_2=(K_2,L_2)$ be a spherical fuzzy neutrosophic cubic graph of $G^*_2=(V^*_2,E^*_2)$. Then the composition of G_1 and G_2 is denoted by $G_1[G_2]$ and defined as follows

$$G_1[G_2]=(K_1,L_1)[K_2,L_2]$$

$$=\{(K_1[K_2],L_1[L_2]\}$$

$$=\{(M_1,\varphi_1)[M_2,\varphi_2],(N_1,\delta_1)[N_2,\delta_2]\}$$

$$=\{(M_1[M_2],\varphi_1[\varphi_2]),(N_1[N_2],\delta_1[\delta_2])\}$$

$$=\left\{\begin{matrix}\left\langle((T^P_{M_1}\circ T^P_{M_2}),(T^P_{\varphi_1}\circ T^P_{\varphi_2})),((I^P_{M_1}\circ I^P_{M_2}),(I^P_{\varphi_1}\circ I^P_{\varphi_2})),((F^P_{M_1},F^P_{M_2}),(F^P_{\varphi_1},F^P_{\varphi_2}))\right\rangle\\ \left\langle((T^P_{N_1}\circ T^P_{N_2}),(T^P_{\delta_1}\circ T^P_{\delta_2})),((I^P_{N_1}\circ I^P_{N_2}),(I^P_{\delta_1}\circ I^P_{\delta_2})),((F^P_{N_1},F^P_{N_2}),(F^P_{\delta_1},F^P_{\delta_2}))\right\rangle\end{matrix}\right\}$$

$1.\forall(\mu,\nu)\in(\nu_1,\nu_2)=V$

$(T^P_{M_1}\circ T^P_{M_2})(\mu,\nu)=\min(T^P_{M_1}(\mu),T^P_{M_2}(\nu)),(T^P_{\varphi_1}\circ T^P_{\varphi_2})(\mu,\nu)=\max(T^P_{\varphi_1}(\mu),T^P_{\varphi_2}(\nu))$

$(I^P_{M_1}\circ I^P_{M_2})(\mu,\nu)=\min(I^P_{M_1}(\mu),I^P_{M_2}(\nu)),(I^P_{\varphi_1}\circ I^P_{\varphi_2})(\mu,\nu)=\max(I^P_{\varphi_1}(\mu),I^P_{\varphi_2}(\nu))$

$(F^P_{M_1}\circ F^P_{M_2})(\mu,\nu)=\max(F^P_{M_1}(\mu),F^P_{M_2}(\nu)),(F^P_{\varphi_1}\circ F^P_{\varphi_2})(\mu,\nu)=\min(F^P_{\varphi_1}(\mu),F^P_{\varphi_2}(\nu))$

$2.\forall\mu\in V_1$ and $\nu_1\nu_2\in E$

$(T^P_{N_1}\circ T^P_{N_2})((\mu,\nu_1)(\mu,\nu_2))=\min(T^P_{M_1}(\mu),T^P_{M_2}(\nu_1\nu_2)),$
$(T^P_{\delta_1}\circ T^P_{\delta_2})((\mu,\nu_1)(\mu,\nu_2))=\max(T^P_{\varphi_1}(\mu),T^P_{\varphi_2}(\nu_1\nu_2))$

$(I^P_{N_1}\circ I^P_{N_2})((\mu,\nu_1)(\mu,\nu_2))=\min(I^P_{M_1}(\mu),I^P_{M_2}(\nu_1\nu_2)),$
$(I^P_{\delta_1}\circ I^P_{\delta_2})((\mu,\nu_1)(\mu,\nu_2))=\max(I^P_{\varphi_1}(\mu),I^P_{\varphi_2}(\nu_1\nu_2))$

$(F^P_{N_1}\circ F^P_{N_2})((\mu,\nu_1)(\mu,\nu_2))=\max(F^P_{M_1}(\mu),F^P_{M_2}(\nu_1\nu_2)),$
$(F^P_{\delta_1}\circ F^P_{\delta_2})((\mu,\nu_1)(\mu,\nu_2))=\min(F^P_{\varphi_1}(\mu),F^P_{\varphi_2}(\nu_1\nu_2))$

$3.\forall\nu\in V_2$ and $\mu_1\mu_2\in E_1$

$(T^P_{N_1}\circ T^P_{N_2})((\mu_1,\nu)(\mu_2,\nu))=\min(T^P_{N_1}(\mu_1\mu_2),T^P_{N_2}(\nu)),$
$(T^P_{\delta_1}\circ T^P_{\delta_2})((\mu_1,\nu)(\mu_2,\nu))=\max(T^P_{\delta_1}(\mu_1\mu_2),T^P_{\delta_2}(\nu))$

$(I^P_{N_1}\circ I^P_{N_2})((\mu_1,\nu)(\mu_2,\nu))=\min(I^P_{N_1}(\mu_1\mu_2),I^P_{N_2}(\nu)),$
$(I^P_{\delta_1}\circ I^P_{\delta_2})((\mu_1,\nu)(\mu_2,\nu))=\max(I^P_{\delta_1}(\mu_1\mu_2),I^P_{\delta_2}(\nu))$

$(F^P_{N_1}\circ F^P_{N_2})((\mu_1,\nu)(\mu_2,\nu))=\max(F^P_{N_1}(\mu_1\mu_2),F^P_{N_2}(\nu)),$
$(F^P_{\delta_1}\circ F^P_{\delta_2})((\mu_1,\nu)(\mu_2,\nu))=\min(F^P_{\delta_1}(\mu_1\mu_2),F^P_{\delta_2}(\nu))$

$4.\forall(\mu_1,\nu_1)(\mu_2,\nu_2)\in E^\circ-E$

$(T^P_{N_1}\circ T^P_{N_2})((\mu_1,\nu_1)(\mu_2,\nu_2))=\min(T^P_{M_2}(\nu_1),T^P_{M_2}(\nu_2),T^P_{N_1}(\mu_1,\mu_2)),$
$(T^P_{\delta_1}\circ T^P_{\delta_2})((\mu_1,\nu_1)(\mu_2,\nu_2))=\max(T^P_{\varphi_2}(\nu_1),T^P_{\varphi_2}(\nu_2),T^P_{\delta_1}(\mu_1,\mu_2))$

$(I^P_{N_1}\circ I^P_{N_2})((\mu_1,\nu_1)(\mu_2,\nu_2))=\min(I^P_{M_2}(\nu_1),I^P_{M_2}(\nu_2),I^P_{N_1}(\mu_1,\mu_2)),$
$(I^P_{\delta_1}\circ I^P_{\delta_2})((\mu_1,\nu_1)(\mu_2,\nu_2))=\max(I^P_{\varphi_2}(\nu_1),I^P_{\varphi_2}(\nu_2),I^P_{\delta_1}(\mu_1,\mu_2))$

$$(F^P_{N_1} \circ F^P_{N_2})((\mu_1, \nu_1)(\mu_2, \nu_2)) = \max(F^P_{M_2}(\nu_1), F^P_{M_2}(\nu_2), F^P_{N_1}(\mu_1, \mu_2)),$$
$$(F^P_{\delta_1} \circ F^P_{\delta_2})((\mu_1, \nu_1)(\mu_2, \nu_2)) = \min(F^P_{\varphi_2}(\nu_1), F^P_{\varphi_2}(\nu_2), F^P_{\delta_1}(\mu_1, \mu_2))$$

Example 3.10

Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two spherical fuzzy neutrosophic cubic graphs, where V_1 = (a, b) and V_2 = (c, d). Suppose K_1 and K_2 be the spherical fuzzy neutrosophic cubic set representations of V_1 and V_2. Also L_1 and L_2 be the spherical fuzzy neutrosophic cubic set representations of E_1 and E_2 defined as follows:

$$K_1 = \left\langle \begin{matrix} \{a, ([0.2,0.6],0.7), ([0.1,0.4],0.8), ([0.5,0.7],0.9)\} \\ \{b, ([0.3,0.1],0.5), ([0.6,0.8],0.9), ([0.2,0.5],0.7)\} \end{matrix} \right\rangle$$

$$L_1 = \langle \{ab, ([0.2,0.1],0.7), ([0.1,0.4],0.9), ([0.2,0.5],0.9)\} \rangle$$

$$K_2 = \left\langle \begin{matrix} \{c, ([0.2,0.4],0.8), ([0.1,0.2],0.7), ([0.4,0.6],0.9)\} \\ \{d, ([0.3,0.5],0.7), ([0.4,0.6],0.8), ([0.3,0.5],0.8)\} \end{matrix} \right\rangle$$

$$L_2 = \langle \{cd, ([0.2,0.4],0.8), ([0.1,0.2],0.8), ([0.3,0.5],0.9)\} \rangle$$

For the two spherical fuzzy neutrisophic cubic graph $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ the vertex sets V_1 = (a, b) and V_2 = (c, d) and the edges sets E_1 and E_2 are represented

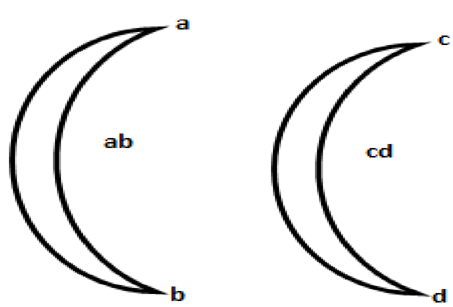

Figure 3.3

The composition of two spherical fuzzy neutrosophic cubic graph G_1 and G_2 is again a spherical fuzzy neutrosophic cubic graph, where

$$K_1[K_2] = \left\langle \begin{array}{l} \{(a,c),([0.2,0.4],0.8),([0.1,0.2],0.8),([0.4,0.6],0.9)\} \\ \{(a,d),([0.2,0.5],0.7),([0.1,0.4],0.8),([0.3,0.5],0.9)\} \\ \{(b,c),([0.2,0.1],0.8),([0.1,0.2],0.9),([0.2,0.5],0.9)\} \\ \{(b,d),([0.3,0.1],0.7),([0.4,0.6],0.9),([0.2,0.5],0.8)\} \end{array} \right\rangle$$

$$L_1[L_2] = \left\langle \begin{array}{l} \{(a,c)(a,d),([0.2,0.4],0.8),([0.1,0.2],0.8),([0.3,0.5],0.9)\} \\ \{(a,d)(b,c),([0.2,0.1],0.8),([0.1,0.2],0.9),([0.2,0.5],0.9)\} \\ \{(b,c)(b,d),([0.2,0.1],0.8),([0.1,0.2],0.9),([0.2,0.5],0.9)\} \\ \{(b,d)(a,c),([0.2,0.1],0.8),([0.1,0.2],0.9),([0.2,0.5],0.9)\} \\ \{(a,d)(b,d),([0.2,0.1],0.7),([0.1,0.4],0.9),(0.2,0.5],0.9)\} \\ \{(b,c)(a,c),([0.2,0.1],0.8),([0.1,0.2],0.9),([0.2,0.5],0.9)\} \end{array} \right\rangle$$

The composition of two spherical fuzzy neutrosophic cubic graph G_1 and G_2.

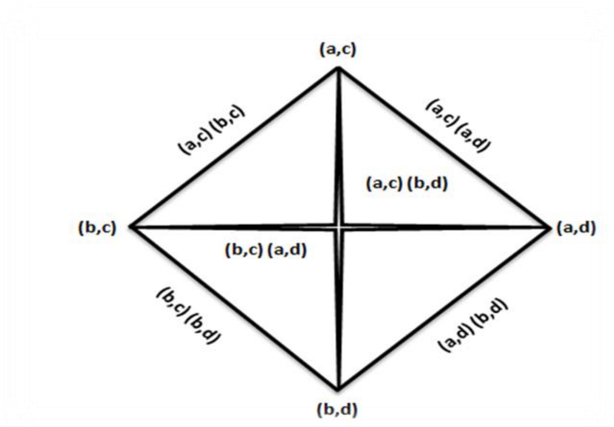

Figure 3.4

Definition 3.11

Let $G_1 = (K_1, L_1)$ and $G_2 = (K_2, L_2)$ be two spherical fuzzy neutrosophic cubic graphs of the graph G_1^* and G_2^* respectively. Then *M*-union is denoted by $G_1 \cup_M G_2$ and defined as

$$G_1 \cup_M G_2 = \{(K_1, L_1) \cup_M (K_2, L_2)\} = \{K_1 \cup_M K_2, L_1 \cup_M L_2\}$$

$$=\left\{\begin{matrix}\left\langle ((T^P_{M_1}\cup_M T^P_{M_2}),(T^P_{\varphi_1}\cup_M T^P_{\varphi_2})),((I^P_{M_1}\cup_M I^P_{M_2}),(I^P_{\varphi_1}\cup_M I^P_{\varphi_2})),((F^P_{M_1}\cup_M F^P_{M_2}),(F^P_{\varphi_1}\cup_M F^P_{\varphi_2}))\right\rangle \\ \left\langle ((T^P_{N_1}\cup_M T^P_{N_2}),(T^P_{\delta_1}\cup_M T^P_{\delta_2})),((I^P_{N_1}\cup_M I^P_{N_2}),(I^P_{\delta_1}\cup_M I^P_{\delta_2})),((F^P_{N_1}\cup_M F^P_{N_2}),(F^P_{\delta_1}\cup_M F^P_{\delta_2}))\right\rangle \end{matrix}\right\}$$

Where

$$(T^P_{M_1}\cup_M T^P_{M_2})(\mu)=\begin{cases}T^P_{M_1}(\mu), & \text{if } \mu\in\nu_1-\nu_2\\ T^P_{M_2}(\mu), & \text{if } \mu\in\nu_2-\nu_1\\ \max\{T^P_{M_1}(\mu),T^P_{M_2}(\mu)\}, & \text{if } \mu\in\nu_1\cap\nu_2\end{cases}$$

$$(T^P_{\varphi_1}\cup_M T^P_{\varphi_2})(\mu)=\begin{cases}T^P_{\varphi_1}(\mu), & \text{if } \mu\in\nu_1-\nu_2\\ T^P_{\varphi_2}(\mu), & \text{if } \mu\in\nu_2-\nu_1\\ \max\{T^P_{\varphi_1}(\mu),T^P_{\varphi_2}(\mu)\}, & \text{if } \mu\in\nu_1\cap\nu_2\end{cases}$$

$$(I^P_{M_1}\cup_M I^P_{M_2})(\mu)=\begin{cases}I^P_{M_1}(\mu), & \text{if } \mu\in\nu_1-\nu_2\\ I^P_{M_2}(\mu), & \text{if } \mu\in\nu_2-\nu_1\\ \max\{I^P_{M_1}(\mu),I^P_{M_2}(\mu)\}, & \text{if } \mu\in\nu_1\cap\nu_2\end{cases}$$

$$(I^P_{\varphi_1}\cup_M I^P_{\varphi_2})(\mu)=\begin{cases}I^P_{\varphi_1}(\mu), & \text{if } \mu\in\nu_1-\nu_2\\ I^P_{\varphi_2}(\mu), & \text{if } \mu\in\nu_2-\nu_1\\ \max\{I^P_{\varphi_1}(\mu),I^P_{\varphi_2}(\mu)\}, & \text{if } \mu\in\nu_1\cap\nu_2\end{cases}$$

$$(F^P_{M_1}\cup_M F^P_{M_2})(\mu)=\begin{cases}F^P_{M_1}(\mu), & \text{if } \mu\in\nu_1-\nu_2\\ F^P_{M_2}(\mu), & \text{if } \mu\in\nu_2-\nu_1\\ \min\{F^P_{M_1}(\mu),F^P_{M_2}(\mu)\}, & \text{if } \mu\in\nu_1\cap\nu_2\end{cases}$$

$$(F^P_{\varphi_1}\cup_M F^P_{\varphi_2})(\mu)=\begin{cases}F^P_{\varphi_1}(\mu), & \text{if } \mu\in\nu_1-\nu_2\\ F^P_{\varphi_2}(\mu), & \text{if } \mu\in\nu_2-\nu_1\\ \min\{F^P_{\varphi_1}(\mu),F^P_{\varphi_2}(\mu)\}, & \text{if } \mu\in\nu_1\cap\nu_2\end{cases}$$

$$(T^P_{N_1}\cup_M T^P_{N_2})(\mu_2\nu_2)=\begin{cases}T^P_{N_1}(\mu_2\nu_2), & \text{if } \mu_2\nu_2\in\nu_1-\nu_2\\ T^P_{N_2}(\mu_2\nu_2), & \text{if } \mu_2\nu_2\in\nu_2-\nu_1\\ \max\{T^P_{N_1}(\mu_2\nu_2),T^P_{N_2}(\mu_2\nu_2)\}, & \text{if } \mu_2\nu_2\in E_1\cap E_2\end{cases}$$

$$(T_{\delta_1}^P \cup_M T_{\delta_2}^P)(\mu_2\,\nu_2) = \begin{cases} T_{\delta_1}^P(\mu_2\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_1 - \nu_2 \\ T_{\delta_2}^P(\mu_2\,\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_2 - \nu_1 \\ \max\{T_{\delta_1}^P(\mu_2\,\nu_2), T_{\delta_2}^P(\mu_2\,\nu_2)\}, & \text{if } \mu_2\,\nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(I_{N_1}^P \cup_M I_{N_2}^P)(\mu_2\,\nu_2) = \begin{cases} I_{N_1}^P(\mu_2\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_1 - \nu_2 \\ I_{N_2}^P(\mu_2\,\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_2 - \nu_1 \\ \max\{I_{N_1}^P(\mu_2\,\nu_2), I_{N_2}^P(\mu_2\,\nu_2)\}, & \text{if } \mu_2\,\nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(I_{\delta_1}^P \cup_M I_{\delta_2}^P)(\mu_2\,\nu_2) = \begin{cases} I_{\delta_1}^P(\mu_2\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_1 - \nu_2 \\ I_{\delta_2}^P(\mu_2\,\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_2 - \nu_1 \\ \max\{I_{\delta_1}^P(\mu_2\,\nu_2), I_{\delta_2}^P(\mu_2\,\nu_2)\}, & \text{if } \mu_2\,\nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(F_{N_1}^P \cup_M F_{N_2}^P)(\mu_2\,\nu_2) = \begin{cases} F_{N_1}^P(\mu_2\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_1 - \nu_2 \\ F_{N_2}^P(\mu_2\,\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_2 - \nu_1 \\ \min\{F_{N_1}^P(\mu_2\,\nu_2), F_{N_2}^P(\mu_2\,\nu_2)\}, & \text{if } \mu_2\,\nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(F_{\delta_1}^P \cup_M F_{\delta_2}^P)(\mu_2\,\nu_2) = \begin{cases} F_{\delta_1}^P(\mu_2\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_1 - \nu_2 \\ F_{\delta_2}^P(\mu_2\,\nu_2), & \text{if } \mu_2\,\nu_2 \in \nu_2 - \nu_1 \\ \min\{F_{\delta_1}^P(\mu_2\,\nu_2), F_{\delta_2}^P(\mu_2\,\nu_2)\}, & \text{if } \mu_2\,\nu_2 \in E_1 \cap E_2 \end{cases}$$

and the *N*-union is denoted by $G_1 \cup_N G_2$ and is defined as follows:

$$G_1 \cup_N G_2 = \{(K_1, L_1) \cup_N (K_2, L_2)\} = \{K_1 \cup_N K_2, L_1 \cup_N L_2\}$$

$$= \begin{Bmatrix} \left\langle ((T_{M_1}^P \cup_N T_{M_2}^P), (T_{\varphi_1}^P \cup_N T_{\varphi_2}^P)), ((I_{M_1}^P \cup_N I_{M_2}^P), (I_{\varphi_1}^P \cup_N I_{\varphi_2}^P)), ((F_{M_1}^P \cup_N F_{M_2}^P), (F_{\varphi_1}^P \cup_N F_{\varphi_2}^P)) \right\rangle \\ \left\langle ((T_{N_1}^P \cup_N T_{N_2}^P), (T_{\delta_1}^P \cup_N T_{\delta_2}^P)), ((I_{N_1}^P \cup_N I_{N_2}^P), (I_{\delta_1}^P \cup_N I_{\delta_2}^P)), ((F_{N_1}^P \cup_N F_{N_2}^P), (F_{\delta_1}^P \cup_N F_{\delta_2}^P)) \right\rangle \end{Bmatrix}$$

Where

$$(T_{M_1}^P \cup_N T_{M_2}^P)(\mu) = \begin{cases} T_{M_1}^P(\mu), & \text{if } \mu \in \nu_1 - \nu_2 \\ T_{M_2}^P(\mu), & \text{if } \mu \in \nu_2 - \nu_1 \\ \max\{T_{M_1}^P(\mu), T_{M_2}^P(\mu)\}, & \text{if } \mu \in \nu_1 \cap \nu_2 \end{cases}$$

$$(T^P_{\varphi_1} \cup_N T^P_{\varphi_2})(\mu) = \begin{cases} T^P_{\varphi_1}(\mu), & \text{if } \mu \in \nu_1 - \nu_2 \\ T^P_{\varphi_2}(\mu), & \text{if } \mu \in \nu_2 - \nu_1 \\ \max\{T^P_{\varphi_1}(\mu), T^P_{\varphi_2}(\mu)\}, & \text{if } \mu \in \nu_1 \cap \nu_2 \end{cases}$$

$$(I^P_{M_1} \cup_N I^P_{M_2})(\mu) = \begin{cases} I^P_{M_1}(\mu), & \text{if } \mu \in \nu_1 - \nu_2 \\ I^P_{M_2}(\mu), & \text{if } \mu \in \nu_2 - \nu_1 \\ \max\{I^P_{M_1}(\mu), I^P_{M_2}(\mu)\}, & \text{if } \mu \in \nu_1 \cap \nu_2 \end{cases}$$

$$(I^P_{\varphi_1} \cup_N I^P_{\varphi_2})(\mu) = \begin{cases} I^P_{\varphi_1}(\mu), & \text{if } \mu \in \nu_1 - \nu_2 \\ I^P_{\varphi_2}(\mu), & \text{if } \mu \in \nu_2 - \nu_1 \\ \max\{I^P_{\varphi_1}(\mu), I^P_{\varphi_2}(\mu)\}, & \text{if } \mu \in \nu_1 \cap \nu_2 \end{cases}$$

$$(F^P_{M_1} \cup_N F^P_{M_2})(\mu) = \begin{cases} F^P_{M_1}(\mu), & \text{if } \mu \in \nu_1 - \nu_2 \\ F^P_{M_2}(\mu), & \text{if } \mu \in \nu_2 - \nu_1 \\ \min\{F^P_{M_1}(\mu), F^P_{M_2}(\mu)\}, & \text{if } \mu \in \nu_1 \cap \nu_2 \end{cases}$$

$$(F^P_{\varphi_1} \cup_N F^P_{\varphi_2})(\mu) = \begin{cases} F^P_{\varphi_1}(\mu), & \text{if } \mu \in \nu_1 - \nu_2 \\ F^P_{\varphi_2}(\mu), & \text{if } \mu \in \nu_2 - \nu_1 \\ \min\{F^P_{\varphi_1}(\mu), F^P_{\varphi_2}(\mu)\}, & \text{if } \mu \in \nu_1 \cap \nu_2 \end{cases}$$

$$(T^P_{N_1} \cup_N T^P_{N_2})(\mu_2 \nu_2) = \begin{cases} T^P_{N_1}(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_1 - \nu_2 \\ T^P_{N_2}(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_2 - \nu_1 \\ \max\{T^P_{N_1}(\mu_2 \nu_2), T^P_{N_2}(\mu_2 \nu_2)\}, & \text{if } \mu_2 \nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(T^P_{\delta_1} \cup_N T^P_{\delta_2})(\mu_2 \nu_2) = \begin{cases} T^P_{\delta_1}(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_1 - \nu_2 \\ T^P_{\delta_2}(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_2 - \nu_1 \\ \max\{T^P_{\delta_1}(\mu_2 \nu_2), T^P_{\delta_2}(\mu_2 \nu_2)\}, & \text{if } \mu_2 \nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(I^P_{N_1} \cup_N I^P_{N_2})(\mu_2 \nu_2) = \begin{cases} I^P_{N_1}(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_1 - \nu_2 \\ I^P_{N_2}(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_2 - \nu_1 \\ \max\{I^P_{N_1}(\mu_2 \nu_2), I^P_{N_2}(\mu_2 \nu_2)\}, & \text{if } \mu_2 \nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(I_{\delta_1}^P \cup_N I_{\delta_2}^P)(\mu_2 \nu_2) = \begin{cases} I_{\delta_1}^P(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_1 - \nu_2 \\ I_{\delta_2}^P(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_2 - \nu_1 \\ \max\{ I_{\delta_1}^P(\mu_2 \nu_2), I_{\delta_2}^P(\mu_2 \nu_2)\}, & \text{if } \mu_2 \nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(F_{N_1}^P \cup_N F_{N_2}^P)(\mu_2 \nu_2) = \begin{cases} F_{N_1}^P(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_1 - \nu_2 \\ F_{N_2}^P(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_2 - \nu_1 \\ \min\{ F_{N_1}^P(\mu_2 \nu_2), F_{N_2}^P(\mu_2 \nu_2)\}, & \text{if } \mu_2 \nu_2 \in E_1 \cap E_2 \end{cases}$$

$$(F_{\delta_1}^P \cup_N F_{\delta_2}^P)(\mu_2 \nu_2) = \begin{cases} F_{\delta_1}^P(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_1 - \nu_2 \\ F_{\delta_2}^P(\mu_2 \nu_2), & \text{if } \mu_2 \nu_2 \in \nu_2 - \nu_1 \\ \min\{ F_{\delta_1}^P(\mu_2 \nu_2), F_{\delta_2}^P(\mu_2 \nu_2)\}, & \text{if } \mu_2 \nu_2 \in E_1 \cap E_2 \end{cases}$$

Example 3.12

Let us consider the two spherical fuzzy neutrosophic cubic graph as $G_1 = (K_1, L_1)$ and $G_2 = (K_2, L_2)$

$$K_1 = \left\langle \begin{matrix} \{(a, ([0.2,0.3],0.7), ([0.4,0.6],0.8), ([0.1,0.5],0.9)\} \\ \{(b, ([0.3,0.5],0.8), ([0.2,0.4],0.6), ([0.2,0.3],0.7)\} \\ \{(c, ([0.1,0.3],0.6), ([0.3,0.5],0.8), ([0.4,0.5],0.8)\} \end{matrix} \right\rangle$$

$$L_1 = \left\langle \begin{matrix} \{ab, ([0.2,0.3],0.8), ([0.2,0.4],0.8), ([0.1,0.3],0.9)\} \\ \{ac, ([0.1,0.3],0.7), ([0.3,0.5),0.8), ([0.1,0.5],0.9)\} \\ \{bc, ([0.1,0.3],0.8), ([0.2,0.4],0.8), ([0.2,0.3],0.8)\} \end{matrix} \right\rangle$$

$$K_2 = \left\langle \begin{matrix} \{a, ([0.3,0.5],0.8), ([0.1,0.4],0.6), ([0.4,0.7],0.9)\} \\ \{b, ([0.2,0.6],0.7), ([0.3,0.6],0.7), ([0.2,0.5],0.8)\} \\ \{c, ([0.3,0.6],0.9), ([0.2,0.4],0.8), ([0.1,0.4],0.7)\} \end{matrix} \right\rangle$$

$$L_2 = \left\langle \begin{matrix} \{ab, ([0.2,0.5],0.8), ([0.1,0.4],0.7), ([0.2,0.5],0.9)\} \\ \{ac, ([0.3,0.5],0.9), ([0.1,0.4],0.8), ([0.1,0.4],0.9)\} \\ \{bc, ([0.2,0.6],0.9), ([0.2,0.4],0.8), ([0.1,0.4],0.8)\} \end{matrix} \right\rangle$$

Here *M*-union of the spherical fuzzy neutrosophic cubic graph $G_1 \cup_M G_2$ as follows:

$$K_1 \cup_M K_2 = \left\langle \begin{array}{l} \{a,([0.1,0.3],0.7),([0.3,0.5],0.8),([0.5,0.4],0.7)\} \\ \{b,([0.4,0.6],0.8),([0.2,0.4],0.9),([0.2,0.5],0.8)\} \\ \{c,([0.2,0.5],0.9),([0.1,0.3],0.7),([0.5,0.8],0.9)\} \end{array} \right\rangle$$

$$L_1 \cup_M L_2 = \left\langle \begin{array}{l} \{ab,([0.1,0.3],0.8),([0.2,0.4],0.9),([0.2,0.5],0.8)\} \\ \{bc,([0.2,0.5],0.9),([0.1,0.3],0.9),([0.2,0.5],0.9)\} \\ \{ac,([0.1,0.3],0.9),([0.1,0.3],0.8),([0.5,0.4],0.9)\} \end{array} \right\rangle$$

Here *N*-union of the spherical fuzzy neutrosophic cubic graph $G_1 \cup_N G_2$ as follows:

$$K_1 \cup_N K_2 = \left\langle \begin{array}{l} \{a,([0.2,0.6],0.6),([0.3,0.5],0.8),([0.5,0.7],0.9)\} \\ \{b,([0.1,0.5],0.7),([0.2,0.4],0.7),([0.4,0.6],0.7)\} \\ \{c,([0.3,0.6],0.9),([0.1,0.3],0.8),([0.2,0.5],0.8)\} \end{array} \right\rangle$$

$$L_1 \cup_N L_2 = \left\langle \begin{array}{l} \{ab,([0.1,0.5],0.7),([0.2,0.4],0.8),([0.4,0.6],0.9)\} \\ \{bc,([0.1,0.5],0.9),([0.1,0.3],0.8),([0.2,0.5],0.8)\} \\ \{ac,([0.2,0.6],0.9),([0.1,0.3],0.8),([02,0.5],0.9)\} \end{array} \right\rangle$$

Proposition 3.13

The *M*-union of the two spherical fuzzy neutrosophic cubic graph is again a spherical fuzzy neutrosophic fuzzy cubic graph.

Definition 3.14

Let $G_1 = (K_1, L_1)$ and $G_2 = (K_2, L_2)$ be two spherical fuzzy neutrosophic cubic graphs of the graphs G_1^* and G_2^* respectively, then *M*-join is denoted by $G_1 +_M G_2$ and is defined as follows:

$$G_1 +_M G_2 = (K_1, L_1) +_M (K_2, L_2) = (K_1 +_M K_2, L_1 +_M L_2)$$

$$= \left\{ \begin{array}{l} \left\langle ((T^P_{M_1} +_M T^P_{M_2}),(T^P_{\varphi_1} +_M T^P_{\varphi_2})),((I^P_{M_1} +_M I^P_{M_2}),(I^P_{\varphi_1} +_M I^P_{\varphi_1})),((F^P_{M_1} +_M F^P_{M_2}),(F^P_{\varphi_1} +_M F^P_{\varphi_2}))\right\rangle \\ \left\langle ((T^P_{N_1} +_M T^P_{N_2}),(T^P_{\delta_1} +_M T^P_{\delta_2})),((I^P_{N_1} +_M I^P_{N_2}),(I^P_{\delta_1} +_M I^P_{\delta_2})),((F^P_{N_1} +_M F^P_{N_2}),(F^P_{\delta_1} +_M F^P_{\delta_2}))\right\rangle \end{array} \right\}$$

Where

(i) if $\mu \in \nu_1 \cup \nu_2$

$$(T^P_{M_1} +_M T^P_{M_2})(\mu) = (T^P_{M_1} \cup_M T^P_{M_2})(\mu), (T^P_{\varphi_1} +_M T^P_{\varphi_2})(\mu) = (T^P_{\varphi_1} \cup_M T^P_{\varphi_2})(\mu)$$

$$(I^P_{M_1} +_M I^P_{M_2})(\mu) = (I^P_{M_1} \cup_M I^P_{M_2})(\mu), (I^P_{\varphi_1} +_M I^P_{\varphi_2})(\mu) = (I^P_{\varphi_1} \cup_M I^P_{\varphi_2})(\mu)$$

$$(F^P_{M_1} +_M F^P_{M_2})(\mu) = (F^P_{M_1} \cup_M F^P_{M_2})(\mu), (F^P_{\varphi_1} +_M F^P_{\varphi_2})(\mu) = (F^P_{\varphi_1} \cup_M F^P_{\varphi_2})(\mu)$$

(ii) if $\mu\nu \in E_1 \cup E_2$

$$(T^P_{N_1} +_M T^P_{N_2})(\mu\nu) = (T^P_{N_1} \cup_M T^P_{N_2})(\mu\nu), (T^P_{\delta_1} +_M T^P_{\delta_2})(\mu\nu) = (T^P_{\delta_1} \cup_M T^P_{\delta_2})(\mu\nu)$$

$$(I^P_{N_1} +_M I^P_{N_2})(\mu\nu) = (I^P_{N_1} \cup_M I^P_{N_2})(\mu\nu), (I^P_{\delta_1} +_M I^P_{\delta_2})(\mu\nu) = (I^P_{\delta_1} \cup_M I^P_{\delta_2})(\mu\nu)$$

$$(F^P_{N_1} +_M F^P_{N_2})(\mu\nu) = (F^P_{N_1} \cup_M F^P_{N_2})(\mu\nu), (F^P_{\delta_1} +_M F^P_{\delta_2})(\mu\nu) = (F^P_{\delta_1} \cup_M F^P_{\delta_2})(\mu\nu)$$

(iii) if $\mu\nu \in E^*$, where E^* is the set of all edges joining the vertices of ν_1 and ν_2.

$$(T^P_{N_1} +_M T^P_{N_2})(\mu\nu) = \min\{T^P_{M_1}(\mu), T^P_{M_2}(\nu)\}, (T^P_{\delta_1} +_M T^P_{\delta_2})(\mu\nu) = \min\{T^P_{\varphi_1}(\mu), T^P_{\varphi_2}(\nu)\}$$

$$(I^P_{N_1} +_M I^P_{N_2})(\mu\nu) = \min\{I^P_{M_1}(\mu), I^P_{M_2}(\nu)\}, (I^P_{\delta_1} +_M I^P_{\delta_2})(\mu\nu) = \min\{I^P_{\varphi_1}(\mu), I^P_{\varphi_2}(\nu)\}$$

$$(F^P_{N_1} +_M F^P_{N_2})(\mu\nu) = \min\{F^P_{M_1}(\mu), F^P_{M_2}(\nu)\}, (F^P_{\delta_1} +_M F^P_{\delta_2})(\mu\nu) = \min\{F^P_{\varphi_1}(\mu), F^P_{\varphi_2}(\nu)\}$$

Definition 3.15

Let $G_1 = (K_1, L_1)$ and $G_2 = (K_2, L_2)$ be two spherical fuzzy neutrosophic cubic graphs of the graphs G_1^* and G_2^* respectively, then *n*-join is denoted by $G_1 +_N G_2$ and is defined as follows:

$$G_1 +_N G_2 = (K_1, L_1) +_N (K_2, L_2) = (K_1 +_N K_2, L_1 +_N L_2)$$

$$= \left\{ \begin{array}{l} \left\langle ((T^P_{M_1} +_N T^P_{M_2}), (T^P_{\varphi_1} +_N T^P_{\varphi_2})), ((I^P_{M_1} +_N I^P_{M_2}), (I^P_{\varphi_1} +_N I^P_{\varphi_1})), ((F^P_{M_1} +_N F^P_{M_2}), (F^P_{\varphi_1} +_N F^P_{\varphi_2})) \right\rangle \\ \left\langle ((T^P_{N_1} +_N T^P_{N_2}), (T^P_{\delta_1} +_N T^P_{\delta_2})), ((I^P_{N_1} +_N I^P_{N_2}), (I^P_{\delta_1} +_N I^P_{\delta_2})), ((F^P_{N_1} +_N F^P_{N_2}), (F^P_{\delta_1} +_N F^P_{\delta_2})) \right\rangle \end{array} \right\}$$

Where

(i) if $\mu \in \nu_1 \cup \nu_2$

$$(T^P_{M_1} +_N T^P_{M_2})(\mu) = (T^P_{M_1} \cup_N T^P_{M_2})(\mu), (T^P_{\varphi_1} +_N T^P_{\varphi_2})(\mu) = (T^P_{\varphi_1} \cup_N T^P_{\varphi_2})(\mu)$$

$$(I^P_{M_1} +_N I^P_{M_2})(\mu) = (I^P_{M_1} \cup_N I^P_{M_2})(\mu), (I^P_{\varphi_1} +_N I^P_{\varphi_2})(\mu) = (I^P_{\varphi_1} \cup_N I^P_{\varphi_2})(\mu)$$

$(F^P_{M_1} +_N F^P_{M_2})(\mu) = (F^P_{M_1} \cup_N F^P_{M_2})(\mu), (F^P_{\varphi_1} +_N F^P_{\varphi_2})(\mu) = (F^P_{\varphi_1} \cup_N F^P_{\varphi_2})(\mu)$

(ii) if $\mu\nu \in E_1 \cup E_2$

$(T^P_{N_1} +_N T^P_{N_2})(\mu\nu) = (T^P_{N_1} \cup_N T^P_{N_2})(\mu\nu), (T^P_{\delta_1} +_N T^P_{\delta_2})(\mu\nu) = (T^P_{\delta_1} \cup_N T^P_{\delta_2})(\mu\nu)$

$(I^P_{N_1} +_N I^P_{N_2})(\mu\nu) = (I^P_{N_1} \cup_N I^P_{N_2})(\mu\nu), (I^P_{\delta_1} +_N I^P_{\delta_2})(\mu\nu) = (I^P_{\delta_1} \cup_N I^P_{\delta_2})(\mu\nu)$

$(F^P_{N_1} +_N F^P_{N_2})(\mu\nu) = (F^P_{N_1} \cup_N F^P_{N_2})(\mu\nu), (F^P_{\delta_1} +_N F^P_{\delta_2})(\mu\nu) = (F^P_{\delta_1} \cup_N F^P_{\delta_2})(\mu\nu)$

(iii) if $\mu\nu \in E^*$, Where E^* is the set of all edges joining the vertices of ν_1 and ν_2.

$(T^P_{N_1} +_N T^P_{N_2})(\mu\nu) = \min\{T^P_{M_1}(\mu), T^P_{M_2}(\nu)\}, (T^P_{\delta_1} +_N T^P_{\delta_2})(\mu\nu) = \max\{T^P_{\varphi_1}(\mu), T^P_{\varphi_2}(\nu)\}$

$(I^P_{N_1} +_N I^P_{N_2})(\mu\nu) = \min\{I^P_{M_1}(\mu), I^P_{M_2}(\nu)\}, (I^P_{\delta_1} +_N I^P_{\delta_2})(\mu\nu) = \max\{I^P_{\varphi_1}(\mu), I^P_{\varphi_2}(\nu)\}$

$(F^P_{N_1} +_N F^P_{N_2})(\mu\nu) = \min\{F^P_{M_1}(\mu), F^P_{M_2}(\nu)\}, (F^P_{\delta_1} +_N F^P_{\delta_2})(\mu\nu) = \max\{F^P_{\varphi_1}(\mu), F^P_{\varphi_2}(\nu)\}$

Proposition 3.16

The M-join and N-join of two spherical fuzzy neutrosophic cubic graphs is again a spherical fuzzy neutrosophic cubic graph.

SUMMARY AND CONCLUSION

SUMMARY AND CONCLUSION

In this dissertation, the concept of Spherical fuzzy set and neutrosophic cubic set is combined to develop a theoretical study spherical fuzzy neutrosophic cubic graph. We also listed out some of the properties like Cartesian product, composition, M-join, N-join, M-union and N-union. We also defined some operations on spherical fuzzy neutrosophic cubic graph.

In future, we will focus our research work to more graph in different areas, which is used in decision making.

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